RICEVIMENTO 1

Titolo nota 11/10/2007

$f : \mathbb{Q} \to \mathbb{Q} \cap \mathbb{R}_{\geq 0} = \mathbb{Q}_{\geq 0}$ $\quad f(x) = x^4$

INIETTIVA : NO perché $f(3) = f(-3)$

(in generale $f(a) = f(-a)$)

SURGETTIVA : NO se lo fosse vorrebbe dire che
$\forall b \in \underset{\text{arrivo}}{\mathbb{Q}_{\geq 0}} \quad \exists a \in \underset{\text{partenza}}{\mathbb{Q}} \quad$ t.c. $f(a) = b$

se prendo $b = 2$ $\quad f(a) = 2 \quad a^4 = 2$ e

non esiste nessun $a \in \mathbb{Q}$ t.c. $a^4 = 2$

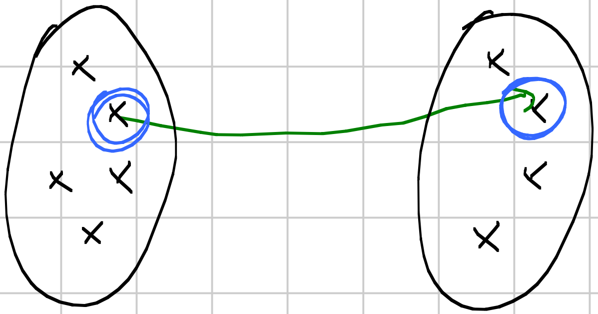
$f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \sqrt{\arcsin\left(4^{-x^2}\right)}$$

$$f\left(\left\{2^{-\frac{1}{2}}\right\}\right) = \left\{f\left(\frac{1}{\sqrt{2}}\right)\right\} = \left\{\sqrt{\arcsin\left(4^{-\frac{1}{2}}\right)}\right\}$$

$$= \left\{\sqrt{\arcsin\left(\frac{1}{2}\right)}\right\}$$

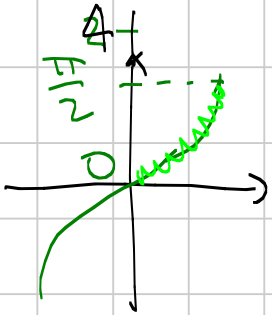
$$= \left\{\sqrt{\pi/6}\right\}$$



$f^{-1}([0,2])$ = cercare i valori di x t.c. $0 \le f(x) \le 2 = \mathbb{R}$

cioè $0 \le \sqrt{\dots} \le 2 \Rightarrow 0 \le \arcsin\left(4^{-x^2}\right) \le 4$

$$0 \le \arcsin(\quad) \le \frac{\pi}{2}$$



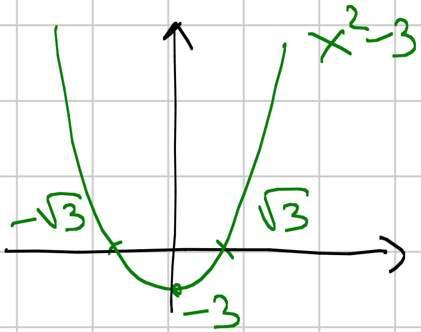
$\Rightarrow 0 \underset{\uparrow \text{sempre}}{\le} 4^{-x^2} \underset{\uparrow \text{sempre}}{\le} 1$ $\quad 4^{-x^2} \le 4^0 \quad -x^2 \le 0$

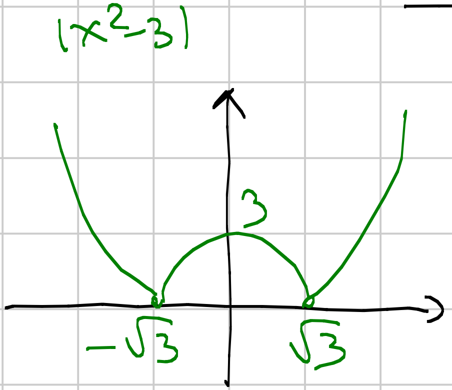
$f : [1,2] \to \mathbb{R}$

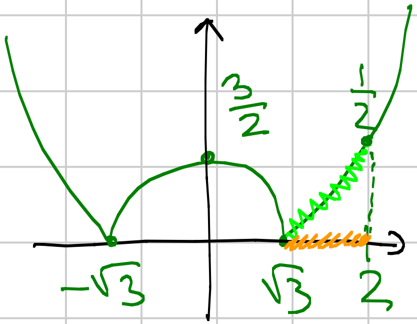
$$f(x) = \arccos\left(\overset{z}{\boxed{\frac{|x^2-3|}{2}}}\right)$$

$f([\sqrt{3},2])$

$$\frac{|x^2-3|}{2}$$

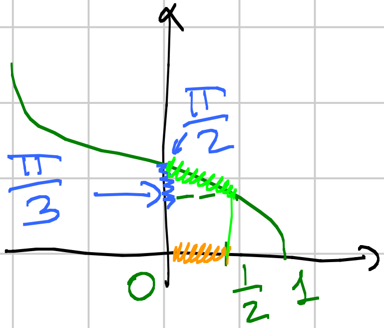






Quando x varia in $[\sqrt{3},2]$, dove varia $\frac{|x^2-3|}{2}$?

Varia in $[0,\frac{1}{2}]$. Quando z varia in $[0,\frac{1}{2}]$, dove varia $\arccos z$?



$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Varia in $[\frac{\pi}{3}, \frac{\pi}{2}]$

$f^{-1}\left(\left[0, \frac{\pi}{3}\right]\right)$ cerco gli x tali che

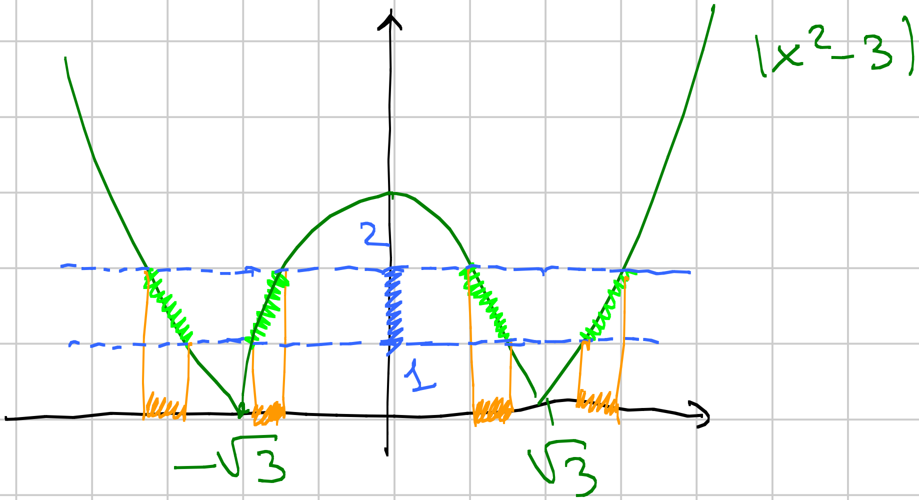
$$0 \le f(x) \le \frac{\pi}{3}$$

$\uparrow$ sempre verificata perché arccos ≥ 0 sempre

Resta da risolvere

$$\arccos\left(\frac{|x^2-3|}{2}\right) \le \frac{\pi}{3} \implies \frac{1}{2} \le \frac{|x^2-3|}{2} \le 1$$

$$\implies 1 \le |x^2-3| \le 2$$



Si tratta di risolvere

$|x^2-3|=1$ $\quad x^2-3=1$

$x^2-3=-1$

$|x^2-3|=2$ $\quad x^2-3=2$

$x^2-3=-2$

$$\frac{|3n^2 - n^3| - 7n}{|6n - 95| + 5n^3}$$

$3n^2 - n^3$ definitivamente è negativo, quindi def.

$$|3n^2 - n^3| = n^3 - 3n^2$$

Definitiv.:

$$\frac{n^3 - 3n^2 - 7n}{6n - 95 + 5n^3} \to \frac{1}{5}$$

— o — o —

$$\frac{(4n)^{2n}}{(3n)!} = \frac{16^n \cdot n^{2n}}{(3n)!}$$

Rapporto: $\dfrac{16^{\cancel{n+1}}(n+1)^{2n+2}}{(3n+3)!} \quad \dfrac{(3n)!}{\cancel{16^{n}} \cdot n^{2n}}$

$$\frac{16\,(n+1)^{2n}(n+1)^2}{n^{2n}} \quad \frac{\cancel{(3n)!}}{(3n+3)(3n+2)(3n+1)\cancel{(3n)!}} =$$

$$16\,(n+1)^2 \left(\frac{n+1}{n}\right)^{2n} \frac{1}{\text{pot. 3° grado}} = 16\,(n+1)^2 \underbrace{\boxed{\left(1+\frac{1}{n}\right)^{2n}}}_{\hookrightarrow e^2} \frac{1}{\text{terzogr.}} \to 0$$

$$\lim_{x\to 0} \frac{\sin(3x)}{x^4} \qquad \lim_{x\to 0^+} = +\infty \qquad \lim_{x\to 0^-} = -\infty$$

$$\frac{\sin(3x)}{x^4} = \underset{\substack{\downarrow \\ 1}}{\boxed{\frac{\sin(3x)}{3x}}} \cdot \underset{\substack{\downarrow \\ \frac{3}{x^3}}}{\boxed{\frac{3x}{x^4}}}$$

$$\frac{\sin(3x)}{x^3} = \underset{\substack{\downarrow \\ 1}}{\boxed{\frac{\sin(3x)}{3x}}} \cdot \underset{\substack{\downarrow \\ +\infty}}{\boxed{\frac{3}{x^2}}} \qquad \left[\frac{3}{0^+}\right]$$

—— o —— o ——

$$\frac{\sqrt[n]{n!}}{n} = \sqrt[n]{\frac{n!}{n^n}}$$

Rapp → rad

$$\frac{(n+1)\cancel{!}}{(n+1)^{n+1}} \cdot \frac{n^n}{\cancel{n!}} = \frac{\cancel{(n+1)} \cdot n^n}{\cancel{(n+1)}(n+1)^n} = \frac{1}{\left(\frac{n+1}{n}\right)^n} = \frac{1}{\left(1+\frac{1}{n}\right)^n} \to \frac{1}{e}$$

$$\lim_{x\to 0} \left(1+\tan x\right)^{\log^2 x}$$

$$e^{\log^2 x \cdot \log(1+\tan x)} \nearrow e^0 = 1$$

basta fare il limite dell'esponente

$$\boxed{\log^2 x \cdot \underbrace{\boxed{\frac{\log(1+\tan x)}{\tan x}}}_{\downarrow\, y=\tan x \atop 1} \cdot \underbrace{\boxed{\frac{\tan x}{x}}}_{\downarrow \atop 1} \cdot x} \to 0$$

—o—o—

$$\binom{3n}{n} - \binom{2n}{n} = \underbrace{\boxed{\binom{3n}{n}}}_{\downarrow \atop +\infty} \left\{ 1 - \underbrace{\boxed{\frac{\binom{2n}{n}}{\binom{3n}{n}}}}_{\downarrow ? \atop 0} \right\}$$

rapporto

← se fosse così, il limite originario è $+\infty$

$$\frac{\binom{2n}{n}}{\binom{3n}{n}} \overset{\text{DEF}}{\underset{\downarrow}{=}} \frac{\dfrac{(2n)!}{n!\,n!}}{\dfrac{(3n)!}{(2n)!\,n!}} = \frac{(2n)!}{n!\,\cancel{n!}} \cdot \frac{(2n)!\,\cancel{n!}}{(3n)!}$$

Rapporto: $$\frac{(2n+2)!\,(2n+2)!}{(n+1)!\,(3n+3)!} \quad \frac{(3n)!\,n!}{(2n)!\,(2n)!}$$

$$= \frac{(2n+2)(2n+1)\cancel{(2n)!}\,(2n+2)(2n+1)\cancel{(2n)!}}{(n+1)\,\cancel{n!}\,(3n+3)(3n+2)(3n+1)\cancel{(3n)!}} \cdot \frac{\cancel{(3n)!}\,\cancel{n!}}{\cancel{(2n)!}\,\cancel{(2n)!}}$$

$$\longrightarrow \frac{16}{27} < 1 \qquad \text{quindi} \qquad \frac{(\ )}{(\ )} \to 0$$

$$\underline{\qquad}\ \circ\ \underline{\qquad}\ \circ\ \underline{\qquad}$$

$$n^{n!} - (n!)^n = \underset{\substack{\downarrow \\ +\infty}}{\boxed{n^{n!}}} \left(1 - \underset{\substack{\downarrow \\ 0}}{\boxed{\frac{(n!)^n}{n^{n!}}}}\right) \quad ? \leftarrow$$

se è così, il limite originario è $+\infty$

$$\frac{(n!)^n}{n^{n!}}$$

Scorciatoia: $(n!) \le n^n$ quindi

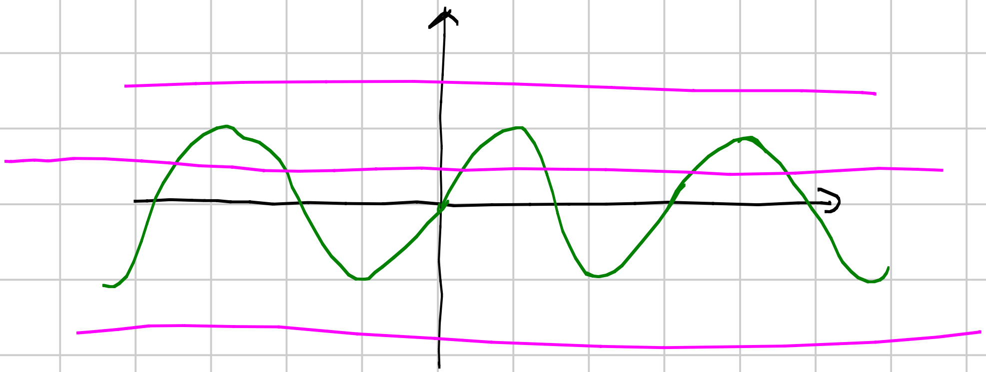
$$(n!)^n \le (n^n)^n = n^{n^2}$$

Divido per $n^{n!}$ e ottengo

$$\underset{\substack{\downarrow \\ 0}}{\boxed{0}} \le \underset{\substack{\rightsquigarrow \\ 0}}{\boxed{\frac{(n!)^n}{n^{n!}}}} \le \frac{n^{n^2}}{n^{n!}} = \underset{\substack{\downarrow \\ 0}}{\boxed{\frac{1}{n^{n!-n^2}}}}$$

—o—o—

$\sin x = \lambda$



$$\left(\sqrt{n^2+3m} - m\right) \qquad \sqrt[3]{m^3+8m^2} - m$$

$$\sqrt{u^2+3m} - \sqrt{m^2} \qquad \frac{\sqrt{}+\sqrt{}}{\sqrt{}+\sqrt{}} \qquad \sqrt[3]{A} - \sqrt[3]{B}$$

$$\sqrt{A}-\sqrt{B} \quad \frac{\sqrt{A}+\sqrt{B}}{\sqrt{A}+\sqrt{B}} = \frac{A-B}{\sqrt{A}+\sqrt{B}}$$

$$(a-b)(a+b) = a^2-b^2$$

$$(a-b)(a^2+ab+b^2) = a^3-b^3$$

$$a = \sqrt[3]{A}$$
$$b = \sqrt[3]{B}$$

$$\left(\sqrt[3]{A}-\sqrt[3]{B}\right)\left(\sqrt[3]{A^2}+\sqrt[3]{AB}+\sqrt[3]{B^2}\right) = A-B$$

$$\left(\sqrt[3]{u^3+8m^2} - \sqrt[3]{m^3}\right)\frac{\sqrt[3]{(u^3+8u^2)^2}+\sqrt[3]{(u^3+8u^2)m^3}+\sqrt[3]{n^6}}{}$$

$$\frac{\cancel{n^3}+8n^2-\cancel{n^3}}{\sqrt[3]{}+\sqrt[3]{}+\sqrt[3]{}} \sim \frac{8n^2}{n^2+n^2+n^2} \sim \frac{8}{3}$$