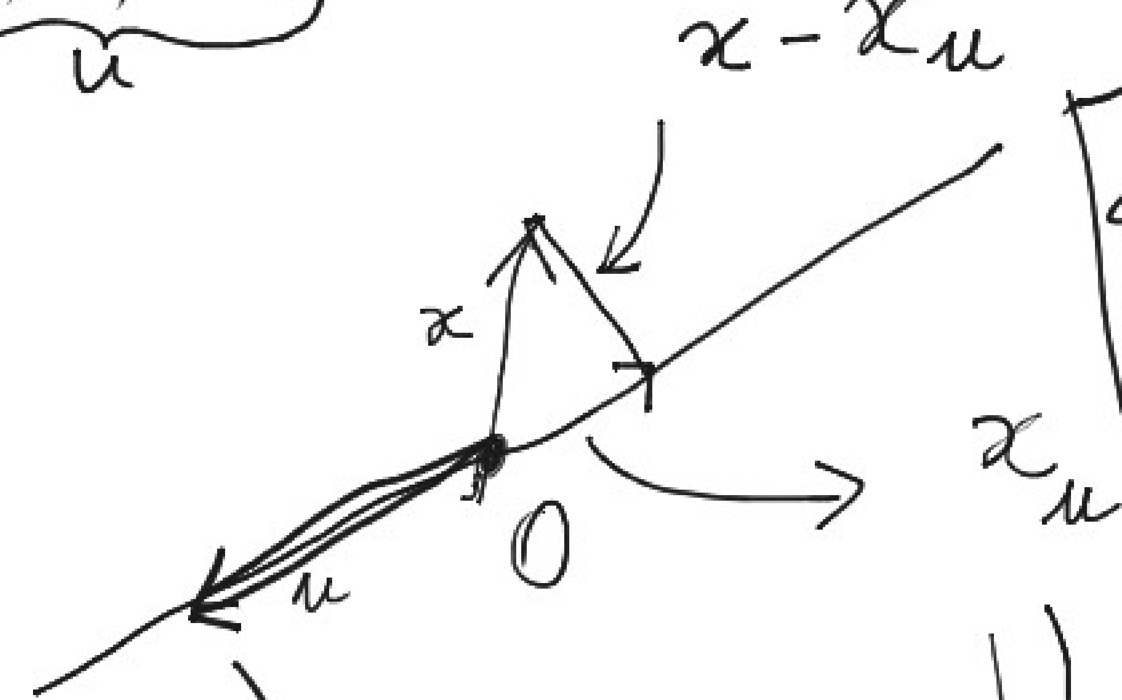
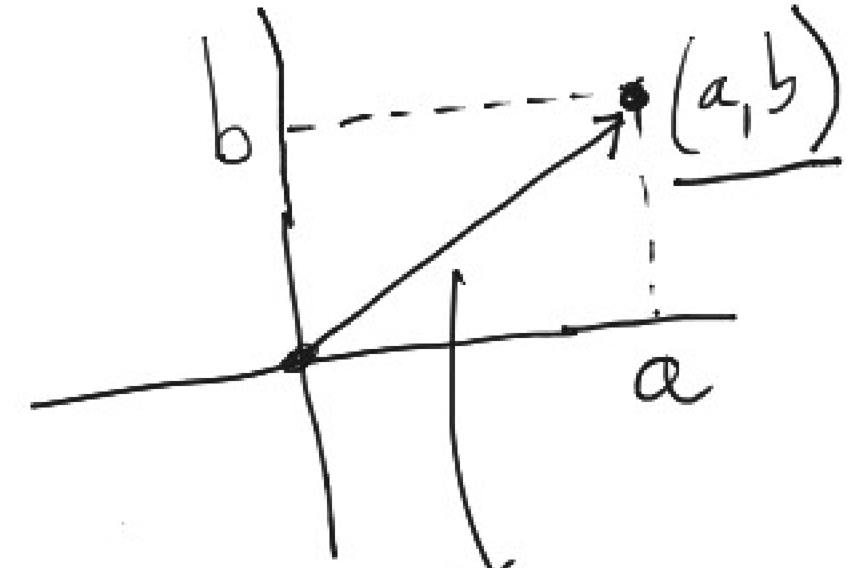
dist. $\underbrace{(1, i, -i)}_{x}$ $\quad \langle \underbrace{(i, 1, i)}_{u} \rangle \rightarrow \boxed{ab = \sum a_i \overline{b_i}} \ \mathbb{C}^n$

$\boxed{ab = \sum_{1}^{n} a_i b_i \quad \mathbb{R}^n}$

$$d = |x - x_u|$$



$$x_u = \frac{(1, i, -i)(i, 1, i)}{|(i, 1, i)|^2}(i, 1, i) =$$

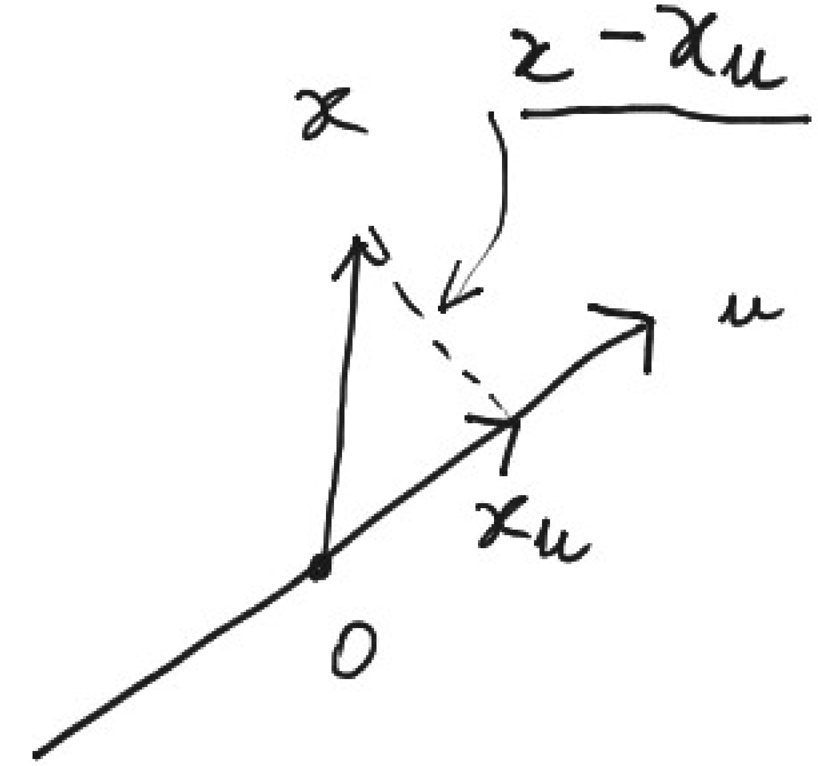


$$= \frac{1(-i) + i \cdot \bar{1} - i(\overline{-i})}{1 + 1 + 1}(i, 1, i) = -\frac{1}{3}(i, 1, i)$$

$$x - x_u = (1, i, -i) - \left(\frac{1}{3} i, \frac{1}{3}, \frac{1}{3} i\right) = \boxed{\left(1 - \frac{1}{3} i, -\frac{1}{3} + i, -\frac{4}{3} i\right)}$$

$$|x - x_u| = \ldots\ldots$$

$$ab = \sum a_i b_i \quad \text{se } a, b \in \mathbb{R}^n$$
$$ab = \sum a_i \overline{b_i} \quad \text{se } a, b \in \mathbb{C}^n$$



$$a_b = \frac{ab}{|b|^2}\, b$$

$$|a_b| = \frac{|ab|}{|b|}$$

Th. Pitagora

$$|x - x_u| = \sqrt{|x|^2 - |x_u|^2} = \sqrt{|x|^2 - \left(\frac{xu}{|u|}\right)^2}$$

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materiali di studio $\longrightarrow$ lezioni on-line

Base nucleo e immagine

$$A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ -1 & 1 & -4 & -3 \\ 1 & 1 & 2 & 1 \end{pmatrix}$$

$$\begin{array}{l} \\ \text{V } 2^a \\ 3 \\ 2^a \\ 3^a \\ \cancel{2^a/3} \end{array}\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ -1 & 1 & -4 & -3 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 3 & -3 & -3 & \\ 0 & -1 & 1 & 1 & \\ \cancel{0} & \cancel{1} & \cancel{-1} & \cancel{-1} & \end{array}$$

$2^a - 3^e$

pivot

$$\begin{array}{l} \\ \\ -3^e \end{array}\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 \end{array}$$

pivot

NON PIVOT parametri

Immagine

$Ae_1 = \text{I colonna}$

$Ae_2 = \text{II colonna}$

$Ae_3 = \text{III colonna}$

$Ae_4 = \text{IV colonna}$

$\operatorname{Im} A = \langle \text{colonne} \rangle$

$Ax = \sum x_i A_i$

colonne

$$\boxed{\text{base di } \operatorname{Im} A \left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \right\}}$$

$$\begin{array}{c|cccc|c} & x_1 & x_2 & x_3 & x_4 & \\ 1^a & 1 & 2 & 1 & 0 & 0 \\ 2^a & 0 & 1 & -1 & -1 & 0 \\ 1^a & 1 & 0 & 3 & 2 & \end{array}$$

$v_{1^a - 2 \cdot 2^a}$

$$\boxed{Ax = 0} \iff x \in \operatorname{Ker} A$$

portando al II membro i non pivot

pivot = parametri (non-pivot)

$$\begin{array}{cc|cc} x_1 & x_2 & x_3 & x_4 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & -3 & -2 \end{array}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -3x_3 - 2x_4 \\ x_3 + x_4 \\ x_3 \\ x_4 \end{pmatrix} = \text{soluzione generale}$$

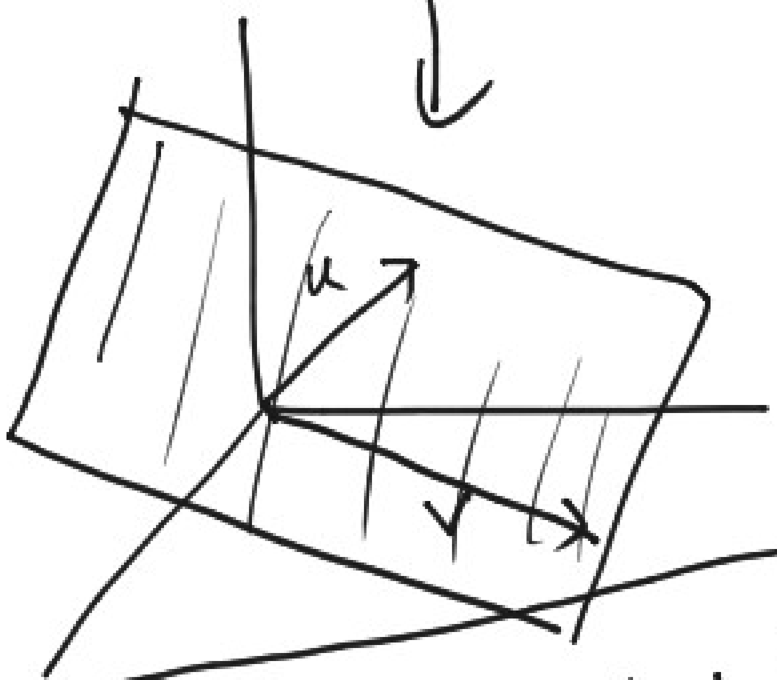
$$= \begin{pmatrix} -3x_3 \\ x_3 \\ x_3 \\ 0 \end{pmatrix} + \begin{pmatrix} -2x_4 \\ x_4 \\ 0 \\ x_4 \end{pmatrix} = x_3 \begin{pmatrix} -3 \\ 1 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

Nucleo ha base $\{(-3,1,1,0), (-2,1,0,1)\}$ ← Ker

AL-1.1 (in fondo)

$$\langle \underbrace{(1,0,1,1)}_{u} \rangle \text{ in } \mathbb{R}^4 \qquad \underbrace{(1,1,1,1)}_{x_0} + \langle \underbrace{(1,0,0,1)}_{v}, \underbrace{(0,1,1,0)}_{w} \rangle \quad \in \mathbb{R}^4$$

$\langle u, v \rangle$



$\boxed{\text{G\_1.6 e G\_1.4}}$

2 dimens.

$$\begin{cases} x_1 = 1 + \alpha + \cancel{0\cdot\beta} \\ x_2 = 1 + \cancel{0\alpha} + \beta \\ x_3 = 1 + \beta \\ x_4 = 1 + \alpha + \cancel{0\beta} \end{cases}$$

eliminare α e β

$\beta = x_3 - 1$

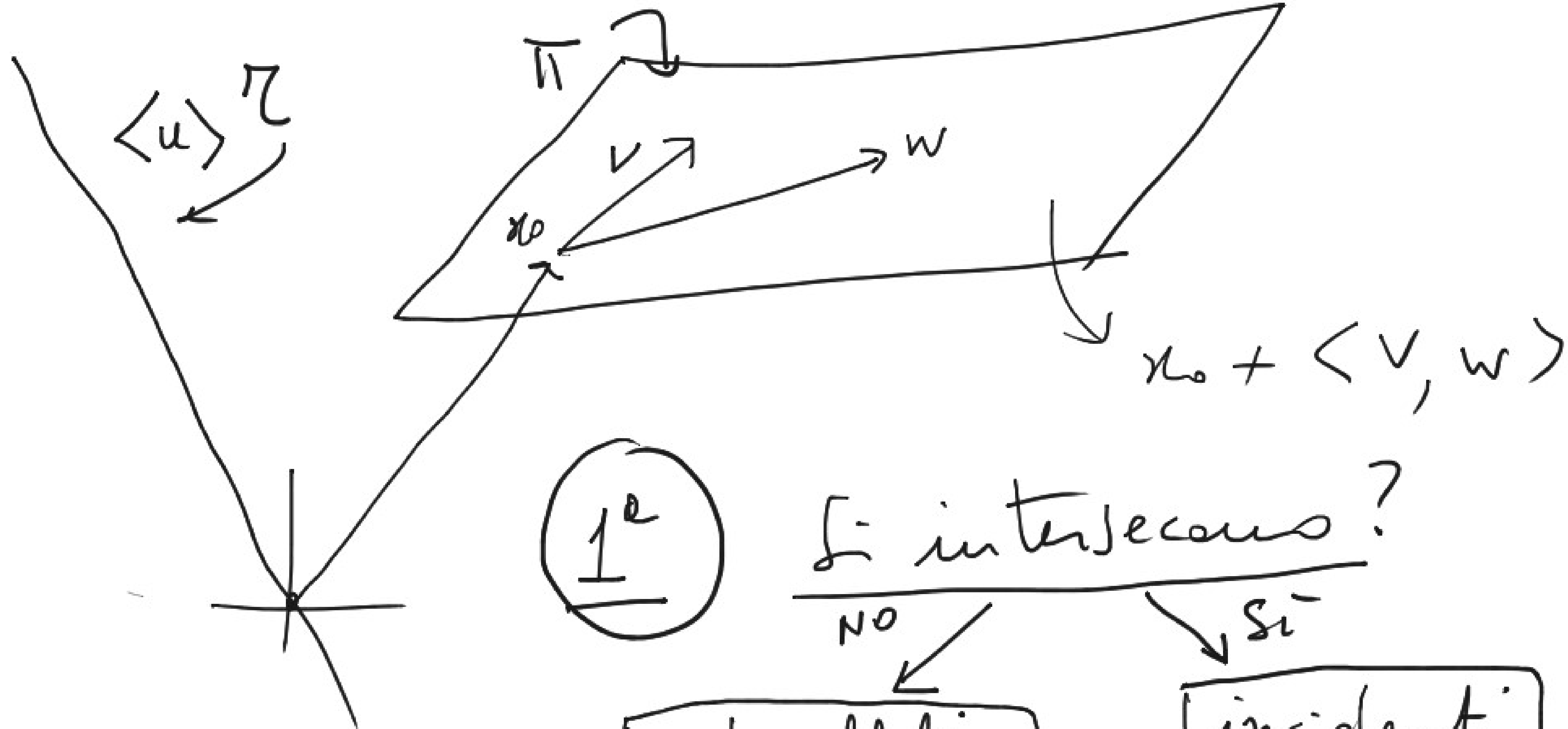
$\alpha = x_1 - 1$

$x_2 = 1 + x_3 - 1 = x_3$

$x_4 = 1 + x_1 - 1 = x_1$

eq. cartesiane

$$\begin{cases} x_2 = x_3 \\ x_4 = x_1 \end{cases}$$



1ª Si intersecano?

NO → parallele (senza punti comuni) oppure sghembi

SÌ → incidenti oppure la retta sta sul piano

2 $\boxed{u \in \langle v, w \rangle}$?

sì → paralleli o $r \subseteq \pi$

NO → incidenti o sghembi

$u \in \langle v, w \rangle$ \ $r \cap \pi$	sì	no
sì	$r \subseteq \pi$	parallele (senza intersez.)
no	incidenti	sghembi

G_1.4 e G_1.6

risolvere questi sistemi

$u \in \langle v, w \rangle \iff \boxed{\exists \alpha, \beta : \ u = \alpha v + \beta w}$

$r \cap \pi \neq \emptyset$ se $\boxed{x_0 + \alpha v + \beta u = \gamma u}$

punto di r generico

punto di π generico

$$
\left[\begin{array}{cc|c}
1 & 0 & 1 \\
0 & 1 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1
\end{array}\right]
$$

$$v \quad w \quad u$$

$u \in \langle v, w \rangle$ se ho soluzioni

$$
\begin{array}{c}
\\
\\
\mathrm{III} \\
\cancel{\mathrm{IV}} \\
\mathrm{III}
\end{array}
\left[\begin{array}{cc|c}
1 & 0 & 1 \\
0 & 1 & 0 \\
0 & 1 & 1 \\
\cancel{1} & \cancel{0} & \cancel{1} \\
0 & 0 & 1
\end{array}\right]
$$

Nes. soluzione

$$
\begin{cases}
1 + \alpha + \cancel{0\beta} = \gamma \\
1 + \cancel{0\alpha} + \beta = \cancel{0\cdot\gamma}\ \gamma \\
1 + \cancel{0\alpha} + \beta = \gamma \\
1 + \alpha + \cancel{0\beta} = \gamma
\end{cases}
$$

$\exists\, \alpha, \beta, \gamma$ tali che

$$x_0 + \alpha v + \beta w = \gamma u$$

$$
\begin{cases}
1 + \alpha + \cancel{0\beta} = \gamma \\
1 + \cancel{0\alpha} + \beta = \cancel{0\cdot\gamma}\ \gamma \\
1 + \cancel{0\alpha} + \beta = \gamma \\
1 + \alpha + \cancel{0\beta} = \gamma
\end{cases}
$$

$\exists\, \alpha, \beta, \gamma$ tali che

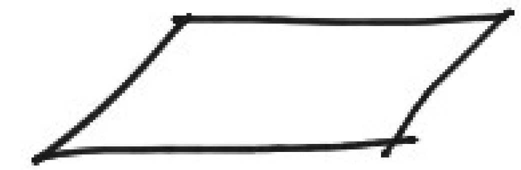
$$
\begin{array}{c|ccc}
 & \alpha & \beta & \gamma \\
\hline
1 & -1 & 0 & 1 \\
1 & 0 & -1 & 0 \\
1 & 0 & -1 & 1 \\
1 & -1 & 0 & 1
\end{array}
$$

se il sistema ha soluzioni ci sono intersezioni fra r e π

Volume del parallelepipedo di spigoli

$$\underset{u}{(1,0,0,0)},\ \underset{v}{(1,0,1,0)},\ \underset{w}{(1,1,1,1)}$$

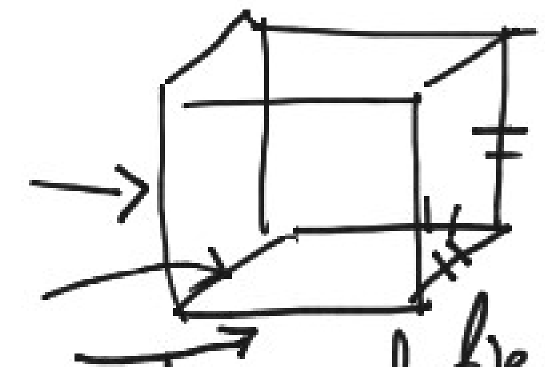
$\mathbb{R}^4$



piano

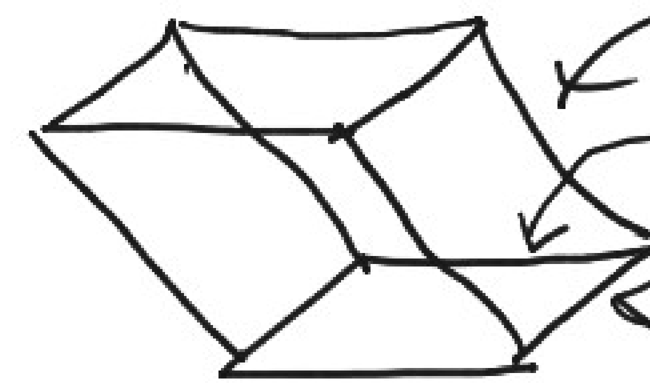
parallelogramma

Vol = Area di base · altezza

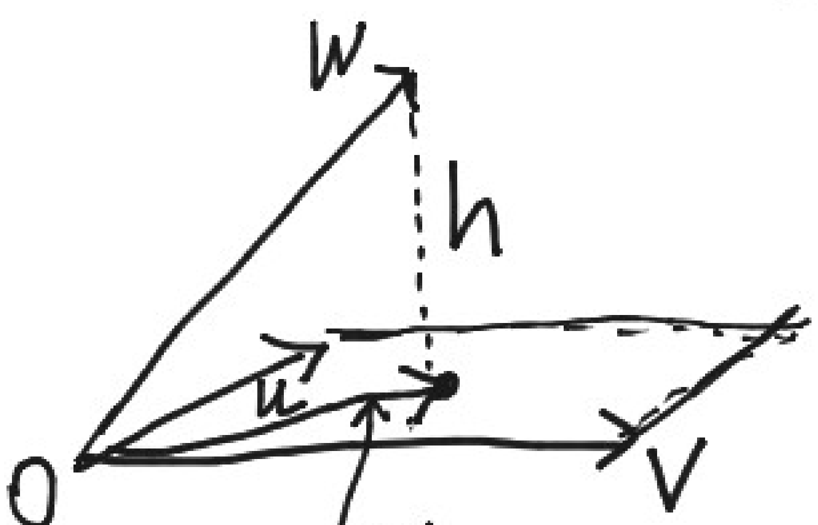


parallelepipedo

base = parallelogramma di lati u e v



spigolo



$w_{\langle u,v\rangle}$ piano della base

$h = w -$

$$\mathrm{Vol} = \overbrace{\sqrt{|u|^2|v|^2-(u\,v)^2}}^{\text{Area base}} \cdot \underbrace{\left|w - w_{\langle u,v\rangle}\right|}_{\text{altezza}}$$

$$w_{\langle u,v\rangle} = \alpha u + \beta v \qquad \alpha, \beta \text{ tali che}$$

$$\begin{cases} (w - \alpha u - \beta v)\,u = 0 \\ (w - \alpha u - \beta v)\,v = 0 \end{cases} \qquad \underline{\text{ha soluzione}}\ (\bar\alpha, \bar\beta)$$

$$w_{\langle u,v\rangle} = \bar\alpha u + \bar\beta v$$

$$\begin{pmatrix} 1 & 1 & 3 \\ -1 & 2 & 0 \\ 1 & 1 & 3 \end{pmatrix}$$

<u>non</u> è simmetrica

Determiniamo gli autovalori

$$0 = \begin{vmatrix} 1-\lambda & 1 & 3 \\ -1 & 2-\lambda & 0 \\ 1 & 1 & 3-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)(3-\lambda) - 3 - (\underbrace{6-3\lambda+\lambda-3}_{3-2\lambda}) =$$

$$= 2(\lambda-3) + (1-\lambda)(2-\lambda)(3-\lambda) =$$

$$= (3-\lambda)\left[(1-\lambda)(2-\lambda)-2\right] = (3-\lambda)(\lambda^2-3\lambda) =$$

$$= -\lambda(3-\lambda)^2$$

$\lambda = 0 \quad \mu = 1$

$\lambda = 3 \quad \mu = 2$

$$\underbrace{\begin{matrix} 1-\lambda & 1 & 3 \\ -1 & 2-\lambda & 0 \\ 1 & 1 & 3-\lambda \end{matrix}}_{A-\lambda I}$$

$\lambda = 3$

$A-$

$$\begin{array}{c|ccc|c} V & -2 & 1 & 3 & 0 \\ & -1 & -1 & 0 & 0 \\ & \cancel{1} & \cancel{1} & \cancel{0} & \cancel{0} \end{array}$$

$A - 3I$

$\lambda = 3$

$$\begin{array}{c|ccc} \text{II} & -2 & 1 & 3 \\ -2\text{I} & 0 & 3 & 3 \end{array}$$

pivot

dim. Ker = 1

numero delle colonne

non pivot = dim Ker

$\boxed{1 < 2}$

μ

NON diagonalizzabile

$\boxed{\text{AL\_7.11 \quad AL\_7.6}}$

Se $\bar{\lambda}$ è semplice

$(\mu = 1)$

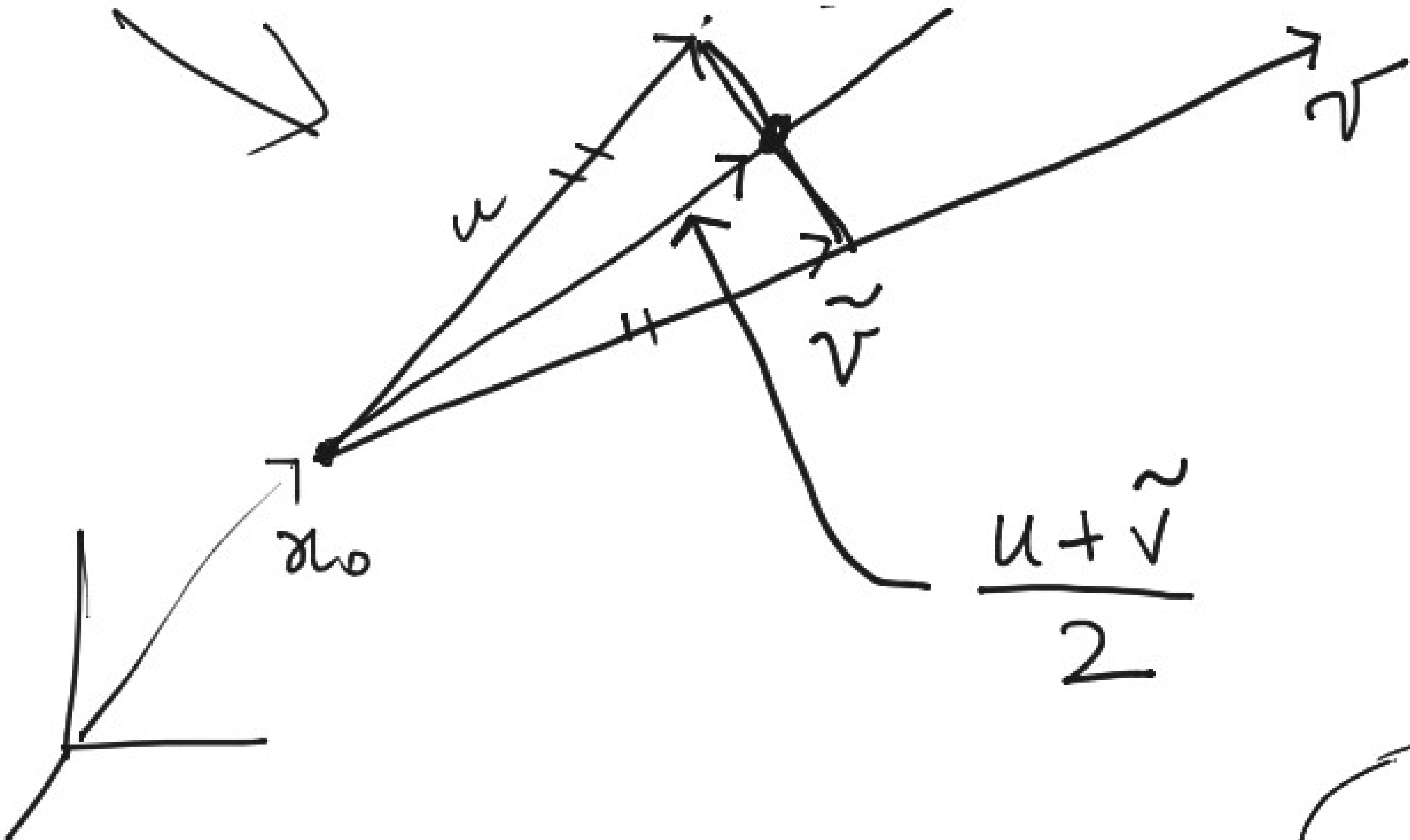
dim $\bar{A}$ ≤ 1 perché

è $\leq$ molteplicità

però ≥ 1 perché $\bar{\lambda}$ è

autovalore e quindi

$V_{\bar{\lambda}}$ contiene autovettori $\neq 0$



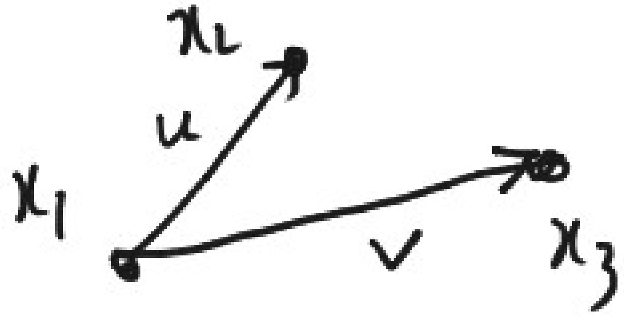
$$\tilde{v} = \frac{|u|}{|v|}\, v$$

$$|\tilde{v}| = \frac{|u|}{|v|}\, |v| = |u|$$

retta bisettrice $\quad x_0 + t\,\frac{u+\tilde{v}}{2}$ $\quad$ oppure

$$x_0 + t\,(u+\tilde{v})$$

multipli hanno la stessa direzione



$u = x_2 - x_1$

$v = x_3 - x_1$