

# CIME course “Rationality problems in Algebraic Geometry”

Preliminary abstracts of the lectures

Arnaud Beauville (Professeur émérite *Université de Nice*)

## The Lüroth problem

**Abstract:** The Lüroth problem

The Lüroth problem asks whether a unirational variety is rational. After a historical survey, I will explain the methods developed in the 70's to get a negative answer, and give some easy examples. Then I will discuss a new method introduced last year by C. Voisin.

Outline of the course:

1. History of the problem: the counter-examples.
2. The candidates: rationally connected varieties, Fano varieties.
3. The intermediate Jacobian: the Clemens-Griffiths criterion; some easy examples.
4. The torsion of  $H^3$  and the Brauer group : the Artin-Mumford example.
5. Beyond Artin-Mumford: the Chow group of 0-cycles.

Brendan Hassett (RICE UNIVERSITY)

## Hodge theory and rationality problems for fourfolds

**Abstract:** The success of the Clemens-Griffiths approach to the irrationality of the cubic threefold raises the question of whether Hodge-theoretic techniques might shed light on higher-dimensional rationality problems. If a fourfold is rational, can we detect this in its middle cohomology? Cubic fourfolds have attracted the most attention--here the relevant Hodge structures have been classified and studied from a variety of points of view. We review these Hodge-theoretic results as well as more recent moduli constructions explaining them geometrically.

Alexander Kuznetsov (Steklov Institut Moscow)

## Derived categories view on rationality problems

**Abstract:** In this course we will discuss the relation between the birational properties of algebraic varieties and the structure of their derived categories of coherent sheaves. We will discuss semiorthogonal decompositions of derived categories, their behavior under basic birational operations as well as the predictions for some particular rationality questions from this perspective.

Alessandro Verra (ROMA 3)

## Classical moduli spaces and rational parametrizations

**Abstract:** Classical algebraic geometers considered rationality problems for a rich collection of moduli of special varieties. These were origins for the study of a wider series of moduli spaces we could define as classical. They parametrize objects which are often interplaying: curves, abelian varieties, K3 surfaces. The course will focus on rational parametrizations of these moduli, building on examples from the following topics:

- 1) Rationality of some classical examples
- 2) Unirationality and moduli of curves
- 3) K3 surfaces and moduli of curves
- 4) Moduli of K3 surfaces with an involution
- 5) Moduli of theta characteristics
- 6) Prym moduli spaces and the Prym map