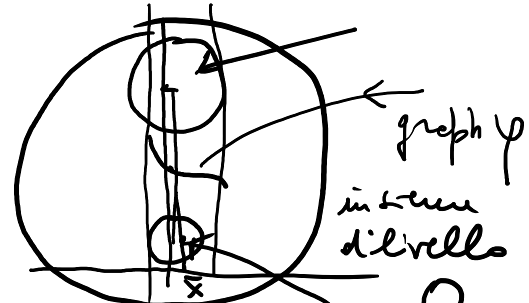
Teorema delle funzioni implicite (conclusione)
Limiti: definizioni

Th. zeri $\quad y \to f(\bar{x}, y)$ su $[y_0-\varepsilon,\ y_0+\varepsilon] = I$

- è continua in I perché $(\bar{x}, y_0-\varepsilon)\,(\bar{x}, y_0+\varepsilon) \subseteq \Omega$
- è definita su I perché $B((x_0,y_0),\rho)$ è convessa e gli estremi vi appartengono
- $f(\bar{x}, y_0+\varepsilon) > 0$ e $f(\bar{x}, y_0-\varepsilon) < 0$



$$\exists\, \bar{y} \in [y_0-\varepsilon,\ y_0+\varepsilon] : \quad f(\bar{x}, \bar{y}) = 0$$

$$\varphi(\bar{x}) = \bar{y}$$

SU OGNI RETTA VERTICALE C'È AL PIÙ UN UNICO ZERO $(\bar{x}, \bar{y})$ di f

$$f(x, \varphi(x)) \equiv 0 \quad \text{su } [x_0-\delta,\ x_0+\delta]$$

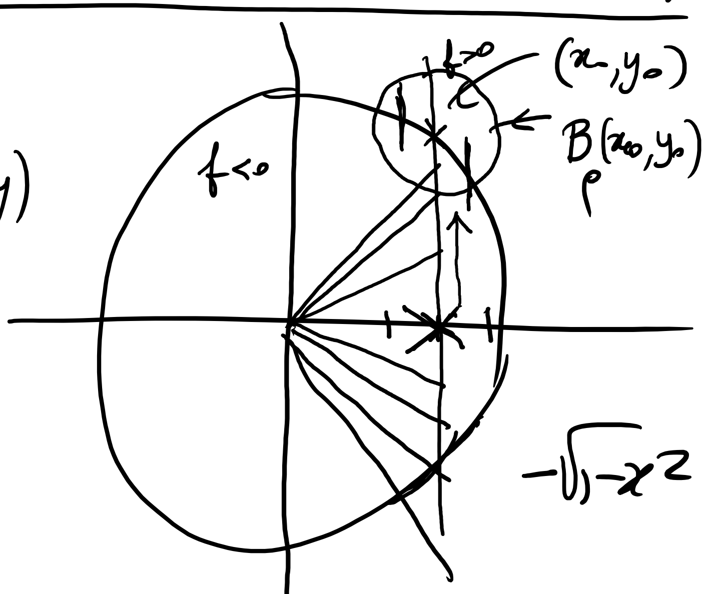
le y per cui $f(x,y)=0$

$f(x_0, y_0) = 0 \qquad \varphi(x_0) = y_0$

$x^2 + y^2 = 1$

$x^2 + y^2 - 1 = f(x,y)$

$|(x,y)|^2 - 1$



φ è continua in x_0

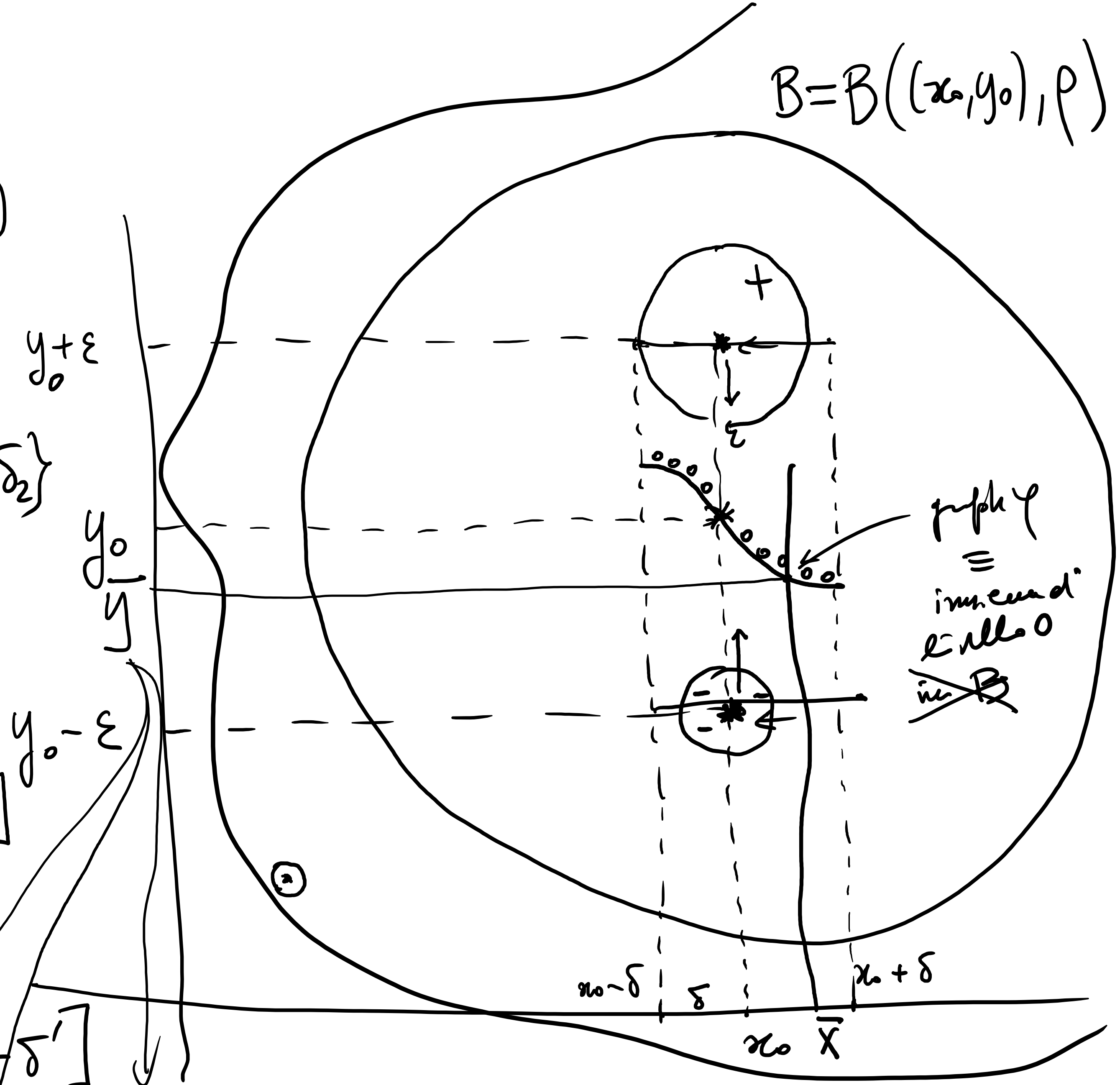
$$\varphi(x) \in [y_0-\varepsilon,\, y_0+\varepsilon]$$

$$\rightarrow |\varphi(x)-\varphi(x_0)| \le \varepsilon$$

$$\forall x \in [x_0-\delta,\, x_0+\delta]$$

$y_0 = \varphi(x_0)$

$\delta = \min\{\delta_1, \delta_2\}$

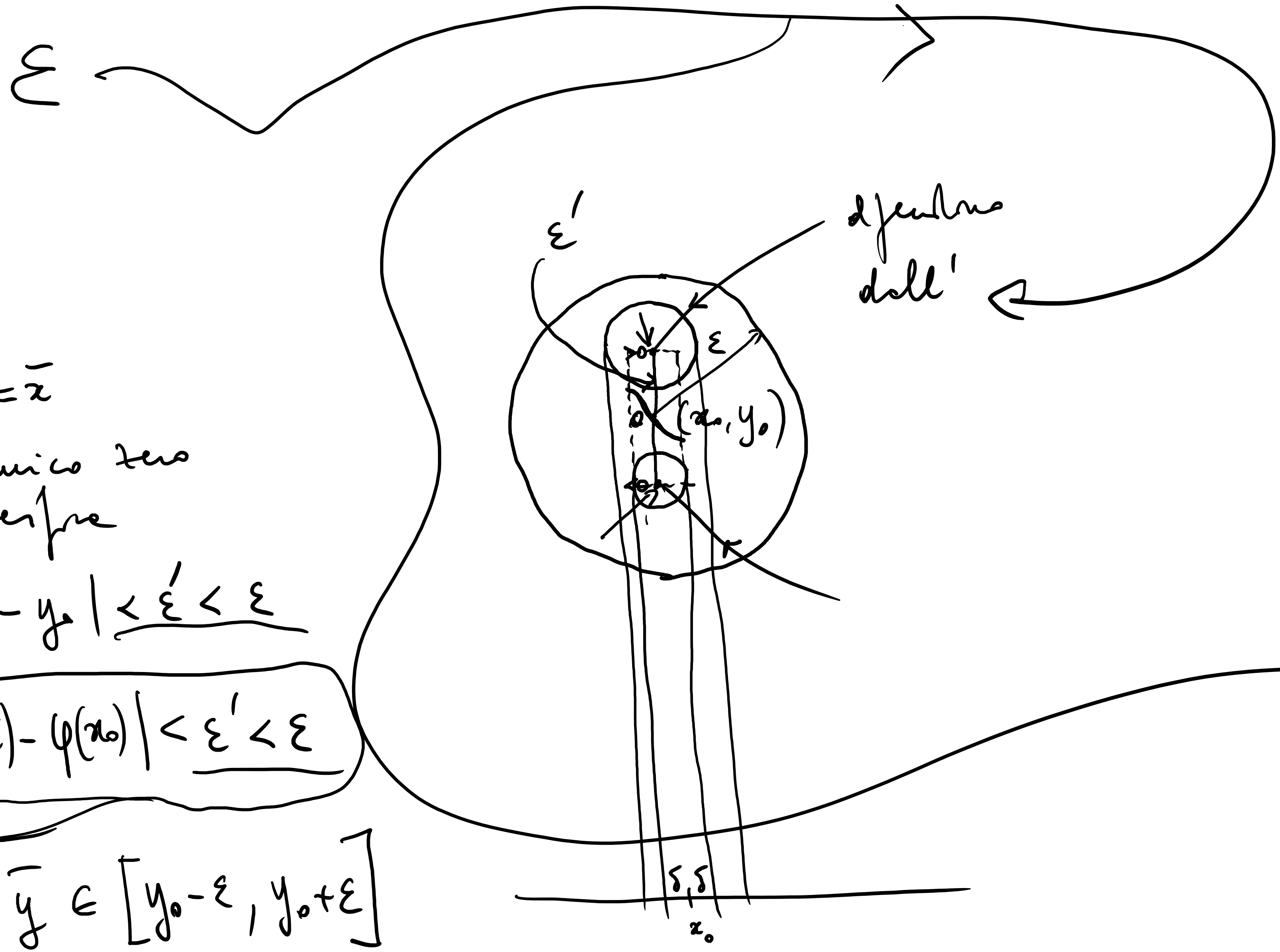


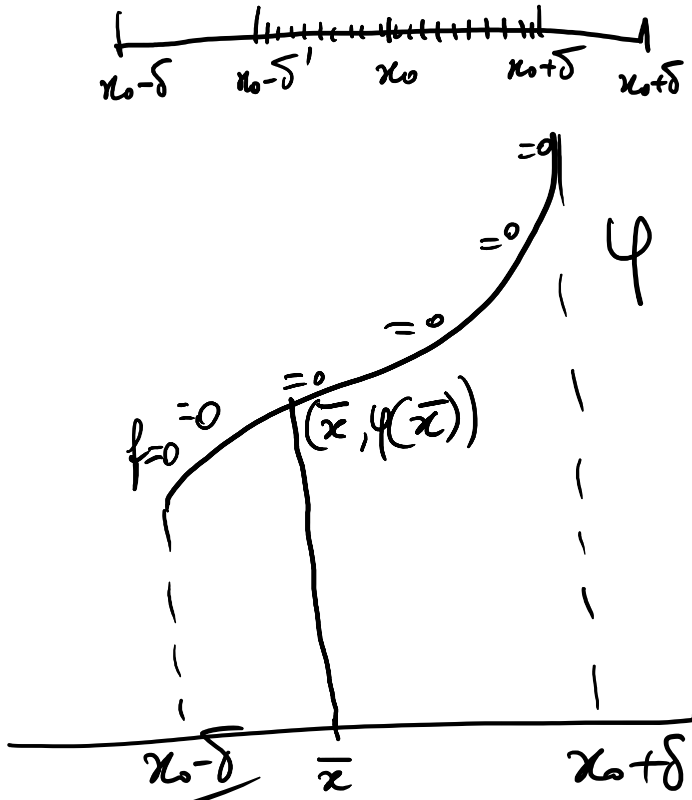
$f(x, \varphi_1(x)) = 0$ su $[x_0-\delta,\, x_0+\delta]$

$f(x, \varphi_2(x)) = 0$ su $[x_0-\delta',\, x_0+\delta']$

$\forall \bar{x} \in$ Intersezione di

$$\varphi_1(\bar{x}) = \bar{y} = \varphi_2(\bar{x})$$





$x = \bar{x}$

l'unico zero verifica

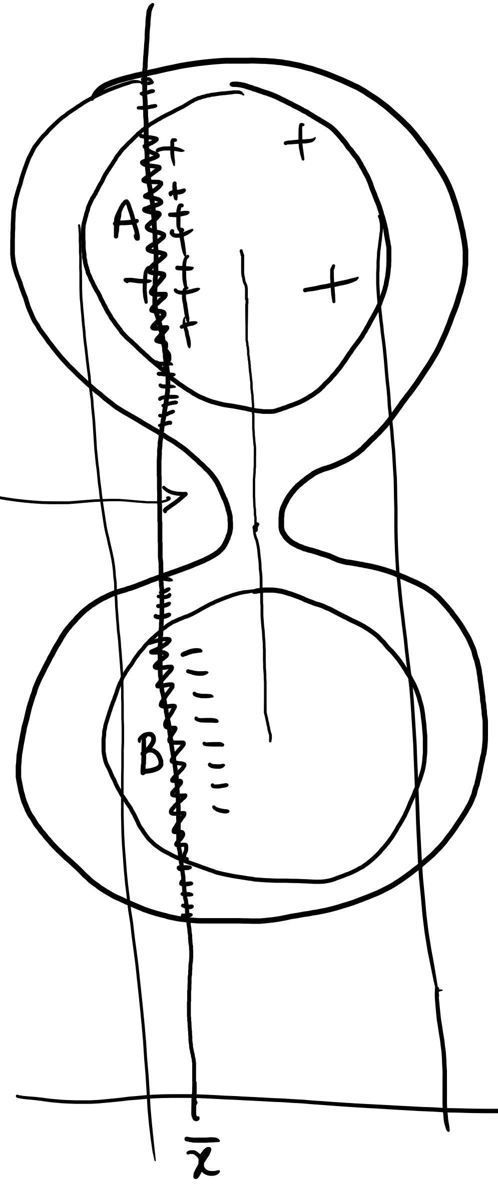
$$|\bar{y} - y_0| < \varepsilon' < \varepsilon$$

$$|\varphi(\bar{x}) - \varphi(x_0)| < \varepsilon' < \varepsilon$$

Ripeto l'intera costruzione con centro $(\bar{x}, \varphi(\bar{x}))$

$$\bar{y} \in [y_0 - \varepsilon, y_0 + \varepsilon]$$

$$\bar{y} = \varphi(\bar{x}) \in]y_0 - \varepsilon, y_0 + \varepsilon[\Longleftarrow \left(f(\bar{x}, y_0 - \varepsilon) < 0 \quad f(\bar{x}, y_0 + \varepsilon) > 0 \right)$$



NON sta nel dominio di

dove $y \to f(\bar{x}, y) = A \cup B$

↑ NON SI PUÒ APPLICARE

IL Th. degli zeri

$$\delta = \min\{\delta_1, \delta_2\}$$

LIMITI

$$\lim_{A} f(x) = B \iff \forall \varepsilon > 0 \ \exists \delta > 0 : \forall x \in \text{dom} f \quad A' \Rightarrow B'$$

DOMINIO	A	A'
$\mathbb{R}^n$, $n \geq 1$	$x \to x_0$	$\lvert x - x_0 \rvert < \delta \quad x \neq x_0$
	∞	$\lvert x \rvert > \delta$
$\mathbb{R}$ (solo lui!)	$x \to x_0^+$	$x_0 < x < x_0 + \delta$
	$x \to x_0^-$	$x_0 - \delta < x < x_0$
	$x \to +\infty$	$x > \delta$
	$x \to -\infty$	$x < -\delta$

CODOMINIO	B	B'
$\mathbb{R}^m$, $n \geq 1$	$L \in \mathbb{R}^m_{m \geq 1}$	$\lvert f(x) - L \rvert < \varepsilon$
	∞	$\lvert f(x) \rvert > \varepsilon$
$\mathbb{R}$ (solo lui!)	$+\infty$	$f(x) > \varepsilon$
	$-\infty$	$f(x) < -\varepsilon$

$$a_n = f(n) \quad \forall n$$

$$\lim_{n \to \infty} a_n = \lim_{n \to +\infty} f(n)$$

$\gamma : \mathbb{R} \to \mathbb{R}^3$

$A =$ " $x \to 1$ " $= x \to x_0$

$B =$ " $(1,2,\pi)$ " $= L$

$$\lim_{x \to 1} \gamma(x) = (1,2,\pi)$$

CONVERGENZA

$$\forall \varepsilon > 0 \ \exists \delta > 0 : \forall x \in dom \ |x-1| < \delta \ x \neq 1 \implies |\gamma(x) - (1,2,\pi)| < \varepsilon$$

DIVERGENZA

$$\lim_{x \to 3^-} \gamma(x) = \infty$$

A $x \to 3^-$

$B = \infty$

$$\forall \varepsilon > 0 \ \exists \delta > 0 : \forall x \in dom \, \gamma \ \underbrace{3-\delta < x < 3}_{A'} \implies \underbrace{|\gamma(x)| > \varepsilon}_{B'}$$