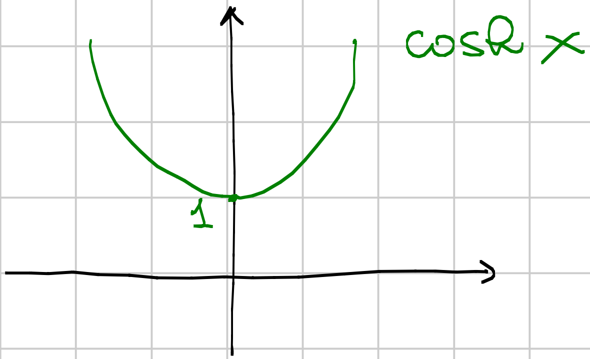
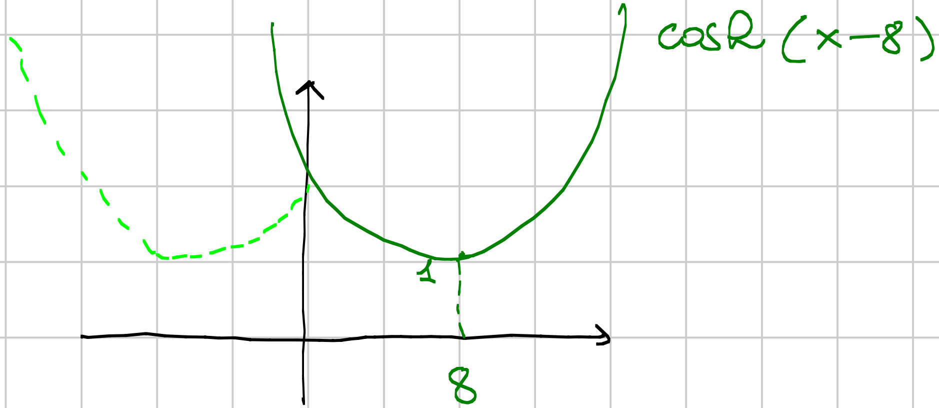
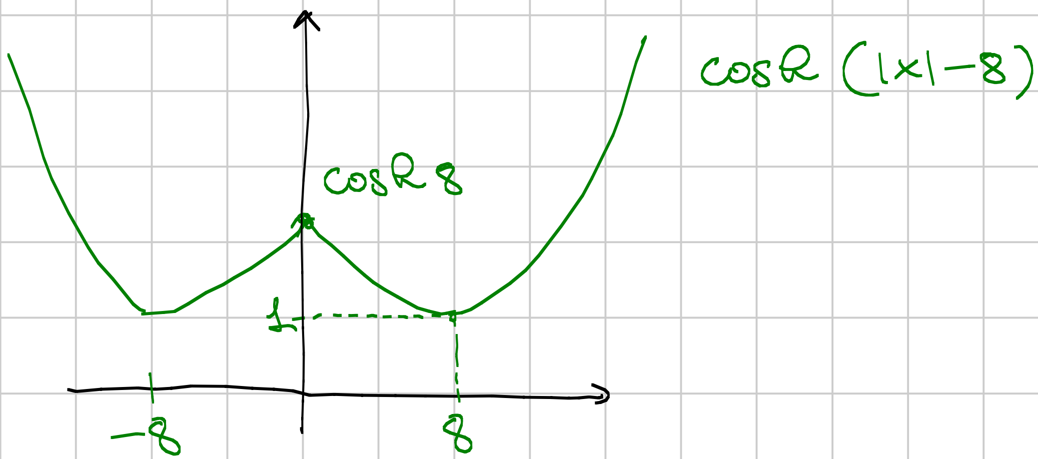
RICEVIMENTO 4

Titolo nota 08/11/2007

$$\cosh(|x|-8)=\lambda$$



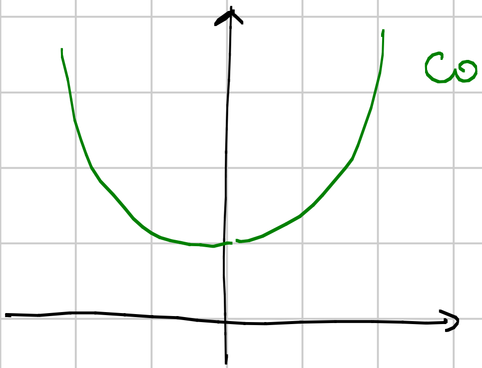




$\cosh(|x|-8)$ è una funzione pari che coincide con $\cosh(x-8)$ per $x\geq 0$

$$\cosh(8)=\cosh(-8)$$

$\cosh(x - \sin\lambda) = \lambda$



$\cosh x$

$\cosh(x - \sin\lambda)$

$\cosh(x - \boxed{\text{Mostro senza } x}) = \lambda$ ha sempre

0 soluz. per $\lambda < 1$

1 " " $\lambda = 1$

2 " " $\lambda > 1$

$\cosh x - \lambda = 8 \rightsquigarrow \cosh x = \overset{a}{\boxed{\lambda+8}}$

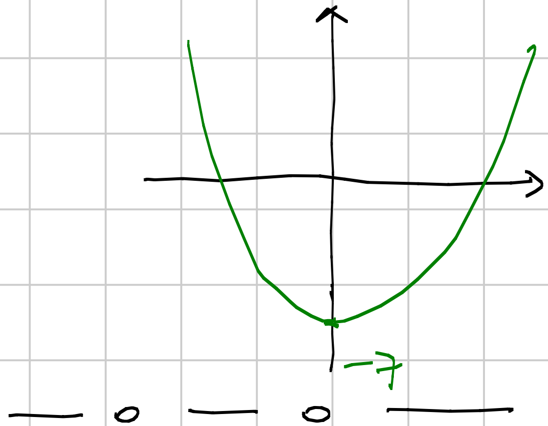
$\rightsquigarrow$ 0 sol. se $\lambda+8 < 1$ cioè $\lambda < -7$

1 sol. se $\lambda - 8 = 1$ " $\lambda = -7$

2 sol. se $\lambda - 8 > 1$ " $\lambda > -7$

$\cosh x - 8 = \lambda$

$\cosh x - 8$



$$\sum_{n=1}^{\infty} \underbrace{\boxed{\left(\sum_{k=n}^{2n} \frac{1}{k^3}\right)}}_{a_n}$$

$a_n \geq 0 \Rightarrow$ posso usare i criteri

$$a_n = \frac{1}{n^3} + \frac{1}{(n+1)^3} + \frac{1}{(n+2)^3} + \dots + \frac{1}{(2n)^3}$$

$$\leq \frac{1}{n^3} + \frac{1}{n^3} + \frac{1}{n^3} + \dots + \frac{1}{n^3}$$

$$= \boxed{\frac{n+1}{n^3}} \quad b_n$$

$\sum b_n$ conv.

$\Downarrow$

$\sum a_n$ conv.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^4+n^\alpha}{n^6+3}$$

converge $\iff \alpha < 6$

converge assolutamente $\iff \alpha < 5$

$$\sum_{n=1}^{\infty} \frac{n^4+n^\alpha}{n^6+3}$$

converge $\iff \alpha < 5$

Per $\alpha < 5$ si fa l'assoluta convergenza.

Per $\alpha \geq 6$ non è verificata la condiz. nec. $\Rightarrow$ non converge

Per $5 \leq \alpha < 6$ o si fa Leibnitz, oppure si aggiunge e toglie il termine dominante

$$\frac{n^4+n^\alpha}{n^6+3} \sim \frac{n^\alpha}{n^6} = \frac{1}{n^{6-\alpha}}$$

$$(-1)^n \frac{n^4+n^\alpha}{n^6+3} = (-1)^n \left[\frac{1}{n^{6-\alpha}} + \frac{n^4+n^\alpha}{n^6+3} - \frac{1}{n^{6-\alpha}} \right]$$

$$= \underbrace{(-1)^n \frac{1}{n^{6-\alpha}}}_{} + \underbrace{(-1)^n \left[\frac{n^4 + n^\alpha}{n^6+3} - \frac{1}{n^{6-\alpha}} \right]}_{}$$

La serie di questo converge per Leibnitz

La serie di questo dovrebbe convergere assolutamente

$$\frac{n^4+n^\alpha}{n^6+3} - \frac{1}{n^{6-\alpha}} = \frac{n^{10-\alpha} + \cancel{n^6} - \cancel{n^6} - 3}{n^{6-\alpha}(n^6+3)} = \frac{n^{10-\alpha} - 3}{n^{6-\alpha}(n^6+3)}$$

$$\sim \frac{n^{10-\alpha}}{n^{12-\alpha}} = \boxed{\frac{1}{n^2}}$$

$\Rightarrow$ CONV. ASSOLUTA

$$\sum_{n=1}^{\infty} (-1)^n \boxed{\frac{1+\frac{(-1)^n}{\sqrt{n}}}{\sqrt{n}}} \to a_n \qquad a_n \sim \frac{1}{\sqrt{n}}$$

Brutalmente: $\sum (-1)^n a_n \sim \sum (-1)^n \frac{1}{\sqrt{n}}$ Leibniz $\Downarrow$ converge

<u>Rigoroso</u>

$$(-1)^n a_n = (-1)^n \left[\frac{1}{\sqrt{n}} + \frac{(-1)^n}{n}\right] = \frac{(-1)^n}{\sqrt{n}} + \frac{1}{n}$$

$$\sum (-1)^n a_n = \boxed{\sum \frac{(-1)^n}{\sqrt{n}}} + \boxed{\sum \frac{1}{n}} \longrightarrow \text{DIVERGE}$$

conv. per Leibniz

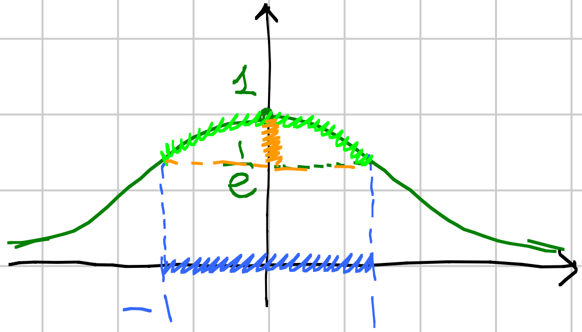
Diverge

$f(x) = e^{-x^2}$

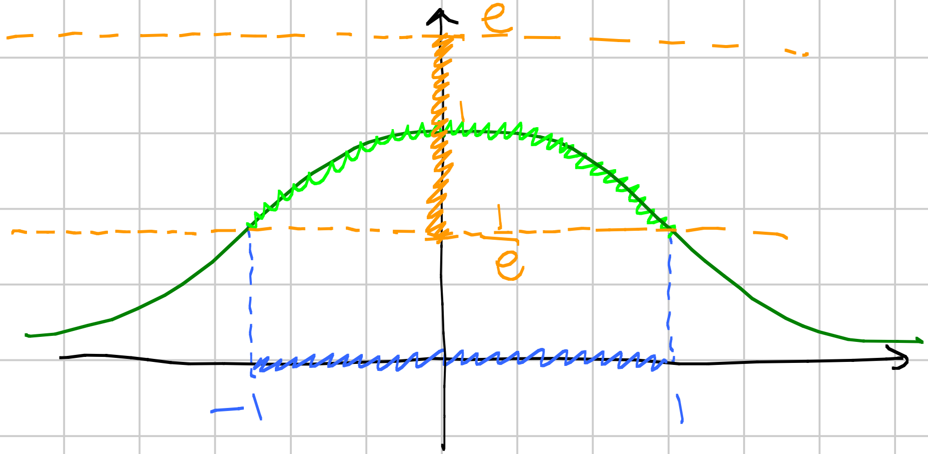
$f([-1,1])$

$f(A)$ con $A = [-1,1]$

$= \left[\frac{1}{e}, 1\right]$



$f^{-1}\left(\left[\frac{1}{e}, e\right]\right)$



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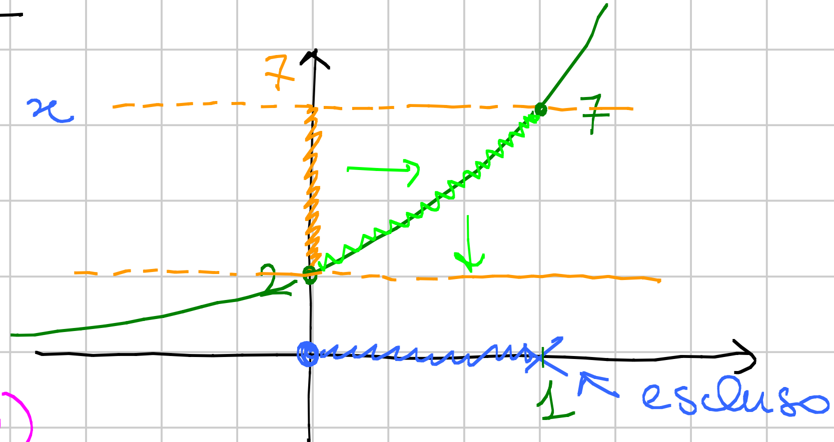
$\{x \in \mathbb{R} : 2 \leq 3^x + 4^x < 7\}$ MONDO x

$2 \leq f(x) < 7$

cerca i valori di x per cui $f(x)$ sta fra 2 e 7

$f^{-1}([2,7))$



$$\{ x \in \mathbb{R} : \ 0 \le \underbrace{x + \sin x}_{f(x)} \le \pi \}$$

$f(x)$ è strett. cresc.