CONNESSIONE. CONVESSITA’.
TEOREMA DELLE FUNZIONI IMPLICITE: (introduzione ed enunciato).

$f : \Omega \to \mathbb{R}$

- Ω connesso
- f continua in Ω
- $\exists\, x_1, x_2 \in \Omega : f(x_1)\, f(x_2) < 0$

TS $\exists\, \bar{x} \in \Omega : f(\bar{x}) = 0$

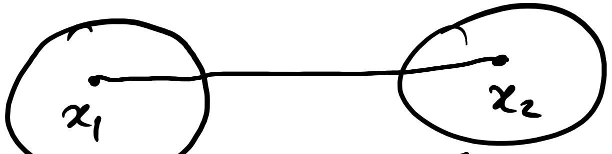
Ω connesso

$\forall x_1, x_2 \in \Omega \quad \exists\, \gamma : [0,1] \to \Omega$ continua

$\gamma(0) = x_1 \quad \gamma(1) = x_2$

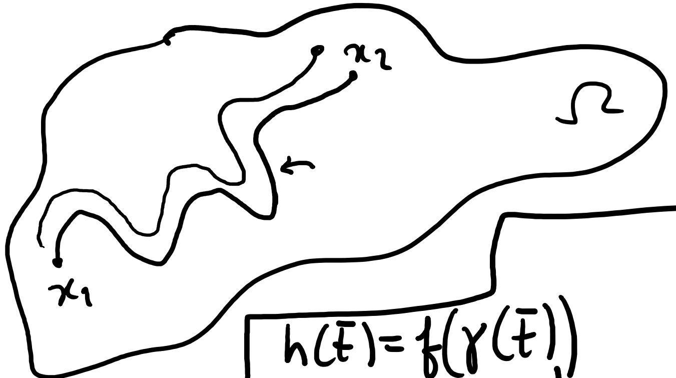
Ω intervallo

sostituisce nel th. 1 variabile



Ω sconnesso

DIM. Scelti x_1, x_2, poiché Ω è connesso, $\exists\, \gamma : [0,1] \to \Omega$, continua e verificante $\gamma(0) = x_1 \quad \gamma(1) = x_2$

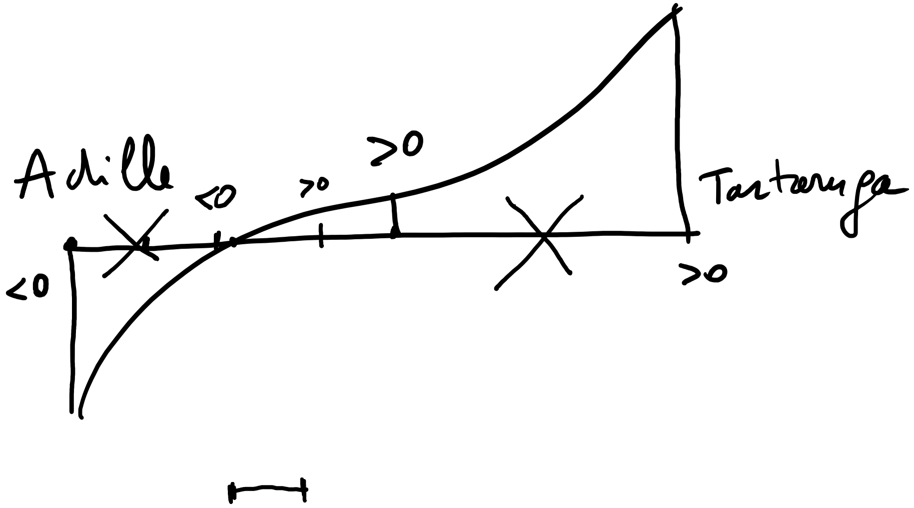


$h(t) \qquad h : [0,1] \to \mathbb{R} \qquad h(t) = f(\gamma(t))$

- h è definita in un intervallo ($[0,1]$)
- h è continua, composta di γ ed f continue.
- $h(0) = f(\gamma(0)) = f(x_1) \qquad h(1) = f(\gamma(1)) = f(x_2)$

$h(\bar{t}) = f(\underbrace{\gamma(\bar{t})}_{\bar{x}})$

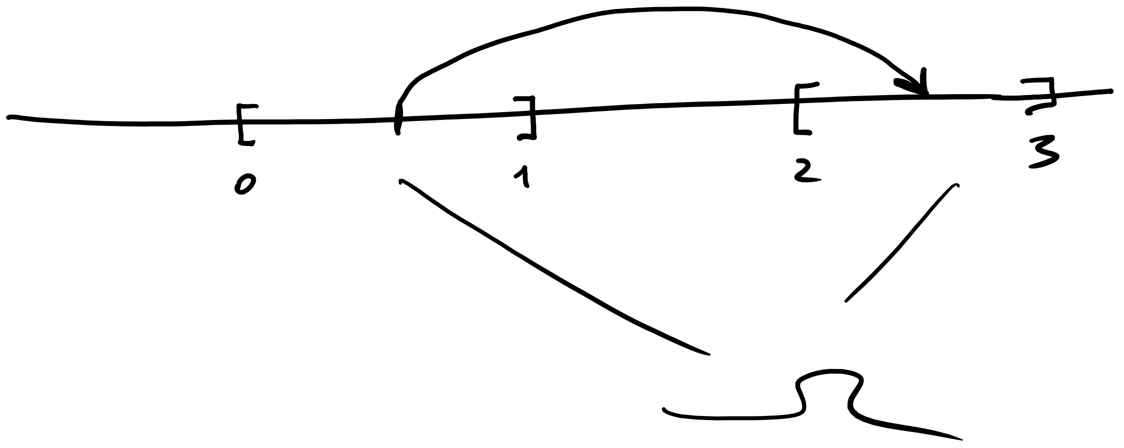
$\exists\, \bar{t} \in [0,1] : h(\bar{t}) = 0$



Achille
Tartaruga
<0
<0
>0
>0
>0

$$\Omega = [0,1] \cup [2,3]$$

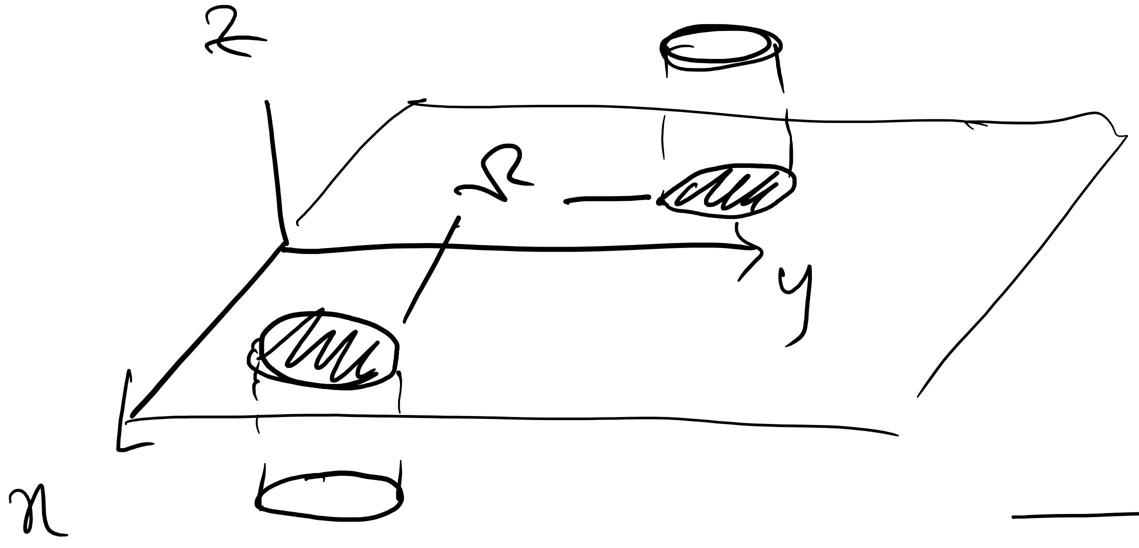
$$\mathcal{F}\Omega = \{0, 1, 2, 3\}$$



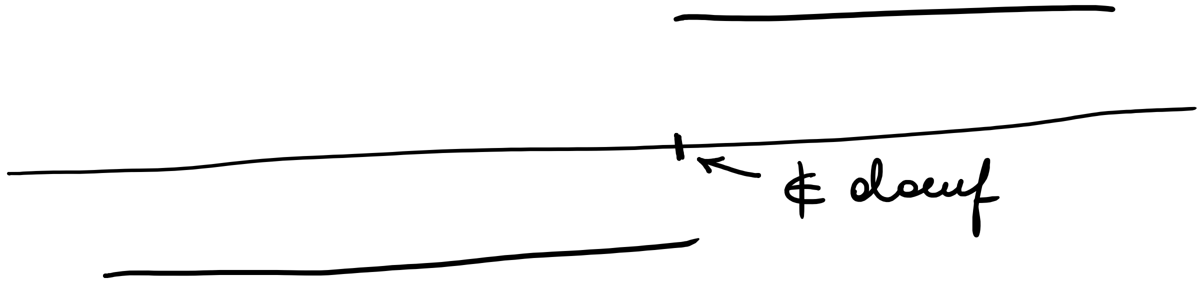
$$B(0,4) = [-4, 4]$$

$$\Omega = [0, +\infty[\qquad \mathcal{F}\Omega = \{0\} \subseteq \Omega$$

$$\Omega = \{0, 1, 2, \dots, n, \dots\} \qquad \mathcal{F}\Omega = \Omega$$

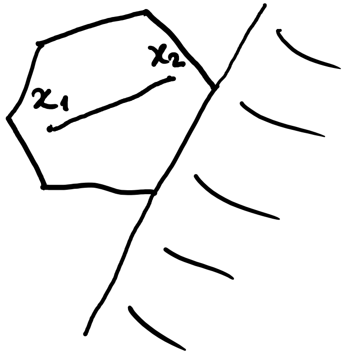


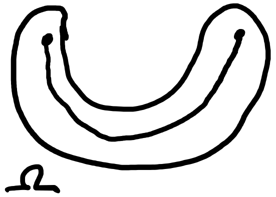
$$f = \frac{x}{|x|}$$



$\notin$ dom f

CONVESSITÀ

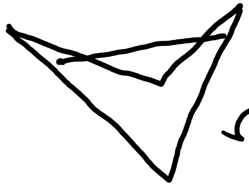




Ω connesso ma non convesso

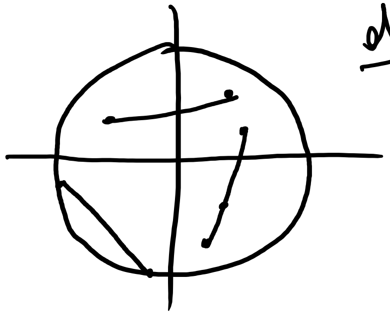
Ω si dirà CONVESSO se

$$\forall x_1, x_2 \in \Omega \quad \forall \lambda \in [0,1] \quad \underbrace{(1-\lambda)x_1 + \lambda x_2}_{\text{funzione continua di } \lambda} \in \Omega$$



Ω quadrilatero non convesso

$$\Omega = B(0,1)$$



$\forall x_1, x_2 \in \Omega \quad \forall \lambda \in [0,1]$

$(1-\lambda)x_1 + \lambda x_2 \in \Omega$

Hp.

$|x_1| < 1 \qquad |x_2| < 1$

Ts

$|(1-\lambda)x_1 + \lambda x_2| < 1$?

DIM.

prodotto scalare

$$|(1-\lambda)x_1 + \lambda x_2|^2 = \underbrace{(1-\lambda)^2 |x_1|^2} + \underbrace{2\lambda(1-\lambda)}_{\geq 0} \boxed{x_1 x_2} + \underbrace{\lambda^2 |x_2|^2} \leq$$

prop. prodotto scalare

dis. Schwarz

$$\leq (1-\lambda)^2 |x_1|^2 + 2\lambda(1-\lambda)|x_1||x_2| + \lambda^2 |x_2|^2 =$$

$$= \Big[\overset{>0}{(1-\lambda)}\underbrace{|x_1|}_{<1} + \overset{>0}{\lambda}\underbrace{|x_2|}_{<1}\Big]^2 <$$

$$< \big[(1-\lambda)1 + \lambda 1\big]^2 = 1$$

$$x_1 x_2 \leq |x_1 x_2| \underset{\text{schwarz}}{\leq} |x_1||x_2|$$

$$x^2+y^2-1=0$$

$$\Updownarrow$$

$$f(x,y)=0$$

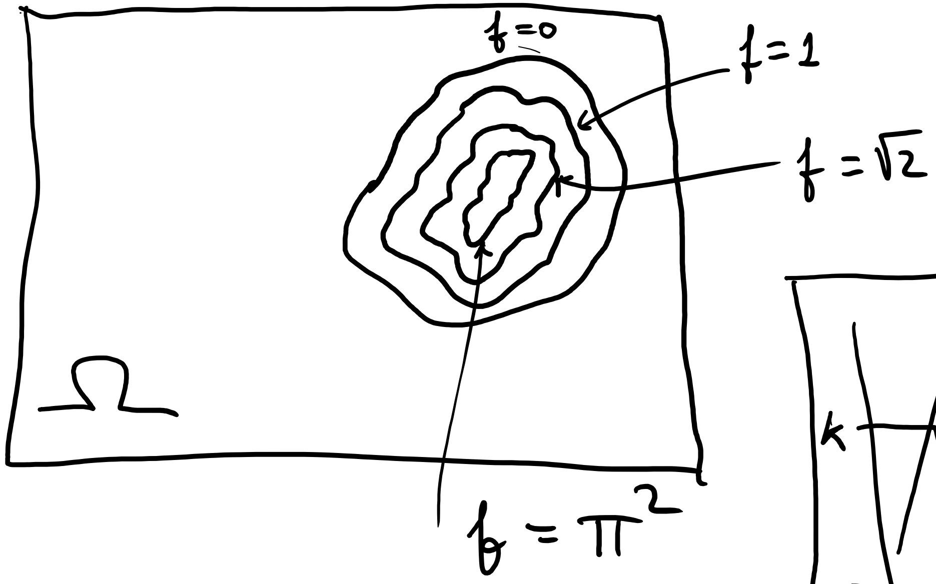
$$f:\Omega\to\mathbb{R}$$

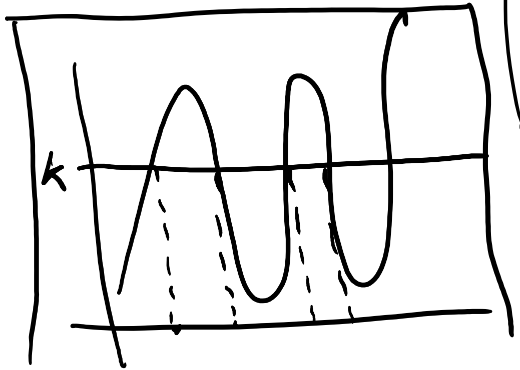
$$\Omega\subseteq\mathbb{R}^2$$

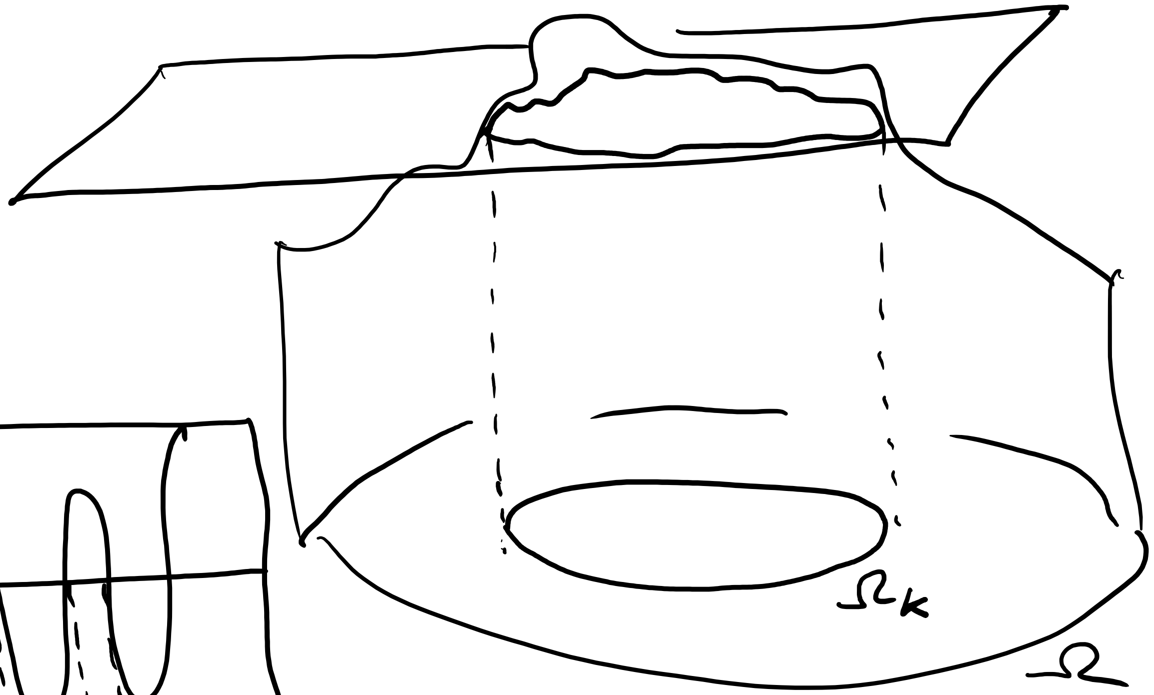
$$f(x,y)=x^2+y^2-1$$

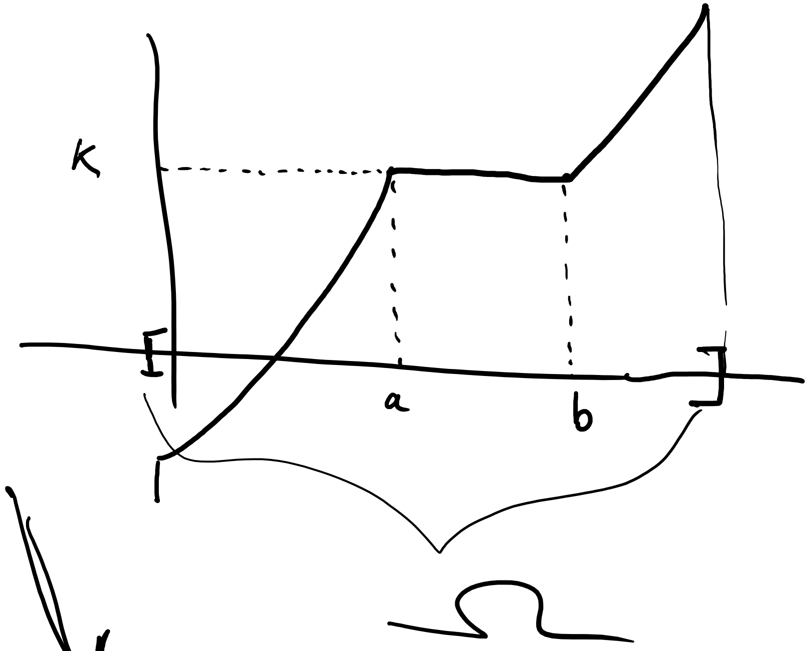
Insieme di livello $k\in\mathbb{R}$ per una funzione $f:\Omega\to\mathbb{R}$ e

ULISSE DINI $\longrightarrow$ $\Omega^k \equiv \{(x,y)\in\underline{\Omega} : f(x,y)=k\}$

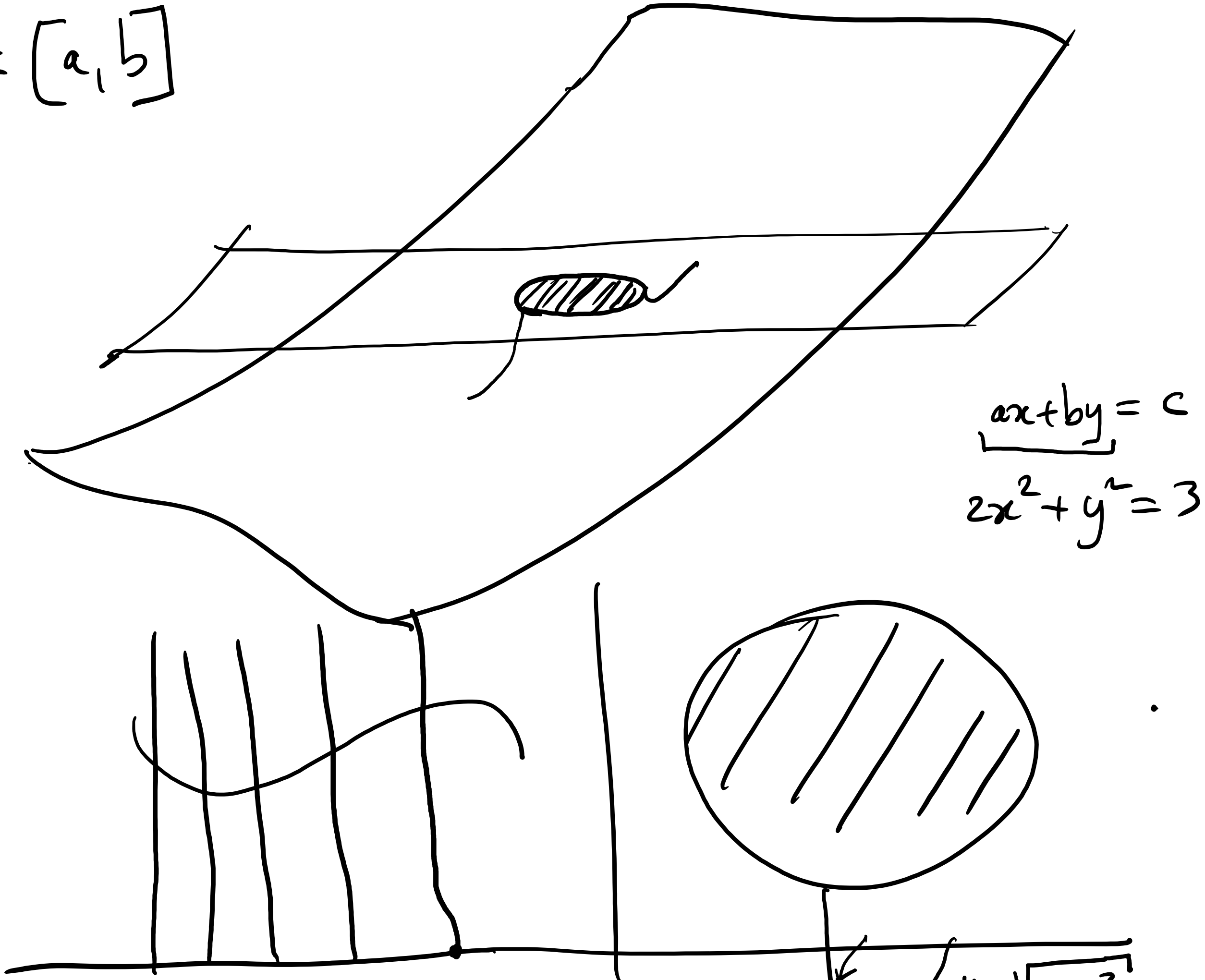








$$\Omega_k = [a,b]$$

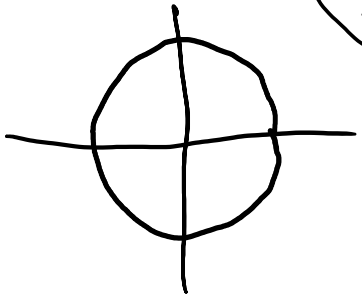


$$\underline{ax+by} = c$$

$$2x^2+y^2=3$$

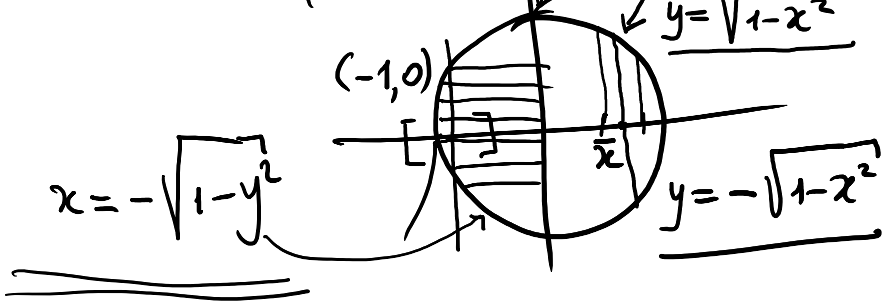
$$\boxed{x^2+y^2-1} = 0$$

$$y^2 = 1-x^2$$



$$y = \pm\sqrt{1-x^2}$$

funzione implicita

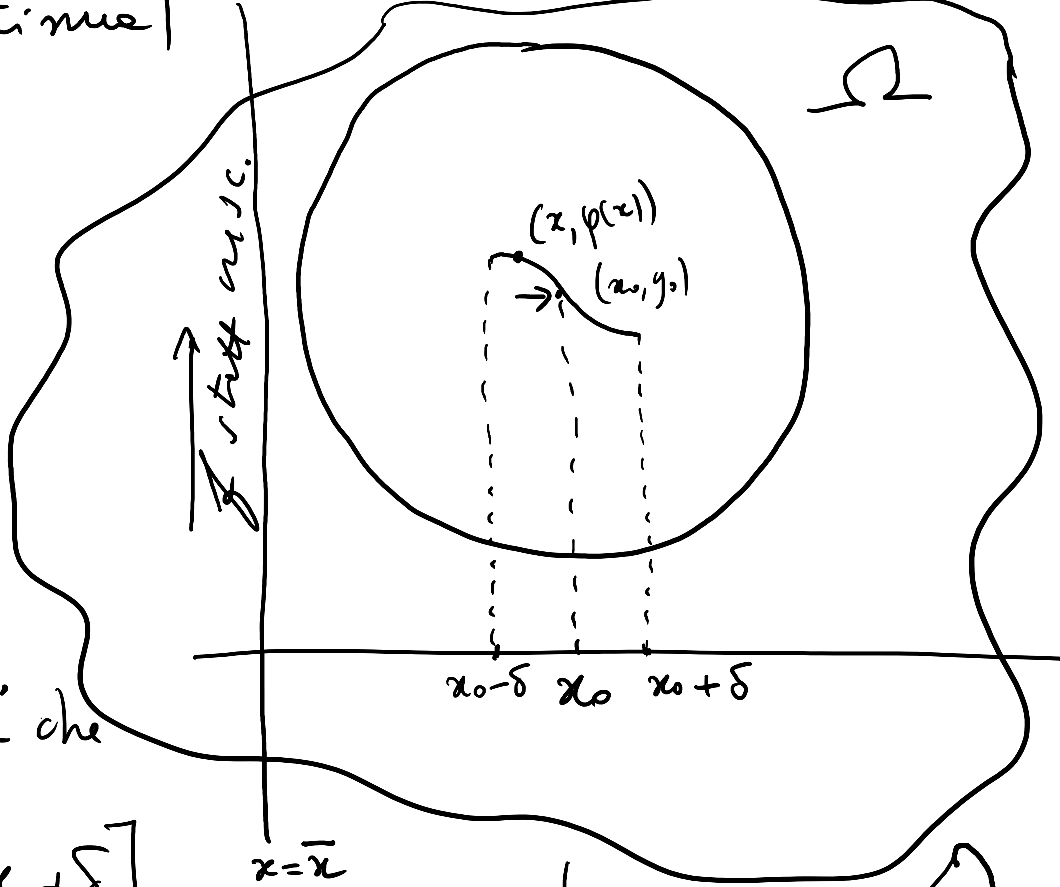


$$f(x, y) = f\left(x, \sqrt{1-x^2}\right) = x^2 + \left(\sqrt{1-x^2}\right)^2 = x^2 + 1 - x^2 = 1$$

formule risolutive dell'equazione $\underline{y^2 = 1 - x^2}$

verifica che $y = \sqrt{1-x^2}$ risolve $x^2 + y^2 - 1 = 0$

TEOREMA DELLE FUNZIONI IMPLICITE (caso f continua)

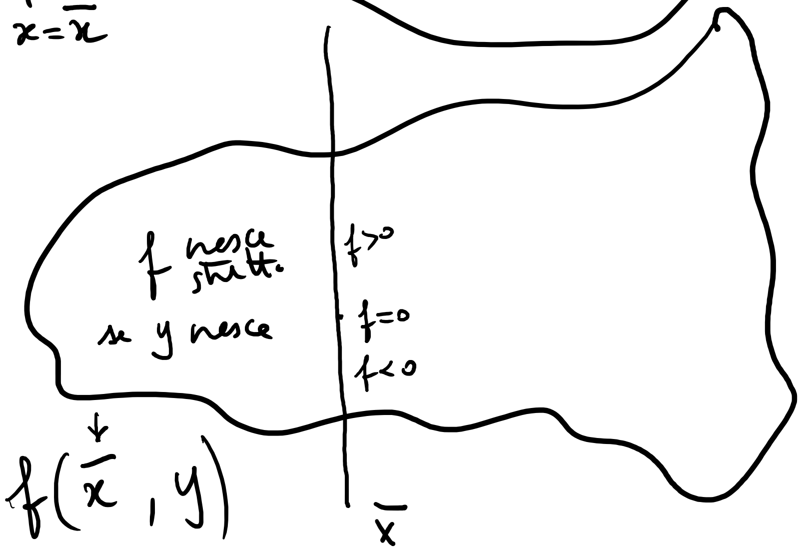


Hp

1) $f : \Omega \to \mathbb{R}$ continua in Ω
2) $f(x_0, y_0) = 0$
3) (x_0, y_0) INTERNO ad Ω
4) $y \to f(x,y)$ è strett. crescente $\forall (x,y) \in \Omega$

TS

1) $\exists \delta > 0 \quad \varphi : [x_0 - \delta, x_0 + \delta] \to \mathbb{R}$ tale che
2) $f(x, \varphi(x)) = 0 \quad \forall x \in [x_0 - \delta, x_0 + \delta]$
3) $\varphi(x_0) = y_0$
4) φ è continua in $[x_0 - \delta, x_0 + \delta]$



$y \longrightarrow f(\bar{x}, y)$