$$\sum_{n=1}^{\infty} \underbrace{\boxed{\left(1+\frac{1}{n}\right)^n}}_{a_n}$$

$a_n \to e \Rightarrow$ NO cond. nec. $\Rightarrow$ non converge

$$\sum \frac{|2n^2-n^3| - |n^3-1000|}{n^4-8}$$

$2n^2-n^3 \leq 0$ defin.

$|2n^2-n^3| = -2n^2+n^3$ defin.

$n^3-1000 \geq 0$ defin.

$|n^3-1000| = n^3-1000$ defin.

Defin.

$$\frac{||-||}{n^4-8} = \frac{-2n^2+\cancel{n^3}-\cancel{n^3}+1000}{n^4-8}$$

$\sim \frac{1}{n^2} \Rightarrow$ converge

$$n^3 \left\{ \sin\frac{1}{n^2} - \sin\frac{1}{n^2+n} \right\} =$$

$$n^3 \left\{ \frac{1}{n^2} - \frac{1}{n^2+n} + o\left(\frac{1}{n^4}\right) \right\}$$

$$\underset{\substack{\downarrow \\ 1}}{\boxed{n^3 \frac{\cancel{n^2}+n-\cancel{n^2}}{n^2(n^2+n)}}} + \underset{\substack{\downarrow \\ 0}}{\boxed{n^3\, o\left(\frac{1}{n^4}\right)}}$$

$$n^3 o\left(\frac{1}{n^4}\right) = \cancel{n^3} \cdot \underset{\substack{\downarrow \\ 0}}{\boxed{\frac{1}{n^{\cancel{4}}}}}\ \underset{\substack{\downarrow \\ 0}}{\boxed{\omega(n)}}$$

$$\sin x \sim x + o(x^2)$$

$$\sin\frac{1}{n^2} \sim \frac{1}{n^2} + o\left(\frac{1}{n^4}\right)$$

$$\sin\frac{1}{n^2+n} \sim \frac{1}{n^2+n} + o\left(\frac{1}{n^4}\right)$$

$$n^3\, o\left(\frac{1}{n^4}\right) = o\left(\frac{1}{n}\right)$$

$$n^3\, o\left(\frac{1}{n^4}\right) = \frac{o\left(\frac{1}{n^4}\right)}{\frac{1}{n^3}} = \underset{\substack{\downarrow \\ 0}}{\boxed{\frac{o\left(\frac{1}{n^4}\right)}{\frac{1}{n^4}}}} \cdot n \to 0$$

$$\lim_{x\to\frac{\pi}{3}} \frac{2\cos x - 1}{2\sin x - \sqrt{3}} =$$

$y = x - \frac{\pi}{3}$ $\quad$ $x = y + \frac{\pi}{3}$

Quando $x \to \frac{\pi}{3}$ ho che $y \to 0$

$$= \lim_{y\to 0} \frac{2\cos\left(y+\frac{\pi}{3}\right) - 1}{2\sin\left(y+\frac{\pi}{3}\right) - \sqrt{3}} = \lim_{y\to 0} \frac{2\cos y \cos\frac{\pi}{3} - 2\sin y \sin\frac{\pi}{3} - 1}{2\sin y \cos\frac{\pi}{3} + 2\cos y \sin\frac{\pi}{3} - \sqrt{3}}$$

$$= \lim_{y\to 0} \frac{\cos y - \sqrt{3}\sin y - 1}{\sin y + \sqrt{3}\cos y - \sqrt{3}} =$$

$$= \lim_{y\to 0} \frac{\cancel{y}\left[\overset{\nearrow 0}{\boxed{\frac{\cos y - 1}{y}}} - \sqrt{3}\overset{\nearrow 1}{\boxed{\frac{\sin y}{y}}}\right]}{\cancel{y}\left[\underset{\downarrow 1}{\boxed{\frac{\sin y}{y}}} + \sqrt{3}\underset{\downarrow 0}{\boxed{\frac{\cos y - 1}{y}}}\right]} = -\sqrt{3}$$

$$\left(2+\frac{1}{n}\right)^n - 2^n = \left[2\left(1+\frac{1}{2n}\right)\right]^n - 2^n =$$

$$= 2^n\left(1+\frac{1}{2n}\right)^n - 2^n = 2^n\left[\left(1+\frac{1}{2n}\right)^n - 1\right]$$

$$= 2^n\left[e^{n\log\left(1+\frac{1}{2n}\right)} - 1\right] \text{“}\longrightarrow\text{”}\ 2^{\infty}\left(e^{\frac{1}{2}}-1\right) = +\infty$$

$$n\log\left(1+\frac{1}{2n}\right) = \underbrace{\boxed{\frac{\log\left(1+\frac{1}{2n}\right)}{\frac{1}{2n}}}}_{\substack{\downarrow \\ 1}} \cdot 2 \longrightarrow \frac{1}{2}$$

$$n\log\left(1+\frac{1}{2n}\right) = \underbrace{n\left[\frac{1}{2n}+\cdots\right]}_{\substack{\downarrow \\ \frac{1}{2}}}$$

$$\log(1+t) = t + \cdots$$

$$\lim_{x\to 0} \frac{40^x - \cos x}{2x} \qquad \cos x = 1 + o(x)$$

$$40^x = e^{x\log 40} = 1 + x\log 40 + o(x)$$

$$\frac{40^x - \cos x}{2x} = \frac{\cancel{1} + x\log 40 - \cancel{1} + o(x)}{2x} \to \frac{1}{2}\log 40$$

$$\frac{40^x - \cos x}{2x} = \underbrace{\boxed{\frac{40^x - 1}{2x}}}_{\log 40} + \underbrace{\boxed{\frac{1-\cos x}{2x^2}}}_{\frac{1}{2}} \cdot \underbrace{\boxed{x}}_{0} \to \frac{1}{2}\log 40$$

— o —

$$n\left(\sqrt[n]{3} - \sqrt[n]{2}\right) = \qquad 3^{\frac{1}{n}} = e^{\frac{1}{n}\log 3} = 1 + \frac{1}{n}\log 3 + o\left(\frac{1}{n}\right)$$

$$= n\left(\cancel{1} + \frac{\log 3}{n} - \cancel{1} - \frac{\log 2}{n} + o\left(\frac{1}{n}\right)\right) \to \log 3 - \log 2$$

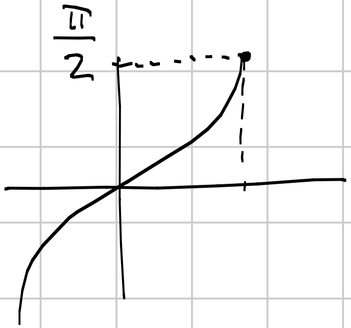
$$n\, o\left(\frac{1}{n}\right) = \frac{o\left(\frac{1}{n}\right)}{\frac{1}{n}} \to 0$$

$$n\left(\sqrt[n]{3}-1+1-\sqrt[n]{2}\right) = \frac{\sqrt[n]{3}-1}{\frac{1}{n}} - \frac{\sqrt[n]{2}-1}{\frac{1}{n}}$$

$$= \frac{3^{\frac{1}{n}}-1}{\frac{1}{n}} - \frac{2^{\frac{1}{n}}-1}{\frac{1}{n}}$$

—o—o—

$$\boxed{n\left\{\arcsin\left(1-\frac{1}{n}\right)-\arcsin\left(1-\frac{2}{n}\right)\right\}}$$



$$\arcsin(1-x) = \frac{\pi}{2} - c\sqrt{x} + o(x)$$

$$\frac{\arcsin(1-x)-\frac{\pi}{2}+c\sqrt{x}}{x} \to 0$$

$$\boxed{\frac{\arcsin(1-x)-\arcsin(2-x)}{x}}$$

$$\lim_{x \to -\infty} (\underset{\substack{\downarrow \\ +\infty}}{x^2} \underset{\substack{\downarrow \\ +\infty}}{\boxed{-4x}}) = +\infty$$

$$\underset{\substack{\downarrow \\ 0}}{\boxed{-\frac{n}{n^2+2}}} \leq \underset{\substack{\rightsquigarrow \\ 0}}{\boxed{(-1)^n \frac{n}{n^2+2}}} \leq \underset{\substack{\downarrow \\ 0}}{\boxed{\frac{n}{n^2+2}}}$$

$$-1 \leq (-1)^n \leq 1$$ moltiplico per $\frac{n}{n^2+2}$

$$\lim_{x \to -\infty} \sqrt{x^2+x+3} + x =$$

$y = -x$

Quando $x \to -\infty$ ho che $y \to +\infty$

$$\lim_{y \to +\infty} \sqrt{y^2-y+3} - y$$

$$\sqrt{y^2-y+3} - y \quad \frac{\sqrt{} + y}{\sqrt{} + y} = \frac{\cancel{y^2} - y + 3 - \cancel{y^2}}{\sqrt{y^2-y+3} + y} \sim \frac{y}{2y} \to \frac{1}{2}$$

$$\frac{y\left(-1+\frac{3}{y}\right)}{\sqrt{y^2\left(1-\frac{1}{y}+\frac{3}{y^2}\right)}+y} = \frac{\cancel{y}\left(-1+\frac{3}{y}\right)}{\cancel{y}\sqrt{1-\frac{1}{y}+\frac{3}{y^2}}+\cancel{y}}$$

$$= \frac{-1+\frac{3}{y}\nearrow 0}{\sqrt{1-\underset{\underset{0}{\downarrow}}{\frac{1}{y}}+\underset{\underset{0}{\downarrow}}{\frac{3}{y^2}}}+1} \longrightarrow -\frac{1}{2}.$$

—o—o—

$$n^2\log\left(\frac{n^2+1}{n^2-1}\right) = n^2\log\left(\frac{n^2-1+2}{n^2-1}\right)$$

$$= n^2\log\left(1+\frac{2}{n^2-1}\right) = \boxed{n^2}\;\underset{\underset{1}{\downarrow}}{\boxed{\frac{\log\left(1+\frac{2}{n^2-1}\right)}{\frac{2}{n^2-1}}}}\cdot\frac{2}{\boxed{n^2-1}} \to 2$$

$$n^2\log\left(1+\frac{2}{n^2-1}\right) \sim n^2\cdot\frac{2}{n^2-1} \to 2.$$

$$\lim_{n\to+\infty} \frac{\log(1+n^2)}{n^2} = 0$$

$$\lim_{x\to+\infty}\left(\frac{x^2-2x-1}{x^2+3}\right)^{\frac{x^2}{x-1}} \qquad [1^\infty]$$

$$= \lim_{x\to+\infty} e^{\frac{x^2}{x-1}\log\left(\frac{x^2-2x-1}{x^2+3}\right)} = e^{-2} = \frac{1}{e^2}$$

Limite dell' esponente:

$$\frac{x^2}{x-1}\log\left(\frac{x^2-2x-1}{x^2+3}\right) = \frac{x^2}{x-1}\log\left(\frac{x^2+3-2x-4}{x^2+3}\right) =$$

$$= \frac{x^2}{x-1} \underset{\substack{\downarrow \\ 1}}{\boxed{\frac{\log\left(1+\frac{-2x-4}{x^2+3}\right)}{\frac{-2x-4}{x^2+3}}}} \quad \frac{-2x-4}{x^2+3} \longrightarrow -2$$