limite di forme quadratiche all'∞

$\boxed{\lim_{\infty} x^2 = \infty}$ $\boxed{\lim_{\infty} x^2 - 2xy + 2y^2 = ?}$

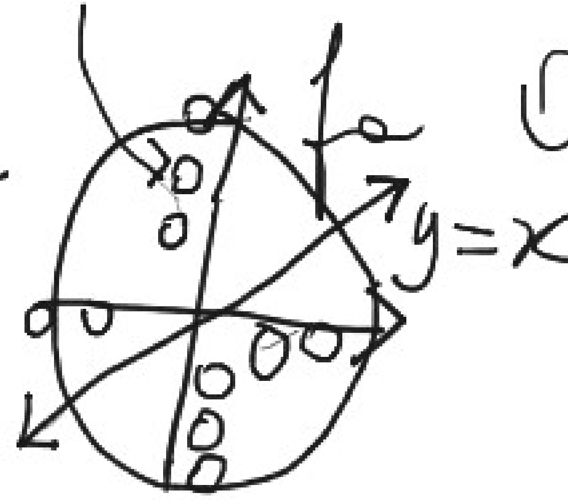
Se $f(x) = \sum_{1}^{n} a_{ij} x_i x_j$ (forma generale)

Se f è <u>definita</u> diverge (se è def. >0 diverge a $+\infty$
se è def. <0 " " $-\infty$

Altrimenti, il limite <u>non esiste</u> (f oscilla).

$f(x,y) = xy$ fa 0 sugli assi e diverge su $y=x$

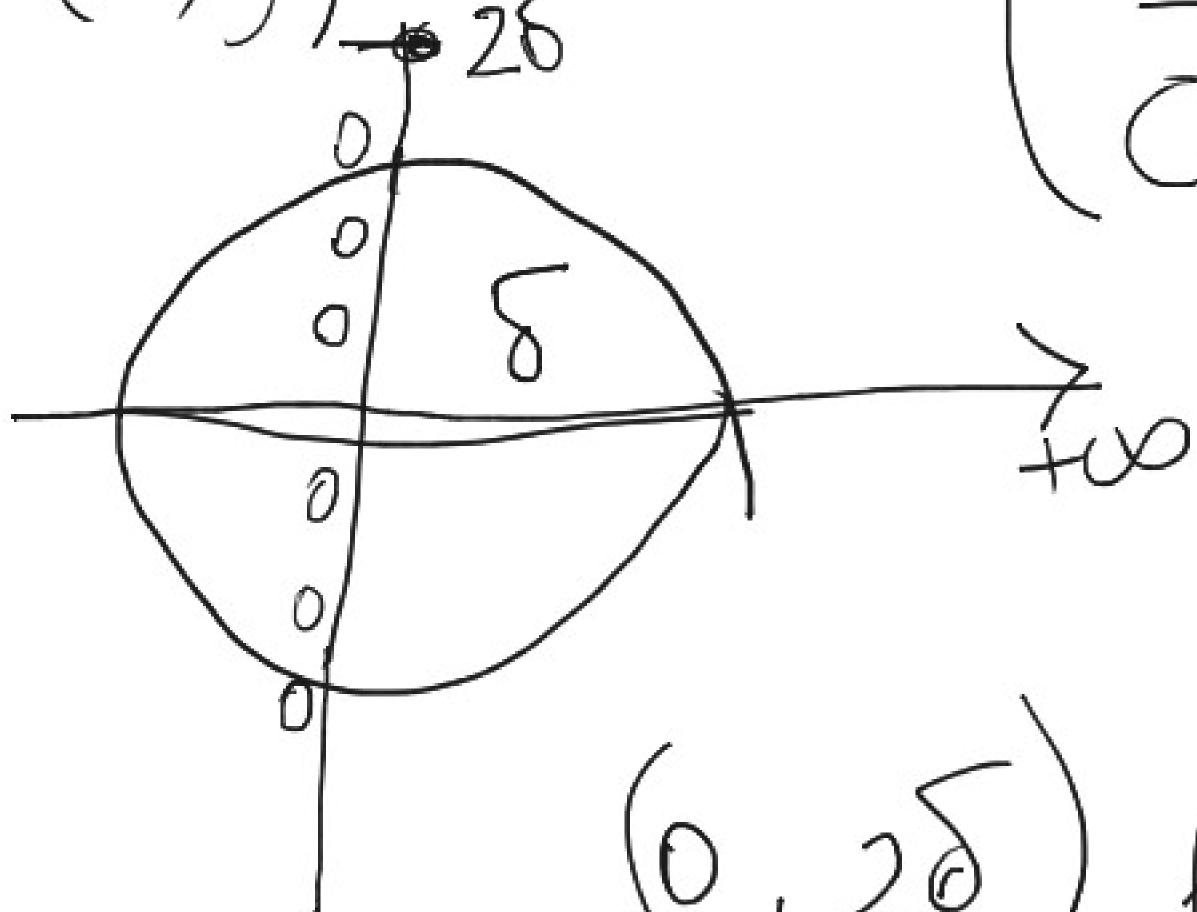
$\begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}$



$f(x,x) = x^2 \nearrow +\infty$

$f(x,y) = x^2$

$\begin{pmatrix} \underline{1} & 0 \\ 0 & \underline{0} \end{pmatrix}$ semidef. > 0



fuori di $\underline{B((0,0),\delta)}$

$\underline{|f(x,y)| > \varepsilon > 0}$

$\underline{\underline{(0,2\delta)}}$ fuori di $B((0,0),\delta)$

$f(0,2\delta) \equiv 0 \quad \forall\delta$

$x^2 + 2y^2$

$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

$\rightarrow \underbrace{x^2 + y^2}_{\| \; |(x,y)|^2} \leq \underbrace{x^2 + 2y^2}_{f} \leftarrow$

$\delta = \sqrt{\varepsilon}$

$\boxed{\begin{array}{l} |(x,y)| > \delta \Downarrow \\ |(x,y)^2| > \delta^2 \end{array}}$

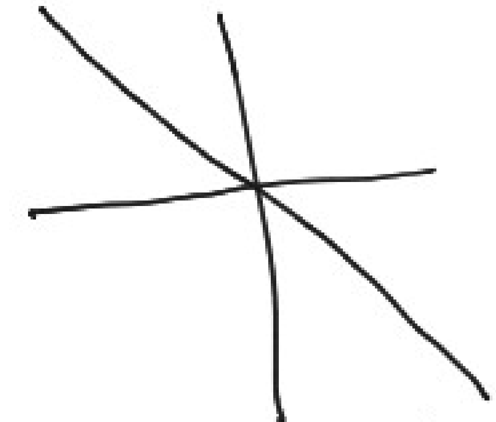
$$-x^2-3y^2$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -3 \end{pmatrix}$$

$$x^2+y^2 \le x^2+3y^2$$

$$\boxed{-x^2-y^2} \ge \underbrace{-x^2-3y^2}_{f(x,y)}$$

$$-x^2-y^2 \to -\infty$$



$y=-x$

$$\boxed{\lambda \underbrace{|x|^2}_{\to +\infty} \le \underbrace{\sum_{1}^{n} a_{ij}x_i x_j}_{f(x)} \le \Lambda \underbrace{|x|^2}_{\to +\infty}} \quad (*)$$

λ: minimo autovalore

Λ: massimo autovalore

$x \in \mathbb{R}^n$

1) f è def. >0 se $\lambda > 0$

2) f è def <0 se $\underline{\Lambda < 0}$

Da $(*)$ segue la tesi per confronto

$$\sum \alpha_i x_i^2 = f(x)$$

$$\alpha_i \quad \lambda_i$$

$$\boxed{\sum_1^n (\min \alpha_i)\, x_i^2 \le \sum_1^n \alpha_i x_i^2 \le \sum (\max \alpha_i)\, x_i^2}$$

>0 $\quad$ >0

minimo autovalore $\quad$ autovalori $\quad$ max autoval.

$$\boxed{x^2+y^2} \le x^2+2y^2 \le 2x^2+2y^2$$

Segno delle forme quadratiche

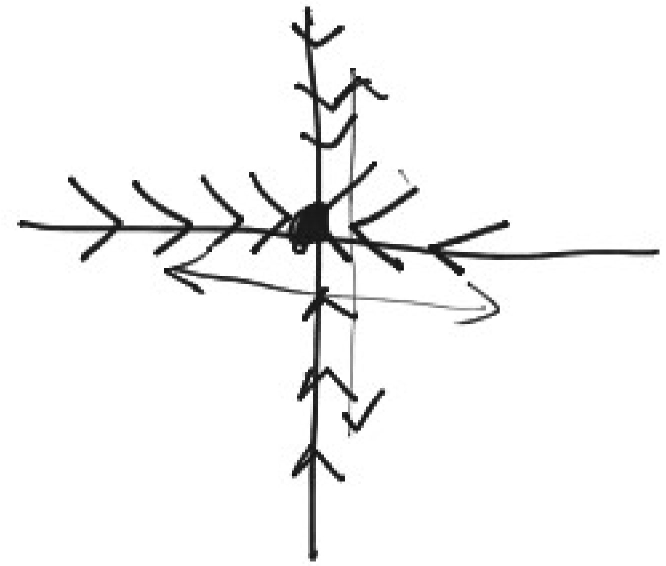
(la più recente)

$$\lim_{(0,0)} \frac{1}{x^2+2y^2} \longrightarrow 0 = (0^2+2\,0^2)$$

1 ← costante

$\boxed{x^2+2y^2}$ funzione continua perché somma di prodotti delle funzioni continue $g(x,y)=x$ e $h(x,y)=y$

$$\frac{1}{\underbrace{x^2 - y^2}_{\to 0}} \longrightarrow \text{NON Esiste}$$



sull'asse x $\quad y \equiv 0 \quad \to \lim\limits_{\substack{(x,y)\to 0 \\ \{y=0\}}} \frac{1}{x^2} = +\infty$

asse y $\quad x = 0 \quad \lim \frac{1}{-y^2} = -\infty$

si comporta esattamente come $\underline{\underline{\frac{1}{x}}}$ in 1 variabile

$$\frac{1}{\sum a_{ij}x_i x_j} \qquad \boxed{x \neq 0}$$

cioè $|x|>0$

$$f(x)=\sum a_{ij}x_i x_j \qquad \lim_{0} \frac{1}{\Lambda|x|^2} = +\infty$$

$$\lambda|x|^2 \leq f(x) \leq \Lambda|x|^2$$

$\lambda > 0 \Rightarrow f$ è defin. >0

$$\frac{1}{f(x)} \geq \boxed{\frac{1}{\Lambda|x|^2}}$$

$+\infty$

ha segno >0 $(x \neq 0)$

0

$$x \to 0 \Leftrightarrow |x| \to 0 \Rightarrow |x|^2 \to 0$$

$$\frac{1}{\Lambda|x|^2} > \varepsilon$$

$f \to +\infty$

$$\Lambda|x|^2 < \frac{1}{\varepsilon}$$

$$|x| < \frac{1}{\sqrt{\varepsilon\Lambda}} = \delta$$

$x \to 0$

f forma quadratica

$\lim_{0} f(x) = 0$

$$\boxed{f(x) = \sum a_{ij} x_i x_j}$$

$|x| \to 0$

$x \to 0$

$x_i \to 0 \ \forall i$

$\lim_{0} \frac{1}{f(x)} = +\infty$ f def. > 0

$-\infty$ f def. < 0

Se f è indef. esistono punti di $\mathbb{R}^n$ su cui assume valori di segno discorde

x' autovettore di λ

x'' autovettore di Λ

versori

f indefinita $\Rightarrow$ $\lambda < 0$ $\Lambda > 0$

$f(x') = \lambda |x'|^2 = \lambda$

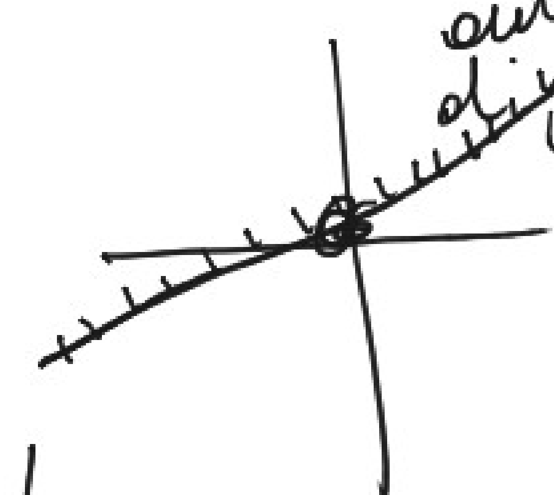
$f(x'') = \Lambda |x''|^2 = \Lambda$

$f(tx) = t^2 f(x)$

$$\frac{1}{f(x)} = \frac{1}{\lambda|x|^2} \quad \text{se } \underline{\underline{x = t x'}} \Rightarrow 1/f < 0$$

$$= \frac{1}{\Lambda|x|^2} \quad \text{se } x = t x'' \Rightarrow \frac{1}{f} > 0$$

$\lim\limits_{0} \frac{1}{f}$ $\qquad f$ <u>semidefinita</u> (≥ 0)



ha senso solo fuori dall'autosp. di $\underline{\underline{0}}$ $\Leftarrow \frac{1}{f(x)} > \varepsilon$

$f(x) < \frac{1}{\varepsilon}$

$0 \leq f(x) \leq \underline{\Lambda|x|^2}$

$\Lambda|x|^2 < 1/\varepsilon$

$|x| < \boxed{(\varepsilon\Lambda)^{-1/2}} = \delta$

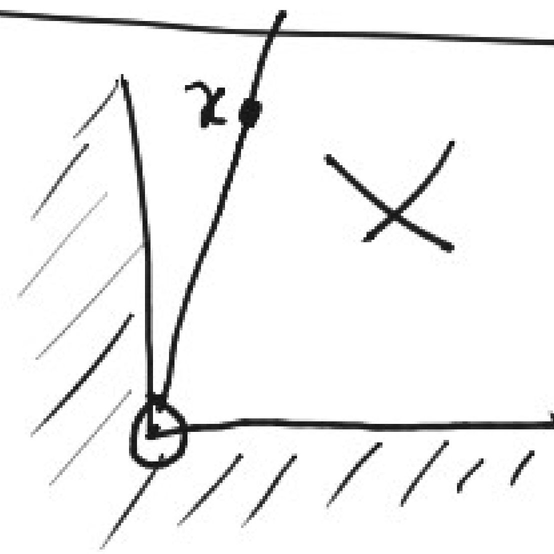
$X \subseteq \mathbb{R}^n$ cono (di vertice 0) se

$\boxed{X \text{ CONO}}$

$\forall x \in X \ \forall \underline{t > 0} \quad tx \in X$

$X = \{x > 0,\ y > 0\}$

I Quadrante



$(x,y) \in X \iff \begin{matrix} x > 0 \\ y > 0 \end{matrix}$

$t(x,y) \equiv (tx, ty) \in X$

$t > 0,\ x > 0,\ y > 0$

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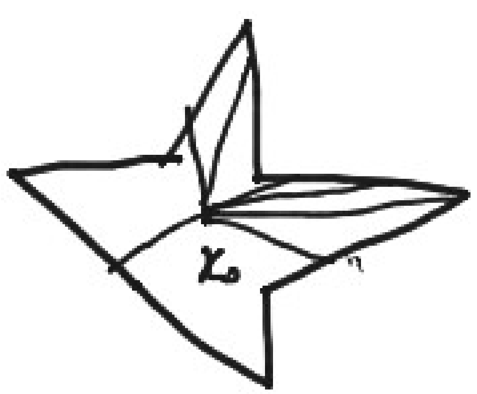
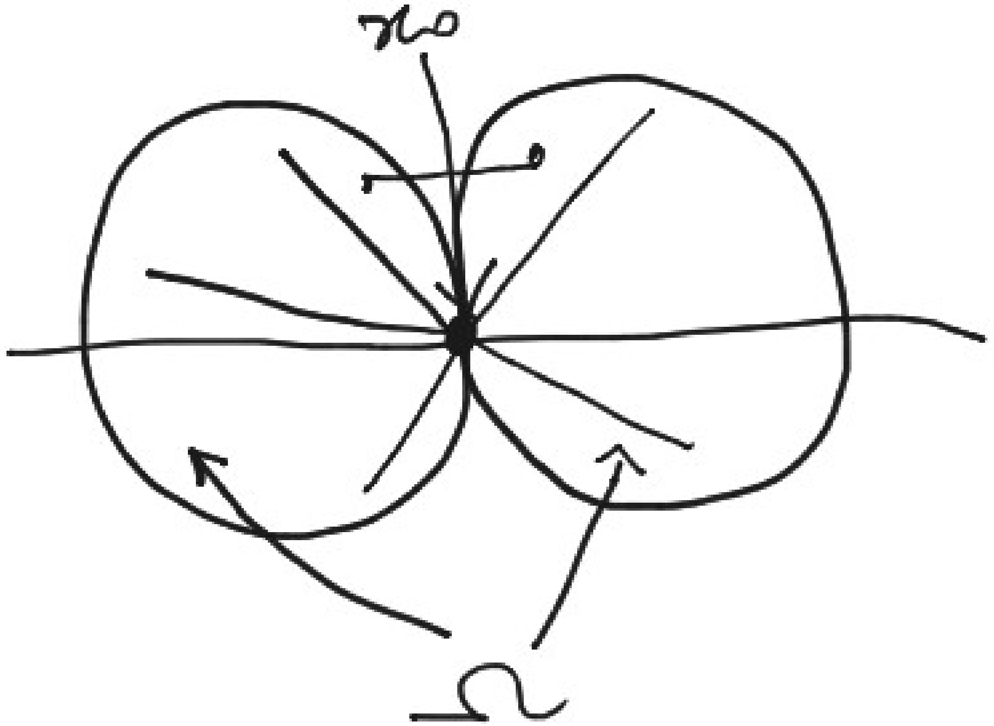
x_0 è il $\underline{\text{POLO}}$

X stella rispetto a $x_0 \in X$ se

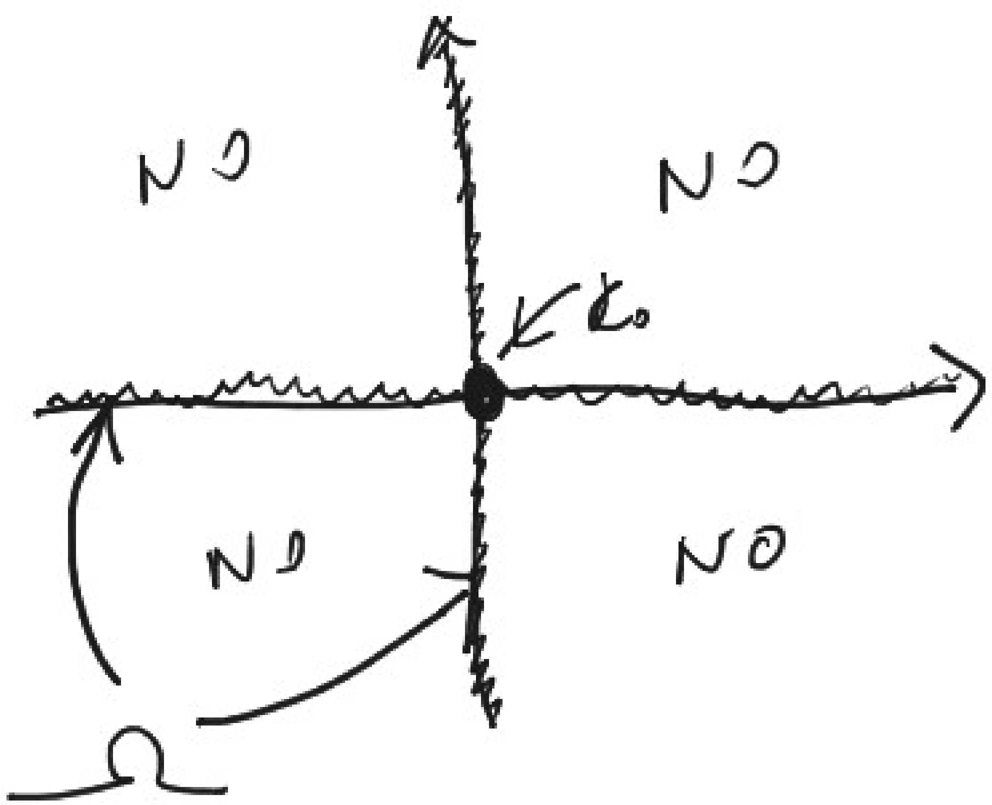
$\forall x \in X \ (1-\lambda)x_0 + \lambda x \in X \ \forall \lambda \in [0,1]$



"un faro in x_0 illumina tutto X"



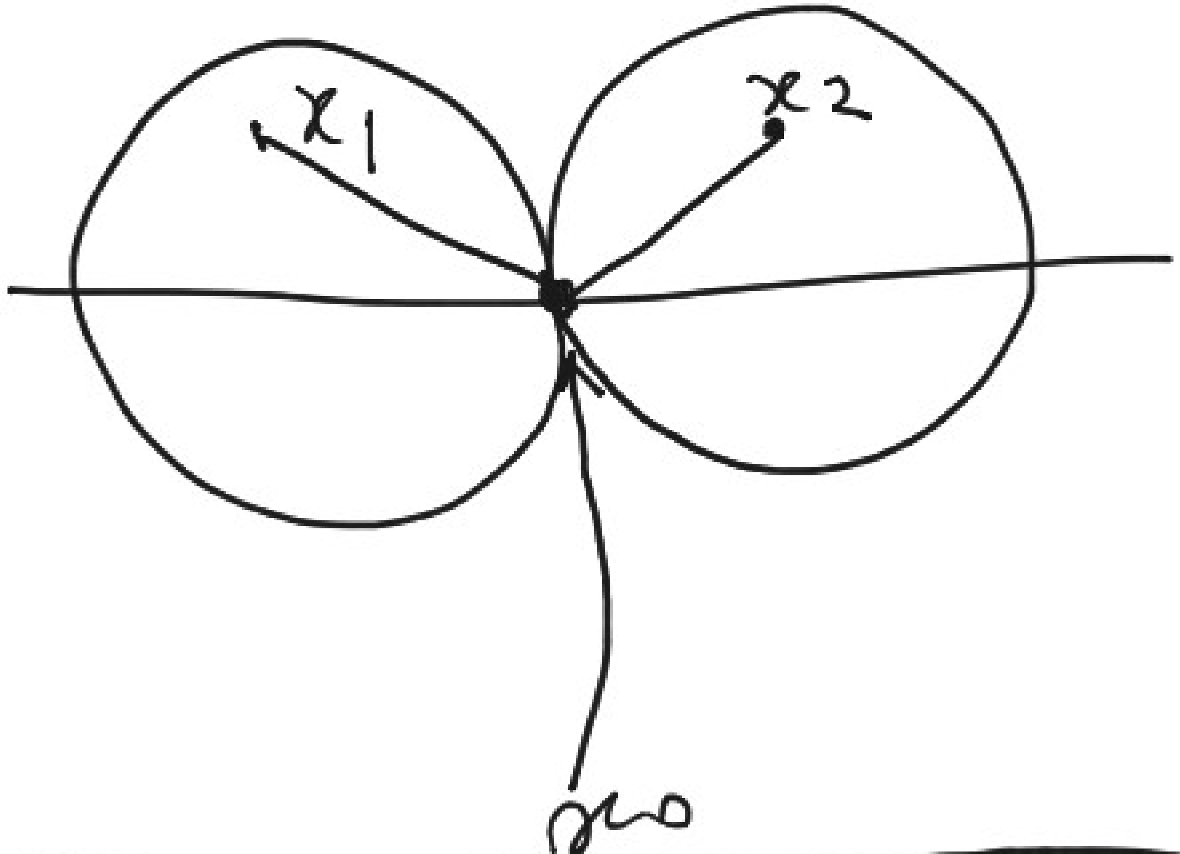
Ω è stella rispetto a x_0
ma non è convesso



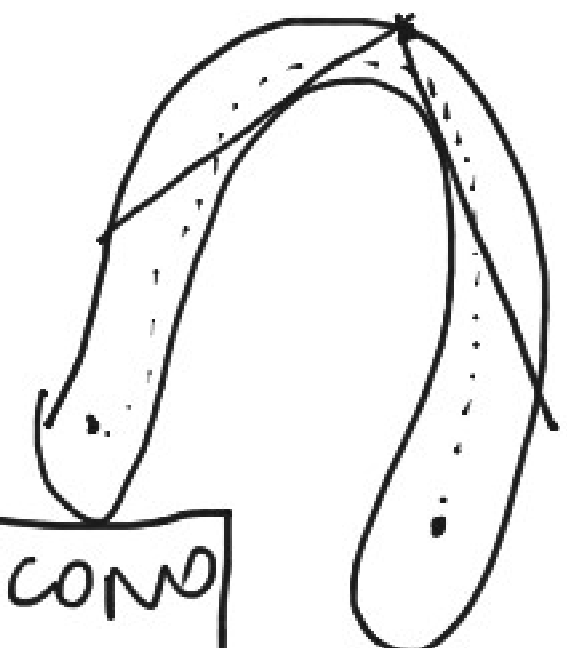
$\Omega = \{ xy = 0 \}$ = unione degli assi cartesiani

è stella rispetto a $(0,0)$
ma non è convesso

stella $\Rightarrow$ connesso (via x_0)



convesso $\Rightarrow$ stella $\Rightarrow$ connesso



CONNESSO NON STELLA

FUNZIONI OMOGENEE SU UN CONO

$f : X \to \mathbb{R}$, X cono, si dice α-OMOGENEA (omogenea di grado α) se

$$f(tx) = t^{\alpha} f(x) \quad \underline{\forall x \in X} \quad \underline{\forall t > 0}$$

$f(x) = |x| = \sqrt{\sum x_i^2}$ è 1-omogenea

Dobbiamo provare che $f(tx) = t f(x) \quad \forall t > 0$

$$f(tx) = |tx| \underset{\substack{\uparrow \\ \text{assioma di} \\ \text{omogeneità}}}{=} |t||x| \underset{t>0}{=} t|x|$$

$$f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}$$

$t > 0$
$(\neq 0)$

$t(x,y) = (tx, ty)$

$$f(tx, ty) = \frac{t^2(x^2 - y^2)}{t^2(x^2 + y^2)} = f(x,y) = t^0 f(x,y)$$

0-omogenea

$$\frac{x^2+y^2}{x-y}$$

1) <u>dominio</u> (è un cono?)

2) grado di omogeneità?

$$\frac{x^2+y^2}{x^3-y^3}$$