MATEMATICA I

ORA 18

Titolo nota 11/10/2007

Criterio funzioni → successioni

" Cambio di variabili da una successione ad una funzione "

$$\lim_{n \to +\infty} \underbrace{\boxed{n}}_{+\infty} \cdot \underbrace{\boxed{\sin\left(\frac{1}{n}\right)}}_{0} = \lim_{n \to +\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} =$$

Pongo $\frac{1}{n} = x$

Quando $n \to +\infty$, ho che $x \to 0$

$$= \lim_{x \to 0} \frac{\sin x}{x} = 1$$

Esempio 2

$$\lim_{n \to +\infty} \underbrace{\boxed{n}}_{+\infty} \underbrace{\left(\sqrt[n]{12} - 1\right)}_{0} = \lim_{n \to +\infty} \frac{12^{\frac{1}{n}} - 1}{\frac{1}{n}} \underset{\frac{1}{n} = x}{=}$$

$$= \lim_{x \to 0} \frac{12^x - 1}{x} = \log 12$$

Esempio 3

$$\lim_{x\to+\infty} \frac{2^{\sqrt{x}}}{x^{1000}}$$

$$2^{\sqrt{x}} = e^{\sqrt{x}\log 2}$$

$$2^{\sqrt{x}} = e^{\log(2^{\sqrt{x}})}$$

$$\frac{2^{\sqrt{x}}}{x^{1000}} = \frac{e^{\sqrt{x}\log 2}}{e^{1000\log x}}$$

$$= e^{\sqrt{x}\log 2 - 1000\log x}$$

basta fare il limite dell'esponente

$$\sqrt{x}\log 2 - 1000\log x = \underbrace{\boxed{\sqrt{x}}}_{\to +\infty}\Big(\log 2 - 1000 \underbrace{\boxed{\frac{\log x}{\sqrt{x}}}}_{\to 0}\Big)$$

$x^{1/2}$

logaritmo contro potenza

$$\to +\infty$$

Sottoprodotto $\displaystyle \lim_{n\to+\infty} \frac{2^{\sqrt{n}}}{n^{1000}} = +\infty$

Esempio 4

$$\lim_{x\to+\infty} \frac{\log(x^2+2^x)}{3x+4} \qquad \left[\frac{\infty}{\infty}\right]$$

$\boxed{\text{logaritmo contro potenza} \longrightarrow \text{tende a } 0}$

NO !!!!!!!!

$$\frac{\log(x^2+2^x)}{3x+4} = \frac{\log\left[2^x\left(1+\frac{x^2}{2^x}\right)\right]}{x\left(3+\frac{4}{x}\right)}$$

$$= \frac{\log(2^x) + \log\left(1+\frac{x^2}{2^x}\right)}{x\left(3+\frac{4}{x}\right)} =$$

$$= \underbrace{\boxed{\frac{\cancel{x}\log 2}{\cancel{x}\left(3+\frac{4}{x}\right)}}}_{\frac{1}{3}\log 2} + \underbrace{\frac{\boxed{\log\left(1+\frac{x^2}{2^x}\right)} \to \log 1 = 0}{\underbrace{\boxed{x\left(3+\frac{4}{x}\right)}}_{+\infty}}}_{\to 0} \longrightarrow \frac{1}{3}\log 2$$

Brutalmente!
$$\frac{\log(x^2+2^x)}{3x+4} \sim \frac{\log(2^x)}{3x} = \frac{\cancel{x}\log 2}{3\cancel{x}} = \frac{\log 2}{3}$$

— o — o —

$$\lim_{x\to+\infty} \frac{e^{\log x+7}}{x^2} = 0$$

$$\frac{e^{\log x+7}}{x^2} = \frac{e^{\log x}\cdot e^7}{x^2} = \frac{x\cdot e^7}{x^2} = \frac{e^7}{x} \to 0$$

— o — o —

Esempio 6

$$\lim_{x\to 0} \frac{7^x - \cos x}{x+\sin x} \qquad \frac{7^0-\cos 0}{0} = \frac{1-1}{0} = \frac{0}{0}$$

$$\frac{7^x-\cos x}{x+\sin x} = \frac{7^x-1+1-\cos x}{x+\sin x} = \frac{7^x-1}{x+\sin x} + \frac{1-\cos x}{x+\sin x} =$$

$$= \underbrace{\boxed{\frac{7^x-1}{x}}}_{\log 7} \underbrace{\boxed{\frac{x}{x+\sin x}}}_{\frac{1}{2}} + \underbrace{\boxed{\frac{1-\cos x}{x^2}}}_{\frac{1}{2}} \cdot \underbrace{\boxed{\frac{x^2}{x+\sin x}}}_{0} \to \frac{1}{2}\log 7$$

$$\frac{7^x-1}{x} \to \log 7 \; ; \quad \frac{x}{x+\sin x} = \frac{1}{\frac{x+\sin x}{x}} = \frac{1}{1+\frac{\sin x}{x}} \to \frac{1}{2}$$

$$\frac{1-\cos x}{x^2} \to \frac{1}{2} \; ; \quad \frac{x^2}{x+\sin x} = \underbrace{\boxed{x}}_{\downarrow\, 0} \; \underbrace{\boxed{\frac{x}{x+\sin x}}}_{\downarrow\, \frac{1}{2}} \to 0$$

— o — o —

$$\lim_{x \to 1^+} \frac{x^2-3x+2}{\sqrt{x-1}} = \left[\frac{0}{0}\right]$$

Faccio un cambio di variabile per riportare il limite a 0.

$y = x-1$ Quando $x \to 1$, ho che $y \to 0$ $x = y+1$

$$= \lim_{y\to 0^+} \frac{(y+1)^2-3(y+1)+2}{\sqrt{y}} = \lim_{y\to 0^+} \frac{y^2+2y+\cancel{1}-3y-\cancel{3}+\cancel{2}}{\sqrt{y}} =$$

$$= \lim_{y\to 0^+} \frac{y^2-y}{\sqrt{y}} = \lim_{y\to 0^+} \frac{y(y-1)}{\sqrt{y}} = \lim_{y\to 0^+} \sqrt{y}\,(y-1) = 0^-$$

Esempio 8

$$\lim_{x\to 0} \frac{e^{2\sin x} + \arctan(\log(1+x)) - 1}{\sin(\sin x) + \arcsin(3x)}$$

$$\frac{\cancel{x}\,\frac{e^{2\sin x}-1}{x} + \cancel{x}\,\frac{\arctan(\log(1+x))}{x}}{\cancel{x}\,\frac{\sin(\sin x)}{x} + \cancel{x}\,\frac{\arcsin(3x)}{x}} \longrightarrow \frac{2+1}{1+3} = \frac{3}{4}$$

$$\frac{e^{2\sin x}-1}{x} = \boxed{\frac{e^{2\sin x}-1}{2\sin x}} \cdot \boxed{\frac{2\sin x}{x}}$$

$y = 2\sin x$

$$\frac{e^{2\sin x}-1}{2\sin x} \to \frac{e^{y}-1}{y} \to 1 \qquad \frac{2\sin x}{x} \to 2$$

$$\frac{\sin(\sin x)}{x} = \boxed{\frac{\sin(\sin x)}{\sin x}} \cdot \boxed{\frac{\sin x}{x}} \to 1$$

$$y = \sin x \qquad \boxed{\frac{\sin y}{y}} \to 1 \qquad \frac{\sin x}{x} \to 1$$

$$\frac{\arcsin(3x)}{x} = \boxed{\frac{\arcsin(3x)}{3x}} \cdot 3 \to 3 \qquad \frac{\arcsin(3x)}{3x} \to 1$$

$$\frac{\arctan(\log(1+x))}{x} = \boxed{\frac{\arctan(\log(1+x))}{\log(1+x)}} \cdot \boxed{\frac{\log(1+x)}{x}} \to 1 \qquad \frac{\log(1+x)}{x} \to 1$$

$y = \log(1+x)$

Quando $x \to 0$, ho che $y \to \log 1 = 0$

$$\boxed{\frac{\arctan y}{y}} \to 1$$