RICEVIMENTO

Titolo nota 21/12/2006

$$2^{|x|} = \lambda x \qquad \frac{2^{|x|}}{x} = \lambda \qquad f(x) = \frac{2^{|x|}}{x}$$

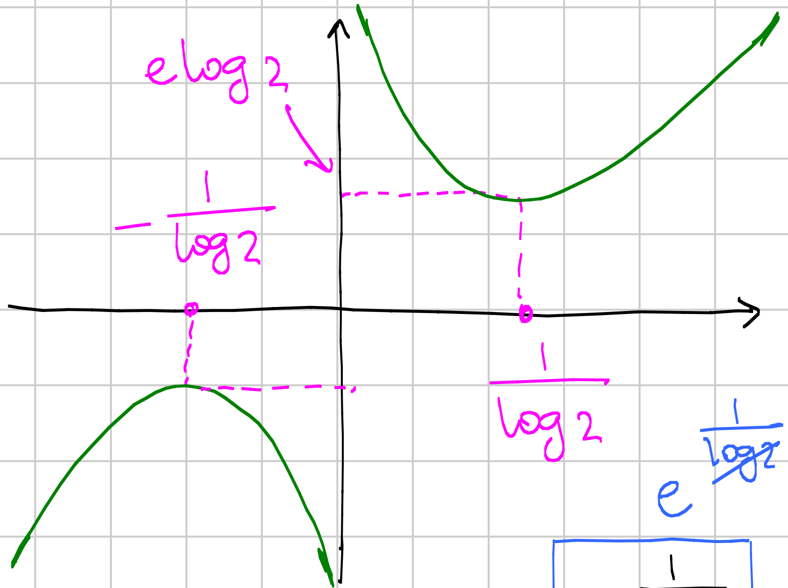
$$f(-x) = \frac{2^{|-x|}}{-x} = -\frac{2^{|x|}}{x} = -f(x)$$

$$f(x) = \frac{2^x}{x}$$

$$f'(x) = \frac{2^x \log 2 \cdot x - 2^x}{x^2}$$

$$f'(x) = 0 \iff 2^x(\log 2 \cdot x - 1) = 0$$

$$x \cdot \log 2 = 1 \iff x = \frac{1}{\log 2}$$



$$e^{\frac{1}{\cancel{\log 2}} \cdot \cancel{\log 2}}$$

$$f\left(\frac{1}{\log 2}\right) = \frac{\boxed{2^{\frac{1}{\log 2}}}}{\frac{1}{\log 2}} =$$

$$= \log 2 \cdot e$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\tan^4 x - 1}{\sqrt{\tan^7 x}}\, dx$$

FARE LA PRIMITIVA

$y = \tan x \qquad x = \arctan y$

$$\frac{dx}{dy} = \frac{1}{1+y^2}$$

$$dx = \frac{dy}{1+y^2}$$

$$\int \frac{\tan^4 x - 1}{(\tan x)^{7/2}}\, dx =$$

$$= \int \frac{y^4 - 1}{y^{7/2}} \frac{dy}{1+y^2} = \int \frac{y^2 - 1}{y^{7/2}}\, dy$$

$$= \int \left(y^{-\frac{3}{2}} - y^{-\frac{7}{2}}\right) dy = -2y^{-\frac{1}{2}} + \frac{2}{5} y^{-\frac{5}{2}}$$

$$= -2 (\tan x)^{-\frac{1}{2}} + \frac{2}{5} (\tan x)^{-\frac{5}{2}}$$

$$\lim_{A \to \frac{\pi}{2}} \int_{\pi/4}^{A} \cdots = \lim_{A \to +\infty} \left[-\underbrace{\frac{2}{(\tan A)^{1/2}}}_{\to 0} + \frac{2}{5} \underbrace{\frac{1}{(\tan A)^{5/2}}}_{\to 0} + 2 - \frac{2}{5} \right] = \frac{8}{5}$$

$y = \tan x \qquad \frac{dy}{dx} = 1 + \tan^2 x \qquad dy = (1+\tan^2 x)\, dx$

$$\int \frac{\tan^4 x - 1}{(\tan x)^{7/2}}\, dx = \int \frac{(\tan^2 x - 1)\overbrace{(\tan^2 x + 1)\, dx}^{dy}}{(\tan\ )^{7/2}}$$

$$= \int \frac{y^2 - 1}{y^{7/2}}\, dy$$

— o — o —

$$\begin{cases} y' = y \log y \\ y(0) = \frac{1}{e} \end{cases}$$

$$\frac{dy}{y \log y} = dt \Rightarrow \log|\log y| = t + c$$

$$|\log y| = e^{t+c} = e^c \cdot e^t$$

$$\log y = \pm e^c \cdot e^t = k e^t$$

$$y = e^{k e^t} \qquad y(0) = e^k = \frac{1}{e} \Leftrightarrow k = -1$$

$$\int_{-2}^{2} \underbrace{\arctan\left(x \cos\left(e^{|x|}+3\right)\right)}_{\uparrow} = 0$$

$f(x)$ è dispari

$$f(-x) = \arctan\left((-x)\cos\left(e^{|-x|}+3\right)\right)$$

$$= \arctan\left(-x\cos\left(e^{|x|}+3\right)\right)$$

$$= -\arctan\left(x\cos\left(e^{|x|}+3\right)\right) = -f(x)$$

— o — o —

$$\begin{cases} y' = y^2 - 4 \\ y(0) = 1 \end{cases} \qquad \frac{dy}{y^2-4} = dt \qquad \int \frac{dy}{y^2-4} = \int dt$$

$$\frac{1}{y^2-4} = -\frac{1}{4}\frac{1}{y-2} + \frac{1}{4}\frac{1}{y+2}$$

$$\int \frac{dy}{y^2-4} = -\frac{1}{4}\log|y-2| + \frac{1}{4}\log|y+2|$$

$$= \log \sqrt[4]{\frac{|y+2|}{|y-2|}} = t+c$$

$$\sqrt[4]{\frac{|y+2|}{|y-2|}} = e^{t+c} \qquad \frac{|y+2|}{|y-2|} = e^{4t+c} = e^{c} \cdot e^{4t}$$

$$\frac{y+2}{y-2} = \pm e^{c} \cdot e^{4t} = k e^{4t}$$

Se $y(0) = 1$ $\quad \frac{1+2}{1-2} = k \iff k = -3$

Se $y(0) = -2$ $\quad \frac{-2+2}{-2-2} = k \quad k=0$

Che sia $y(0) = 2$ oppure $y(0) = -2$, la soluzione è costante.

$$\cosh(2x) = \frac{e^{2x}+e^{-2x}}{2} = 2\cosh^2 x - 1 = 2\sinh^2 x + 1$$

$$= \cosh^2 x + \sinh^2 x$$

$$\cosh^2(x) = \left(\frac{e^x+e^{-x}}{2}\right)^2 = \frac{e^{2x}+e^{-2x}+2}{4}$$

$$\sinh^2(x) = \left(\frac{e^x-e^{-x}}{2}\right)^2 = \frac{e^{2x}+e^{-2x}-2}{4}$$

— o — o —

$$\lim_{n\to+\infty} \sqrt{n\sqrt{2n+1}}\left(\sqrt[4]{n}\sin\frac{1}{n} + \frac{1}{n}\sin\sqrt[4]{n}\right)$$

$$\parallel$$

$$\sqrt[4]{2}\; n^{\frac{3}{4}} \left(\sqrt[4]{n}\,\frac{1}{n} + \underbrace{\frac{\text{roba limitata}}{n}}_{\text{trascurabile}}\right)$$

$$\sqrt[4]{2}\cdot \cancel{n^{\frac{3}{4}}}\left(\frac{1}{\cancel{n^{3/4}}}\right) = \sqrt[4]{2}$$

$$= \sqrt[4]{2}\ \cancel{n^{\frac{3}{4}}}\ \boxed{\sqrt[4]{1+\frac{1}{2n}}}\ \frac{1}{\cancel{n^{3/4}}}\left(\frac{\sin\frac{1}{n}}{\frac{1}{n}} + \frac{1}{\sqrt[4]{n}}\sin\sqrt[4]{n}\right)$$

$$\sqrt[4]{1+\frac{1}{2n}} \to 1 \qquad \frac{\sin\frac{1}{n}}{\frac{1}{n}} \to 1 \qquad \frac{1}{\sqrt[4]{n}}\sin\sqrt[4]{n} \to 0$$

— o — o —

$$\begin{cases} a_{n+1} = n - 3^{a_n} \\ a_0 = 1 \end{cases}$$

$a_0 = 1$

$a_1 = 0 - 3^{a_0} = \boxed{-3}$

$a_2 = 1 - 3^{a_1} = 1 - 3^{(-3)} = 1 - \frac{1}{27}$ un po' meno di 1

$a_3 = 2 - 3^{\text{quasi } 1} = \boxed{\text{neg.}}$

$a_4 = 3 - 3^{\text{neg}} =$ un po' meno di 3

$a_5 = 4 - 3^{\text{un po' meno di } 3} = \boxed{\text{neg.}}$

$a_6 = 5 - 3^{a_5} =$ un po' meno di 5

$\boxed{\text{PIANO}}$ $\quad a_{2n+1} \leq 0 \quad \forall n \in \mathbb{N}$

$\quad a_{2n} \geq 2n-2 \quad \forall n \in \mathbb{N}$

La successione non ha limite

Induzione INSIEME $\quad n=0$ sostituzione

$\boxed{n \Rightarrow n+1}$ Ipotesi è quella scritta

$$a_{2n+2} = (2n+1) - 3^{a_{2n+1}} = (2n+1) - 3^{\text{neg}} \geq 2n$$

$$a_{2n+3} = (2n+2) - 3^{a_{2n+2}} \leq (2n+2) - 3^{2n} \leq 0$$

riga precedente

SI DIMOSTRA A SUA VOLTA PER INDUZIONE

$y' = (y+t)^2$ Pongo $z = y+t$ (cioè $z(t) = y(t) + t$)

$z' = y' + 1 = (y+t)^2 + 1 = z^2 + 1 \Rightarrow \boxed{z' = z^2 + 1}$

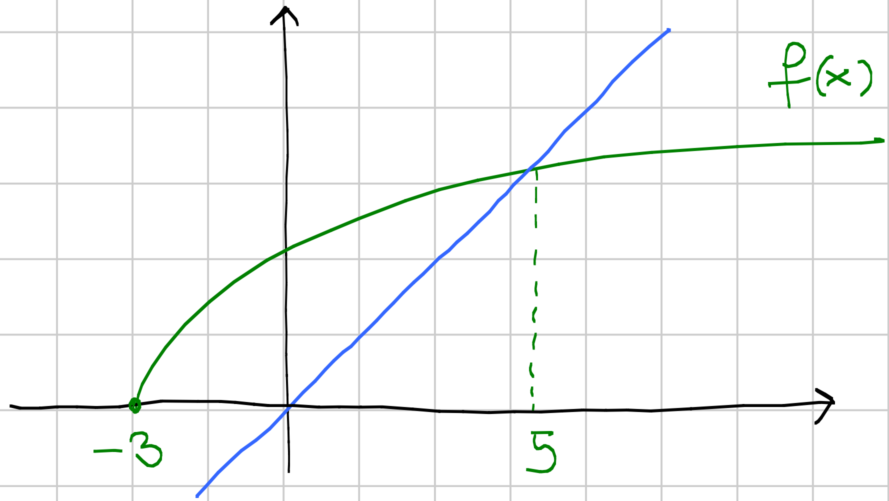
Derivo z — Uso eq. per y' — Definiz. di z

Trovo z

$y = z - t$

— o — o —

$a_{n+1} = f(a_n) \qquad a_0 = \alpha$



$\boxed{\text{Caso } \alpha > 5}$

(i) $a_n \geq 5 \quad \forall n \in \mathbb{N}$

(ii) $a_{n+1} \leq a_n \quad \forall n \in \mathbb{N}$

(iii) $a_n \to \ell \in \mathbb{R}$

(iv) $\ell = 5$

$\boxed{\text{Caso } \alpha < 5}$

(i) $a_n \leq 5 \quad \forall n \in \mathbb{N}$

(ii) $a_{n+1} \geq a_n \quad \forall n \in \mathbb{N}$

(iii) $a_n \to \ell \in \mathbb{R}$

(iv) $\ell = 5$

$\boxed{\text{Caso } \alpha = 5}$ $\quad a_n = 5 \quad \forall n \in \mathbb{N}$

— o —

$$\begin{cases} a_{n+1} = \dfrac{a_n}{\sqrt{n+8}} \\ a_0 = \alpha \end{cases}$$

$\alpha > 0$ Decresci a 0

$\alpha = 0$ sempre $= 0$

$\alpha < 0$ Cresci a 0

$a_{n+1} = (n+8)\, a_n$

Cresci a $+\infty$

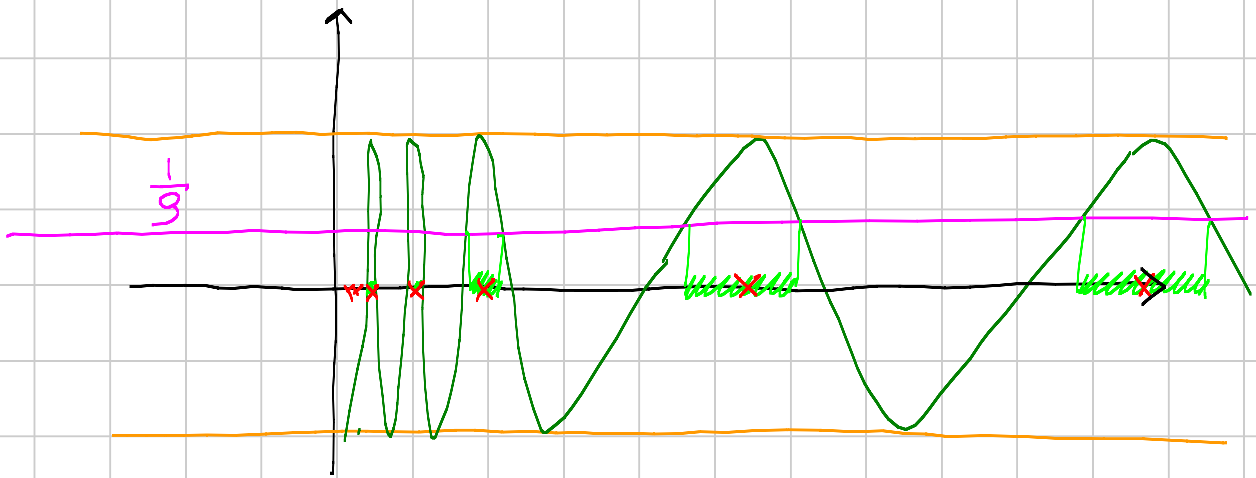
sempre $= 0$

Decresci a $-\infty$

$\left\{ x \in (0,+\infty) \; : \; \sin(\log x) > \frac{1}{9} \right\}$ inf / sup / max / min

$\sup = +\infty$

$\inf = 0$



Vediamo dove $\sin(\log x) = 1$ (dunque $> \frac{1}{9}$)

$\iff \log x = \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z}$

$\iff x = e^{\frac{\pi}{2} + 2k\pi} \quad k \in \mathbb{Z}$

Quando $k \to +\infty$, $x \to +\infty$

Quando $k \to -\infty$, $x \to 0^+$

$\sin(\log x) > \frac{1}{9}$ applico $\arcsin x$ e uso che

$\log x > \arcsin \frac{1}{9}$

$$\boxed{\arcsin(\sin(\alpha)) = \alpha}$$

con $\alpha = \log x$

$\arcsin(\sin(\log x)) > \arcsin \frac{1}{9}$

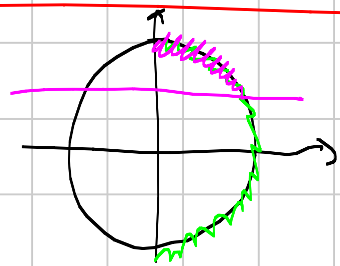
Vale solo se

$\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\sin x > \frac{1}{2}$ applico arcsin

$$\boxed{x > \arcsin \frac{1}{2} = \frac{\pi}{6} \qquad x > \frac{\pi}{6}}$$

NOOOO!!!!!!



$\sin x = \frac{1}{2}$ $x = \arcsin \frac{1}{2} = \frac{\pi}{6} + 2k\pi$

Perso $x = \frac{5\pi}{6} + 2k\pi$