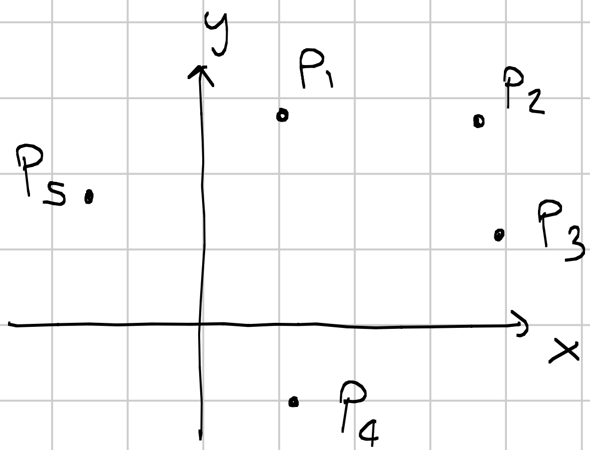
BARICENTRI

In fisica il BARICENTRO di un sistema di n punti nel piano o nello spazio è un pto G di coordinate



nel p.to P_i ci sia una massa m_i

$$G = \left(\frac{\sum m_i x_i}{M} , \frac{\sum m_i y_i}{M} \right)$$

$P_i = (x_i, y_i)$ $\qquad$ M = massa totale $= \sum m_i$

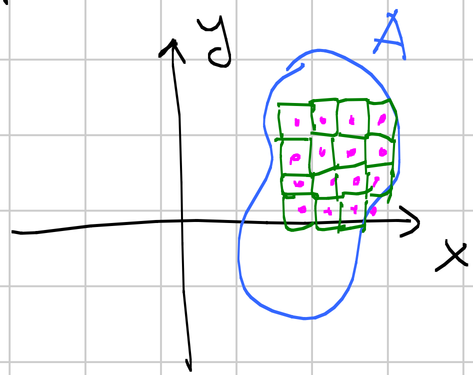
Le coordinate di G sono la media delle coordinate dei vari P_i, pesate con le masse m_i.

Se tutti i punti hanno la stessa massa m, allora

$$G = \left(\frac{x_1 + \dots + x_n}{n}, \frac{y_1 + \dots + y_n}{n} \right)$$

Nello spazio stessa cosa con 3 coordinate.

Se invece di avere n pti abbiamo una figura piana A si può pensare di suddividere la figura in tanti quadratini e considerarli come fossero masse puntiformi concentrate nel centro del quadrato.



Passando al limite si ottiene

$$G = (x_G, y_G) \text{, dove}$$

$$\boxed{x_G = \frac{1}{\text{Area}(A)} \iint_A x \, dx\, dy} \qquad \boxed{y_G = \frac{1}{\text{Area}(A)} \iint_A y \, dx\, dy}$$

supponendo A costruito di un materiale di densità unif.

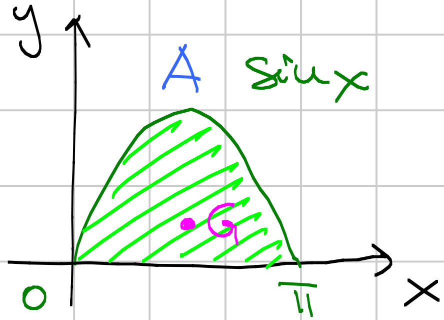
Nello spazio il baricentro di un solido $V \subseteq \mathbb{R}^3$ è il p.to

$G = (x_G, y_G, z_G)$, dove

$$x_G = \frac{1}{\mathrm{Vol}(V)} \iiint_V x \, dx\,dy\,dz$$ e idem per y_G e z_G.

<u>Esempio 1</u> $\quad x_G = \frac{\pi}{2}$ per ragioni di simmetria

(controllare con il calcolo)



Per calcolare y_G serve l'area $\quad$ Area $(A) = 2$

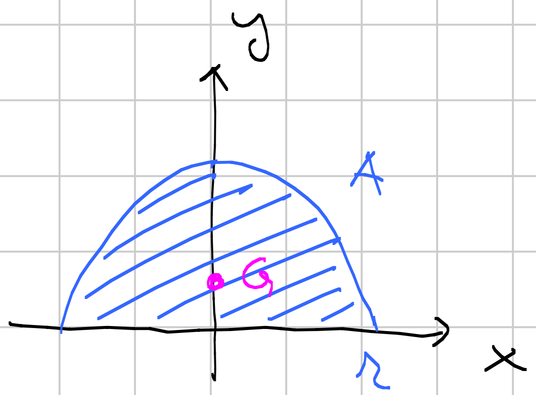
$$\iint_A y \, dx\,dy = \int_0^{\pi} dx \int_0^{\sin x} dy \cdot y = \int_0^{\pi} dx \left[\frac{y^2}{2} \right]_{y=0}^{y=\sin x} =$$

$A = \{ (x,y) \in \mathbb{R}^2 : x \in [0,\pi], \ 0 \leq y \leq \sin x \}$

$$= \frac{1}{2} \int_0^{\pi} \sin^2 x \, dx = \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}$$

Quindi $y_G = \frac{1}{\text{Area}} \cdot \frac{\pi}{4} = \frac{\pi}{8}$

$\boxed{\text{Esempio 2}}$ Semicerchio



$x_G = 0$ per simmetria

$$\text{Area}(A) = \frac{\pi r^2}{2}$$

PAG

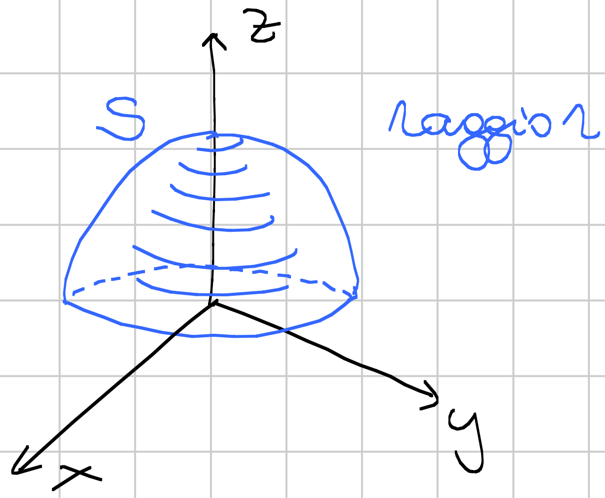
$$\iint_A y \, dx \, dy = \int_0^r d\rho \int_0^{\pi} d\theta \, \rho \sin\theta \cdot \rho = \int_0^r \rho^2 d\rho \underbrace{\int_0^{\pi} \sin\theta \, d\theta}_{2}$$

$$= 2 \int_0^r \rho^2 d\rho = 2 \left[\frac{\rho^3}{3} \right]_{\rho=0}^{\rho=r} = \frac{2}{3} r^3 \Rightarrow y_G = \frac{2}{3} r^3 \cdot \frac{2}{\pi r^2} = \frac{4}{3\pi} r$$

Esempio 3 **Semisfera**

Per simmetria $x_G = y_G = 0$

z_G ci aspettiamo che dipenda da r



Volume$(S) =$

$$= \iiint_S 1 \, dx\,dy\,dz =$$

S: $\rho \in [0, r]$, $\theta \in [0, 2\pi]$

$\varphi \in [0, \pi/2]$

$$= \int_0^r d\rho \int_0^{2\pi} d\theta \int_0^{\pi/2} d\varphi \; 1 \cdot \overbrace{\rho^2 \cos\varphi}^{J} = \int_0^r \rho^2 d\rho \underbrace{\int_0^{2\pi} d\theta}_{2\pi} \underbrace{\int_0^{\pi/2} \cos\varphi \, d\varphi}_{1}$$

$$= 2\pi \int_0^r \rho^2 d\rho = 2\pi \left[\frac{\rho^3}{3}\right]_{\rho=0}^{\rho=r} = \frac{2\pi}{3} r^3$$

$$\iiint_S z\,dx\,dy\,dz = \underbrace{\int_0^r d\rho \int_0^{2\pi} d\theta \int_0^{\pi/2} d\varphi}_{\text{Descrizione di } S} \; \underbrace{\rho \sin\varphi}_{z} \; \underbrace{\rho^2 \cos\varphi}_{J}$$

$$= \int_0^r \rho^3 d\rho \underbrace{\int_0^{2\pi} d\theta}_{2\pi} \int_0^{\pi/2} \sin\varphi \cos\varphi \, d\varphi = 2\pi \int_0^r \rho^3 d\rho \left[\frac{\sin^2\varphi}{2} \right]_{\varphi=0}^{\varphi=\frac{\pi}{2}}$$

$$= \pi \int_0^r \rho^3 d\rho = \pi \left[\frac{\rho^4}{4} \right]_{\rho=0}^{\rho=r} = \frac{\pi}{4} r^4$$

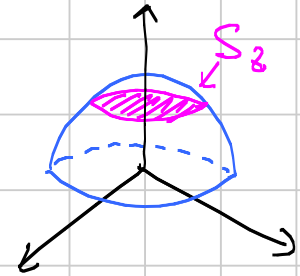
$$z_G = \frac{1}{\mathrm{Vol}(S)} \cdot \iiint_S z\,dx\,dy\,dz = \frac{3}{\cancel{2\pi}\, \cancel{r^3}} \cdot \frac{\cancel{\pi}}{4} r^{\cancel{4}} = \frac{3}{8} r$$

Semisfera $\underbrace{z_G = \frac{3}{8} r}_{+\ \text{BASSO}}$ Semicerchio $y_G = \frac{4}{3\pi} r$

Esercizio 3bis

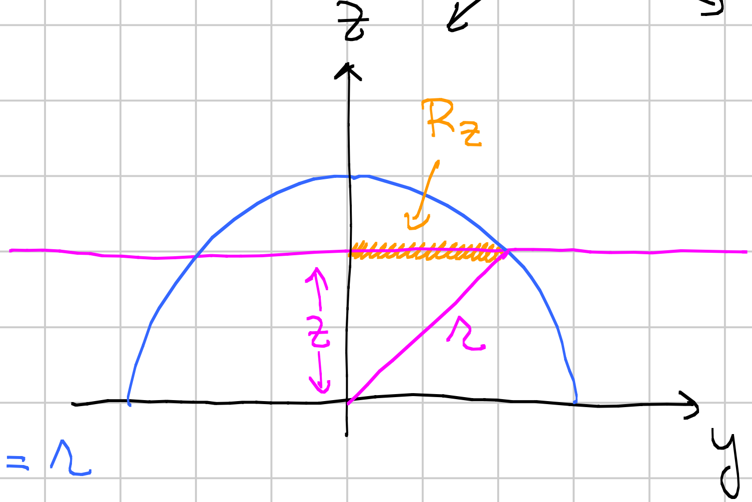
SEZIONI

$$\iiint_S z\,dx\,dy\,dz = \int_{\underset{z_{min}}{0}}^{\overset{z_{max}}{r}} dz \iint_{S_z} \boxed{z}\,dx\,dy = \int_0^r z\,dz \iint_{S_z} dx\,dy$$



$$= \int_0^r z \,\text{Area}(S_z)\,dz = \pi \int_0^r z \cdot R_z^2\,dz =$$

S_z è un cerchio di raggio R_z



Pitagora $\longrightarrow (R_z)^2 + z^2 = r^2$

$$R_z = \sqrt{r^2 - z^2}$$

Controlli: $z=0 \to R_z = r$

$z = r \to R_z = 0$

$$\pi \int_0^r z \cdot (r^2 - z^2)\,dz = \pi \int_0^r (r^2 z - z^3)\,dz = \pi \left[r^2 \frac{z^2}{2} - \frac{z^4}{4} \right]_{z=0}^{z=r}$$

$$= \pi \left(\frac{r^4}{2} - \frac{r^4}{4} \right) = \frac{\pi}{4} r^4$$

Esempio 3 ter

colonne

$$\iiint_S z \, dx\, dy\, dz =$$

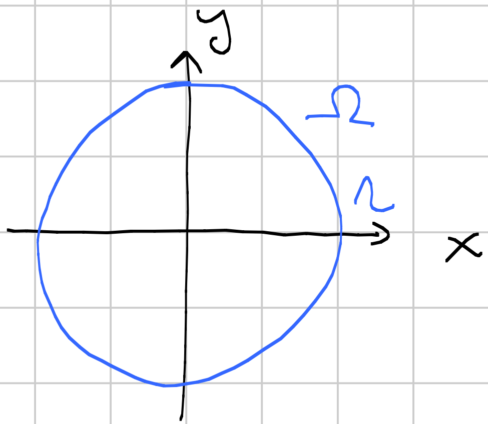
Eq. sfera (parte alta)

$$S = \left\{ (x,y,z) \in \mathbb{R}^3 : (x,y) \in \Omega, \quad 0 \le z \le \sqrt{r^2 - x^2 - y^2} \right\}$$

$$x^2 + y^2 + z^2 = r^2 \to z^2 = r^2 - x^2 - y^2 \to z = \sqrt{r^2 - x^2 - y^2}$$

$$= \iint_\Omega dx\, dy \int_0^{\sqrt{r^2-x^2-y^2}} dz \cdot z = \iint_\Omega dx\, dy \left[\frac{z^2}{2} \right]_{z=0}^{z=\sqrt{r^2-x^2-y^2}}$$

$$= \frac{1}{2} \iint_\Omega (r^2 - x^2 - y^2)\, dx\, dy =$$



$$= \frac{1}{2}\int_0^r d\rho \int_0^{2\pi} d\theta \, (r^2 - \overset{x^2+y^2}{\overset{\downarrow}{\rho^2}})\, \rho = \frac{1}{2}\int_0^r \rho\,(r^2-\rho^2)\, d\rho \underbrace{\int_0^{2\pi} d\theta}_{2\pi}$$

$$= \pi \int_0^r \rho\,(r^2-\rho^2)\, d\rho =$$ stesso integrale del 3bis pur di cambiare ρ con z.