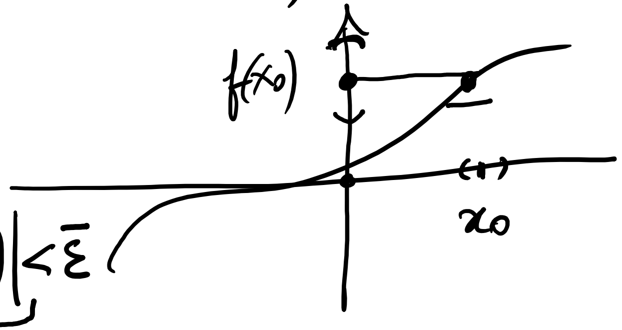
Esempi di teoremi sulle funzioni continue

$f : \Omega \to \mathbb{R}^m$ è continua in $x_0 \in \Omega$ se accade

$$\forall \varepsilon > 0 \ \exists \delta > 0 : \forall x \in \underset{\operatorname{dom} f}{\Omega} \quad |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$$

$\uparrow$

PERMANENZA DEL SEGNO $\quad f : \Omega \to \mathbb{R}$, $\Omega \subseteq \mathbb{R}^n$, continua in x_0, e verifica $f(x_0) > 0$, $\exists \rho > 0 : f(x) > 0 \quad \forall x \in B(x_0, \rho)$



DIM. $\quad \overset{\text{Scelto}}{\bar{\varepsilon}} = f(x_0) \quad \exists \delta > 0 : \forall x \in \underset{\Omega}{\operatorname{dom} f} \ |x - x_0| < \delta \quad |f(x) - f(x_0)| < \bar{\varepsilon}$

$$\rho = \delta$$

$$0 = \underbrace{f(x_0) - \bar{\varepsilon}}_{} < f(x) < f(x_0) + \bar{\varepsilon}$$

$f: \Omega \to \Sigma \qquad g: \Sigma \to H$

$\Omega \subseteq \mathbb{R}^m, \ \Sigma \subseteq \mathbb{R}^n, \ H \subseteq \mathbb{R}^p$

$h: \Omega \to H \qquad h(x) = g(f(x))$

g è continua in $f(x_0)$

f sia continua in x_0, g sia continua in $f(x_0)$

TS $h(x)$ è continua in x_0

DIM $\forall \varepsilon > 0 \ \exists \delta > 0 : \forall x \in \operatorname{dom} h \ (= \Omega) \quad |x - x_0| < \delta$

$$\Big|\underbrace{g(f(x))}_{h(x)} - \underbrace{g(f(\overbrace{x_0}))}_{h(x_0)}\Big| < \varepsilon$$

$g(y)$, $y_0 = f(x_0)$

CONTINUITA' DELLA FUNZIONE COMPOSTA

$\sigma > 0 \ \exists \delta > 0 : \forall x \in \Omega$
$|x - x_0| < \delta \quad |f(x) - f(x_0)| <$
$\qquad\qquad y - y_0$

f è continua in x_0

$\varepsilon > 0 \ \exists \sigma > 0 : \forall y \in \Sigma \ (\operatorname{dom} g) \ |y - f(x_0)| < \sigma$

$|g(y) - g(f(x_0))| < \varepsilon$

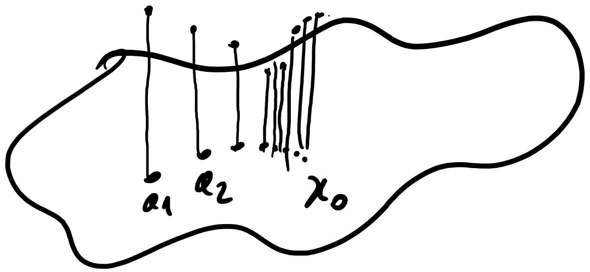
$y \in \operatorname{dom} g$ vera

$|y - f(x_0)| < \sigma$ ← vera

$$\begin{cases} \boxed{a_n \longrightarrow x_0} \\ f \text{ continua in } x_0 \end{cases}$$

$a_n \in \Omega \subseteq \mathbb{R}^M$

$f : \Omega \to \mathbb{R}^N$



$\underline{\mathrm{T}}$ $\quad f(a_n) \to f(x_0)$

$$|f(\underset{\substack{\parallel \\ x}}{a_n}) - f(\underset{x_0}{x_0})| < \varepsilon \qquad |f(x) - f(x_0)|_{\mathbb{R}^N} < \varepsilon$$

$\forall \varepsilon > 0$

$|x - x_0| < \delta \to \delta$ di continuità associato ad ε

$a_n \in \operatorname{dom} f$ $\quad$ vero!

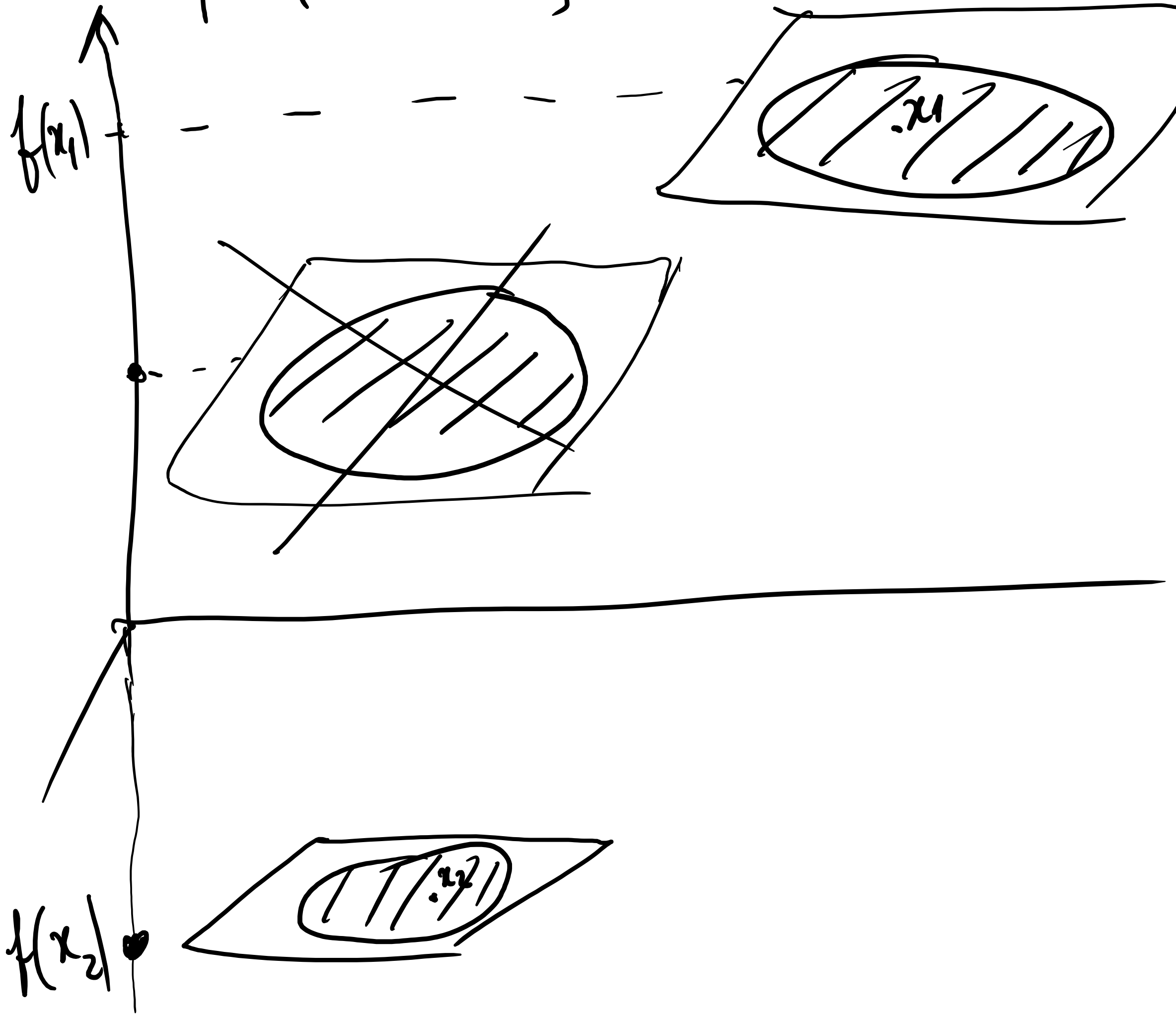
$\forall \delta > 0 \ \exists \nu : \forall n > \nu \ |a_n - x_0| < \delta$

$|a_n - x_0| < \delta$ vero purché $n > \nu$ opportuno

$$\lim_{n \to \infty} a_n = x_0 \ + \text{continuità in } x_0 \implies \lim_{n\to\infty} f(a_n) = f(x_0)$$

$f(x) = \frac{1}{x}$ $\qquad f(1) > 0 \qquad f(-1) < 0$

$\mathrm{dom} f = \{ x \neq 0 \}$

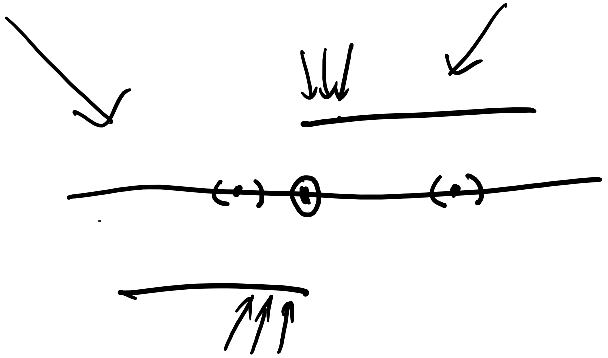


$f : \Omega \to \mathbb{R} \qquad \Omega \subseteq \mathbb{R}$

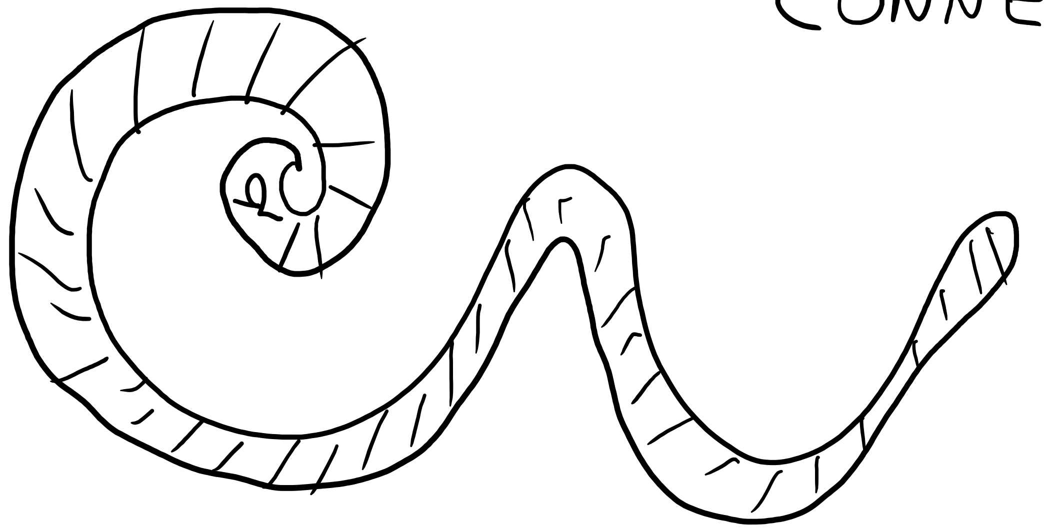
- f continua in Ω
- Ω intervallo
- $\exists\, x_1, x_2 \quad f(x_1) f(x_2) < 0$

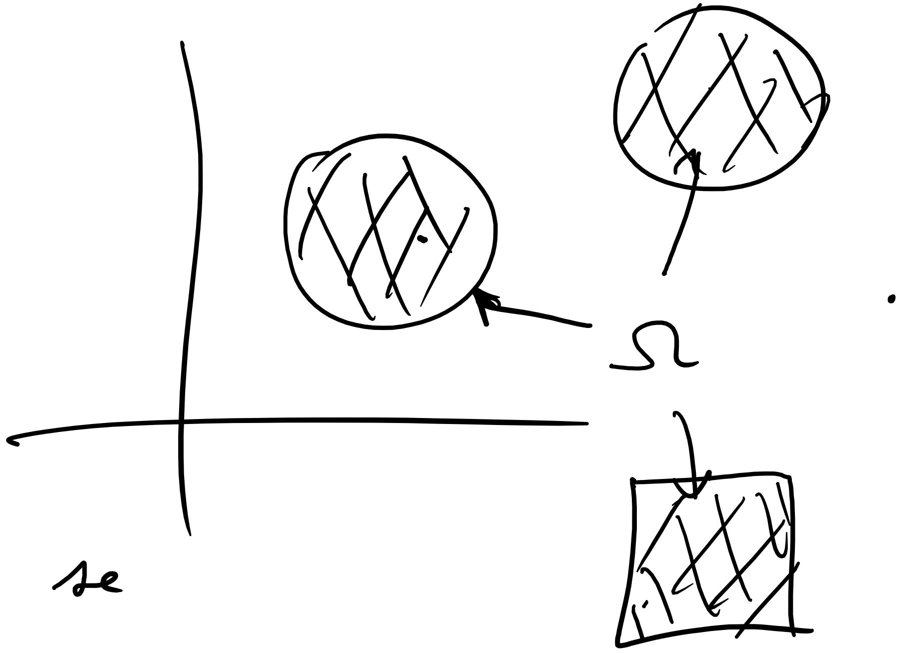
$\Rightarrow \quad \exists\, \bar{x} \in \Omega : f(\bar{x}) = 0$

$$f(x) = \frac{x}{|x|} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$



CONNESSIONE

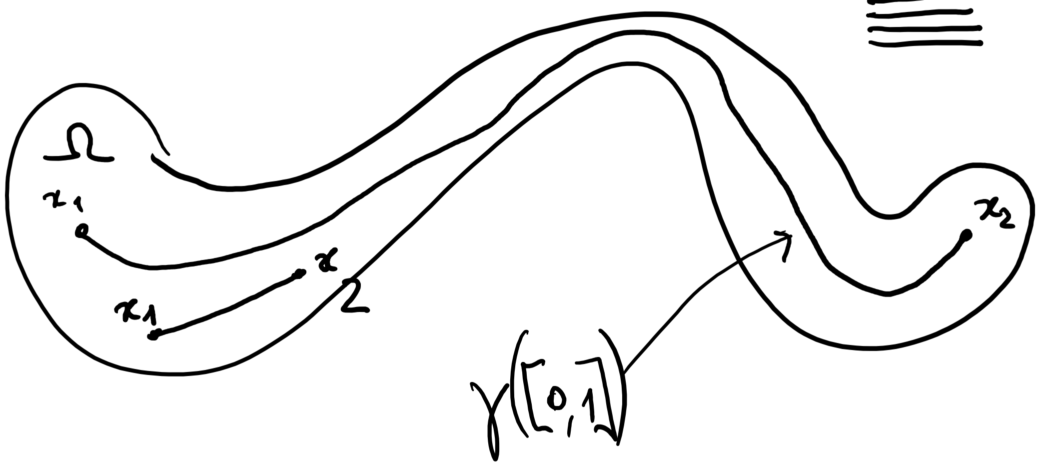


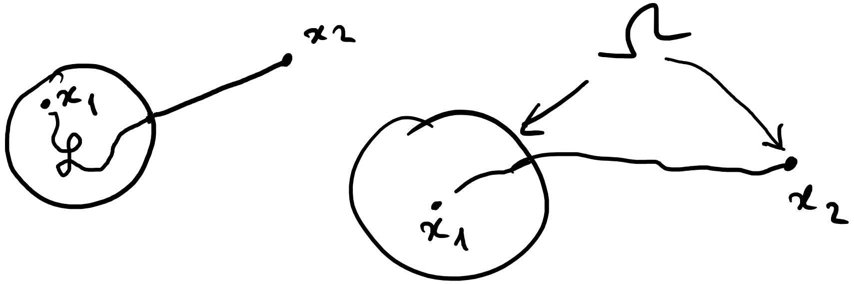


Un insieme $\Omega \subseteq \mathbb{R}^N$ si dirà CONNESSO se

$$\forall x_1, x_2 \in \Omega \quad \exists\, \gamma : [0,1] \longrightarrow \Omega, \text{ continua, tale che } \gamma(0) = x_1$$

$$\gamma(1) = x_2$$





TEOREMA (degli zeri)

$f : \Omega \to \mathbb{R} \qquad \Omega \subseteq \mathbb{R}^N$

- $\exists\, x_1, x_2 \in \Omega \ : \ f(x_1) \cdot f(x_2) < 0$
- f è continua in Ω (in ogni suo punto)
- Ω CONNESSO

Ts. $\quad \exists\, \bar{x} \in \Omega \quad f(\bar{x}) = 0$