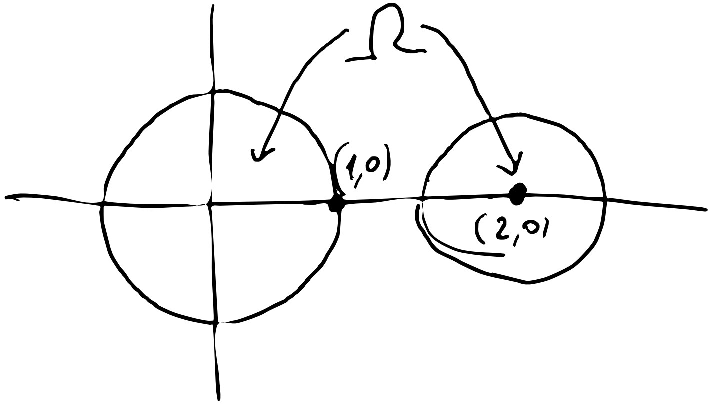
$\Omega = B(0,1) \cup \{(2,0)\}$



(2,0) è isolato

$B\left((2,0), \frac{1}{2}\right) \cap \Omega =$

$= (2,0)$

$$\boxed{\exists\, \delta > 0 : B(x_0, \delta) \cap \Omega = \{x_0\}}$$

x_0 è isolato

$f: \Omega \to \mathbb{R}$ Insieme di livello k è $\{x \in \Omega : f(x) = k\}$

$f^{-1}(k)$

$f: \Omega \to \mathbb{R}$ Pallino

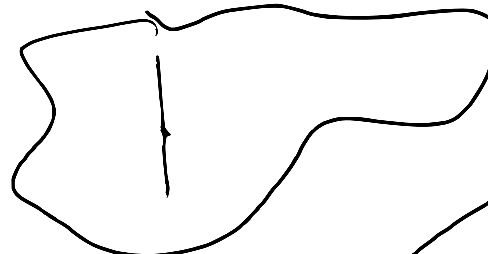
$f: \Omega \to \mathbb{R}$

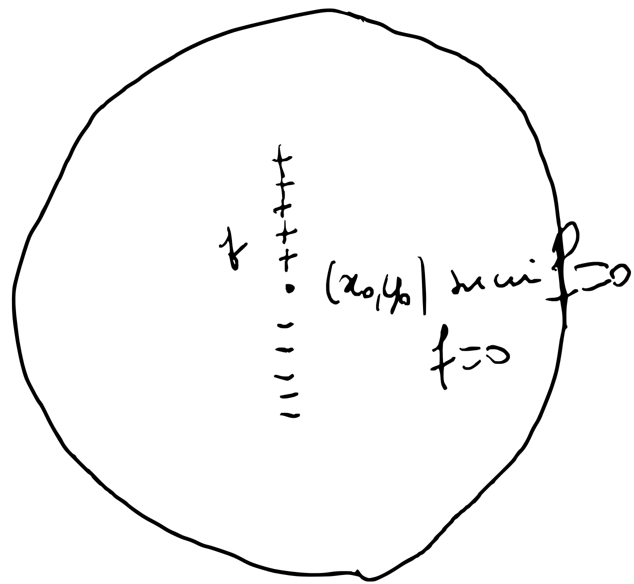
Teorema delle funzioni implicite (I)

$f : \Omega \to \mathbb{R}$

1) continua 2) $\underline{f(x_0, y_0) = 0}$ 3) $(x_0, y_0) \in \mathring{\Omega}$ (interno)

$y \to f(x, y)$ è strett. cresc $\forall (x,y) \in \Omega$



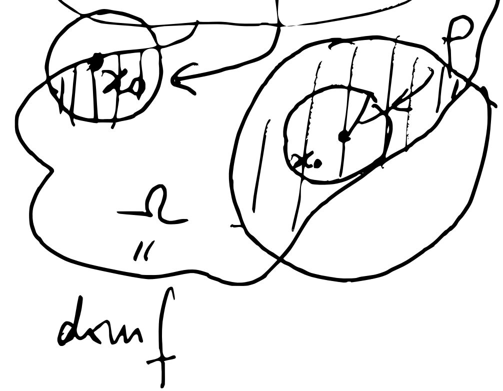


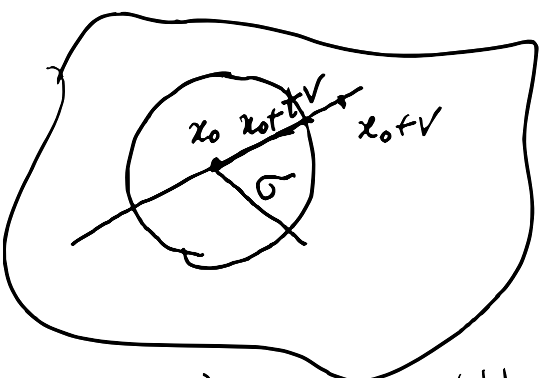
x_0 è di min locale
$\exists \delta : \forall x \in \text{dom} f \cap B(x_0, \delta)$

x_0 è interno
$\exists \rho : B(x_0, \rho) \subseteq \text{dom} f$

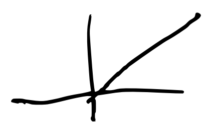
$\sigma = \min\{\delta, \rho\}$

$f(x_0) \leq f(x)$





$\boxed{t \to f(x_0+tv)}$



$f : [0,1] \to \mathbb{R}$

$f(x) = x$

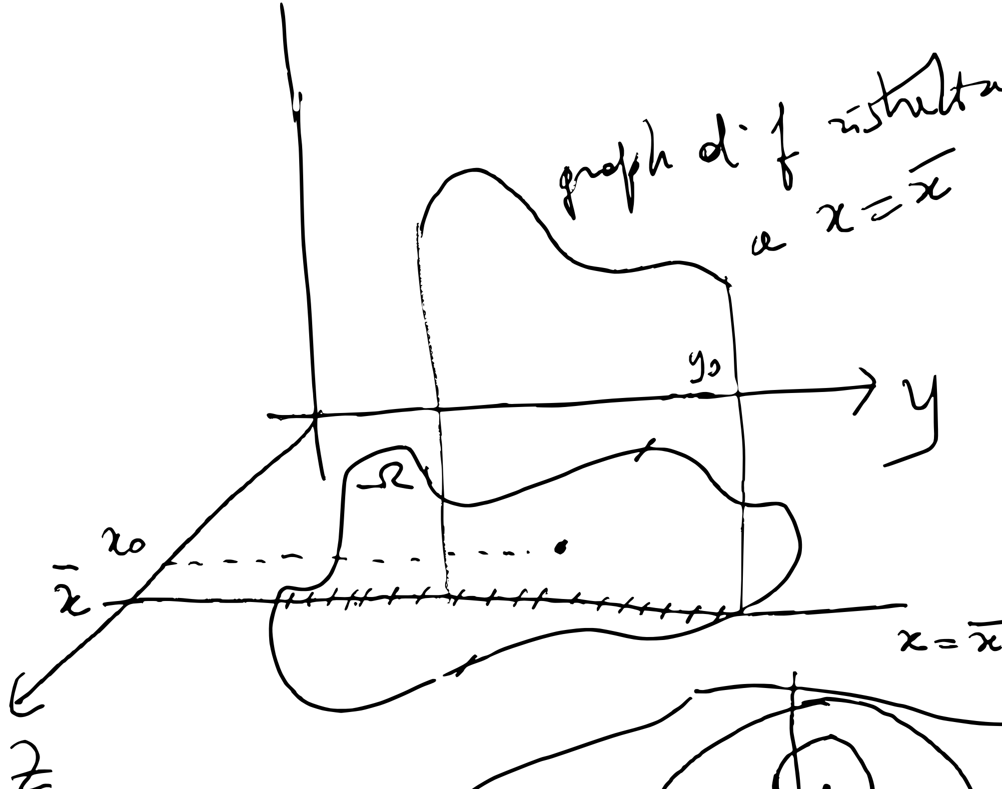
$x_0 = 0$ è

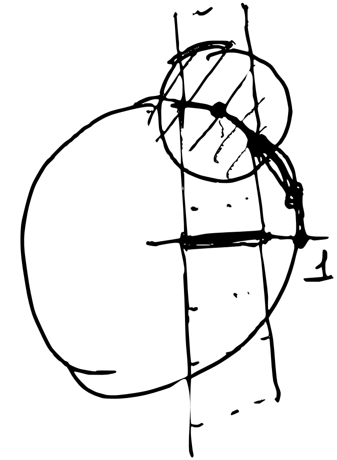
di minimo globale

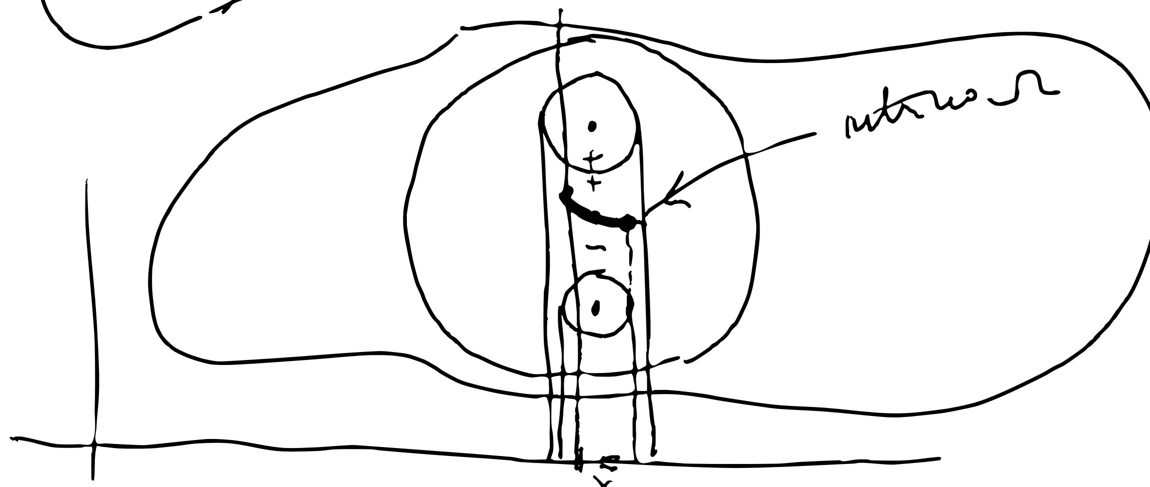
$|(x_0+tv) - x_0| = |t||v| < \sigma$

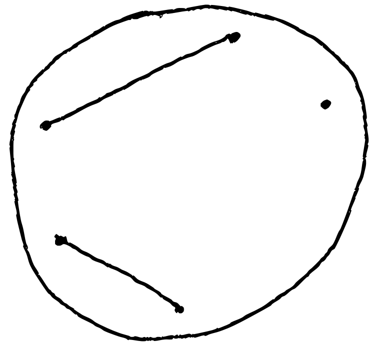
$|t| < \frac{\sigma}{|v|}$

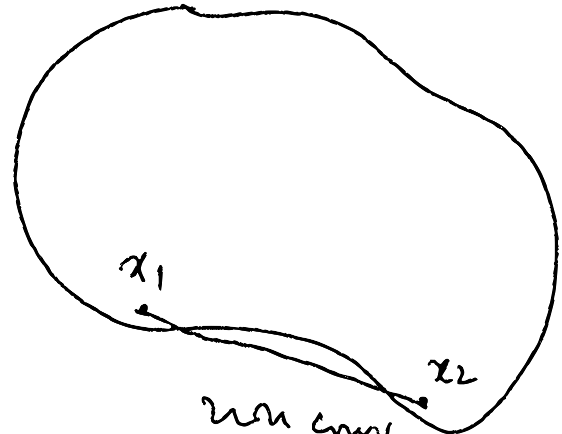
$]-\frac{\sigma}{|v|}, \frac{\sigma}{|v|}[$ 0 è interno


graph di f ristretta
a $x=\bar{x}$
y_0
y
Ω
x_0
$\bar{x}$
$x=\bar{x}$
z


1


interno Ω
$\bar{x}$

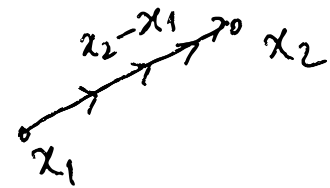

convesso



non conv.

segmento di estremi x_1 e x_2

$\gamma(t) = x_1 + t(x_2 - x_1) \quad t \in [0,1]$



$(1-t)x_1 + t x_2$

$\forall x_1, x_2 \in \Omega \quad (1-t)x_1 + t x_2 \in \Omega \quad \forall t \in [0,1]$

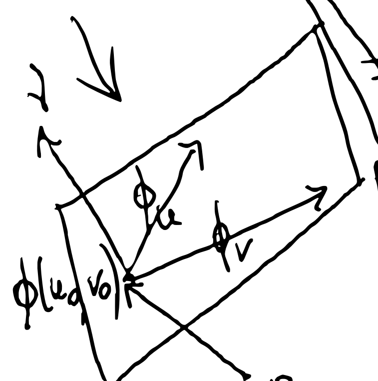
$$\boxed{\phi : \mathbb{R}^2 \to \mathbb{R}^3} \qquad \phi(u,v) = \big(x(u,v),\, y(u,v),\, z(u,v)\big)$$

$$\phi_u = \begin{pmatrix} x_u \\ y_u \\ z_u \end{pmatrix} \qquad \phi_v = \begin{pmatrix} x_v \\ y_v \\ z_v \end{pmatrix}$$

Eq. param. del piano tang.

$$\boxed{(u_0, v_0)} \mapsto (x_0, y_0, z_0)$$

$$\psi(\alpha, \beta) = \phi(u_0, v_0) + \alpha\, \phi_u(u_0, v_0) + \beta\, \phi_v(u_0, v_0)$$



$\nu = \phi_u(u_0, v_0) \times \phi_v(u_0, v_0)$ vettore normale al piano tangente

$\nu = (a, b, c)$

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$ eq. implic. piano tangente

$$\operatorname{graph}(f) = \left\{ (x, y, z) \subset \mathbb{R}^3 : \begin{array}{l} (x,y) \in \operatorname{dom} f \\ z = f(x,y) \end{array} \right\}$$

$$z - f(x_0, y_0) = \nabla f(x_0, y_0)\,(x - x_0,\, y - y_0)$$

$$\boxed{z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)}$$

Piano tangente ad un grafico di funzione

$$|p(z)| = \left|a_n + \sum_{0}^{n-1} a_i z^{i-n}\right| |z|^n$$

$\left|a_n + \sum\ \right| \to |a_n| > 0$ perm. segno $\quad \boxed{\bar{\varepsilon} = \frac{|a_n|}{2}}$

$\exists \bar{\delta} > 0 : \ |z| > \bar{\delta}$

$$\left|a_n + \sum\right| > |a_n| - \bar{\varepsilon} = |a_n| - \frac{|a_n|}{2} = \frac{|a_n|}{2}$$

$$|p(z)| > \frac{|a_n|}{2} |z|^n \to +\infty$$

$$\boxed{\begin{gathered} |z|^n > \varepsilon \\ |z| > \varepsilon^{\frac{1}{n}} = \delta \end{gathered}}$$