RICEVIMENTO

Titolo nota 14/12/2006

$$\begin{cases} a_{n+1} = n - a_n \\ a_0 = 0 \end{cases}$$

$a_0 = 0$, $a_1 = 0 - a_0 = 0$, $a_2 = 1 - a_1 = 1$

$a_3 = 2 - a_2 = 1$

$a_4 = 3 - a_3 = 2$

$a_5 = 4 - a_4 = 2$

$a_6 = 5 - a_5 = 3$

$a_7 = 6 - a_6 = 3$

Sembra essere $0, 0, 1, 1, 2, 2, 3, 3, 4, 4, \dots$

Occorre fare un piano che lo dimostri

$$\boxed{\begin{aligned} a_{2n} &= n \\ a_{2n+1} &= n \end{aligned}}$$

SI DIMOSTRA PER INDUZIONE

$n = 0$ $\boxed{\begin{aligned} a_0 &= 0 \\ a_1 &= 0 \end{aligned}}$ OK

$\boxed{\text{P.I.}}$ Ipotesi $a_{2n} = n$ Tesi: $a_{2n+2} = n+1$

$a_{2n+1} = n$ $a_{2n+3} = n+1$

$$a_{2n+2} = 2n+1 - a_{2n+1} = 2n+1 - n = n+1$$

$$a_{2n+3} = 2n+2 - a_{2n+2} = 2n+2 - (n+1) = n+1$$

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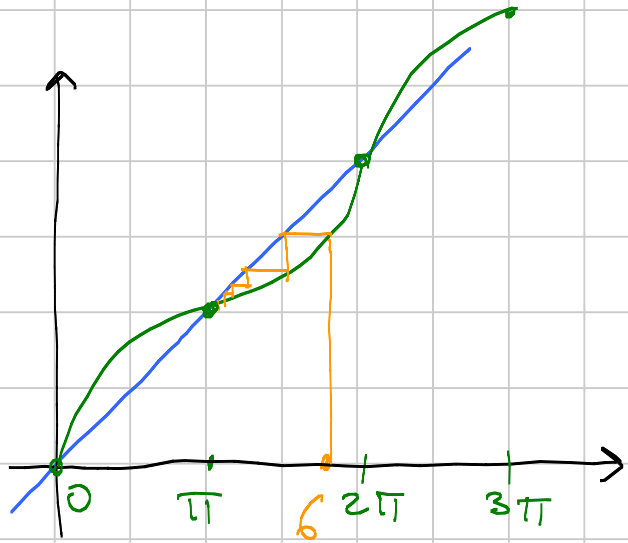
$$\begin{cases} a_{n+1} = a_n + \sin a_n \\ a_0 = 6 \end{cases}$$

$f(x) = x + \sin x$



Risolvere $f(x) = x$ $\cancel{x} + \sin x = \cancel{x}$ $\sin x = 0$

$f(x) > x$

$$f(x) > x \iff \cancel{x} + \sin x > \cancel{x} \iff \sin x > 0$$

$$f'(x) = 1 + \cos x \geq 0 \implies f \text{ cresc. (anche strett.)}$$

Idea: $a_n \to \pi$ decrescendo.

$\boxed{\text{PIANO}}$ (i) $\pi \leq a_n \leq 6 \quad \forall n \in \mathbb{N}$

(ii) $a_{n+1} \leq a_n \quad \forall n \in \mathbb{N}$

(iii) $a_n \to \ell \in \mathbb{R}$ SOLITO

(iv) $\ell = \pi$ SOLITO

(ii) $a_{n+1} \leq a_n \iff f(a_n) \leq a_n$ vero se $a_n \in [\pi, 2\pi]$

(i) Induzione $n=0$ ok

P.I. Ipotesi $\pi \leq a_n \leq 6$ applico f

$$f(\pi) \leq f(a_n) \leq f(6)$$

$$\pi \leq a_{n+1} \leq f(6) \leq 6$$

solito $f(x) \leq x$ vero per $x \in [\pi, 2\pi]$.

$$\int_0^{+\infty} \frac{2^{\sqrt{x}-1}}{\sqrt{x}\,(4^{\sqrt{x}}-1)}\,dx$$

si calcola la primitiva con la sostituzione $y=\sqrt{x}$ e poi $z=2^y$.

$$\int_0^{1} \underbrace{\frac{2^{\sqrt{x}-1}}{\sqrt{x}\,(4^{\sqrt{x}}-1)}}_{f(x)}\,dx$$

Converge o diverge?

$$4^{\sqrt{x}}-1 \sim e^{\sqrt{x}\log 4}-1$$

$$\sim \cancel{1}+\sqrt{x}\log 4-\cancel{1}$$

$$f(x) \sim \frac{1}{2}\,\frac{1}{\sqrt{x}}\,\frac{1}{\sqrt{x}\log 4}$$

$$= \frac{1}{(2\log 4)}\,\frac{1}{x} \implies \text{DIVERGE}$$

$$\iint_A \frac{\log(x+y)}{x+y}\,dx\,dy \qquad A=[1,2]\times[0,1]$$

$$= \int_1^2 dx \int_0^1 \frac{\log(x+y)}{x+y}\,dy = \frac{1}{2}\int_1^2 \left[\log^2(y+x)\right]_{y=0}^{y=1} =$$

$$\int \frac{\log(y+a)}{y+a}\,dy = \frac{1}{2}\log^2(y+a)$$

$$= \frac{1}{2}\int_1^2 \left(\log^2(x+1) - \log^2 x\right)dx$$

$$\int \log^2 x\,dx = \int \underset{f}{1}\cdot \underset{G}{\log^2 x}\,dx = \underset{F}{x}\,\underset{G}{\log^2 x} - \int \underset{F}{\cancel{x}}\cdot \underset{g}{2\log x\cdot\frac{1}{\cancel{x}}}\,dx$$

$$= x\log^2 x - 2\int \log x\,dx = x\log^2 x - 2(x\log x - x)$$

$$\int \log^2(x+1)\,dx \underset{z=x+1}{=} (x+1)\log^2(x+1) - 2\Big((x+1)\log(x+1) - (x+1)\Big)$$

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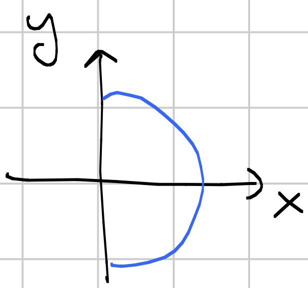
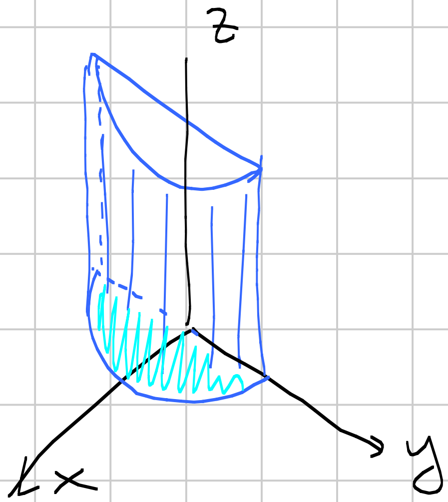
$$\iiint_A z\sqrt{x^2+y^2}\,dx\,dy\,dz \qquad A = \left\{(x,y,z)\in\mathbb{R}^3 : \begin{array}{l} x^2+y^2\le 4 \\ z\in[0,3] \\ x\ge 0 \end{array}\right\}$$

= mezzo cilindro





In coord. cilindriche A si descrive come

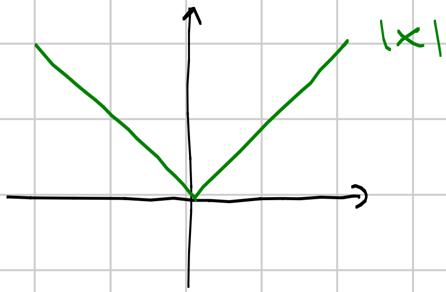
$\rho\in[0,2]$

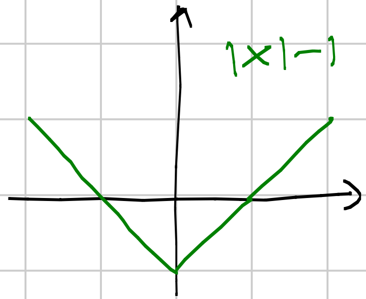
$\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

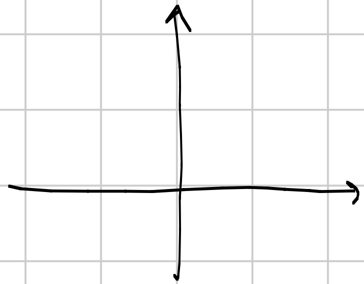
$z\in[0,3]$

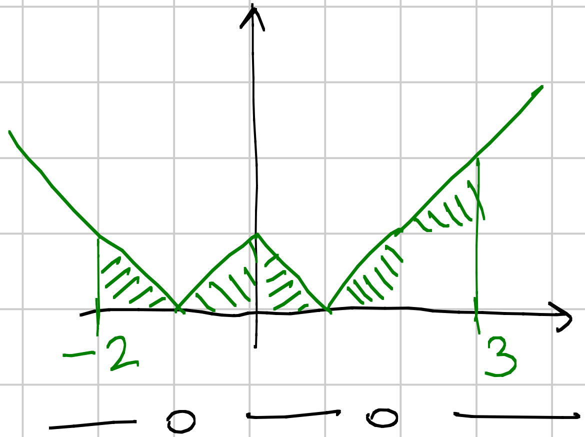
$$= \int_0^2 d\rho \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^3 dz\; z\cdot\overset{\sqrt{x^2+y^2}}{\rho}\;\overset{J}{\rho} = \dots$$

$$\int_{-2}^{3} \left| |x| - 1 \right| \, dx$$









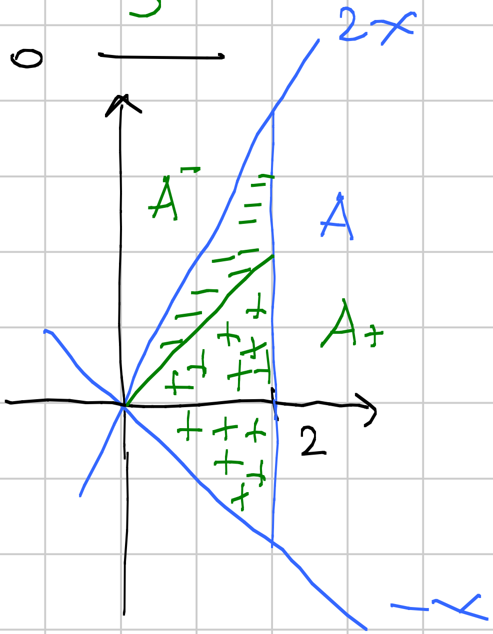
$$\boxed{\frac{7}{2}}$$

$$\iint_A |x^2 - y^2| \, dx \, dy$$

$$x^2 - y^2 \geq 0$$

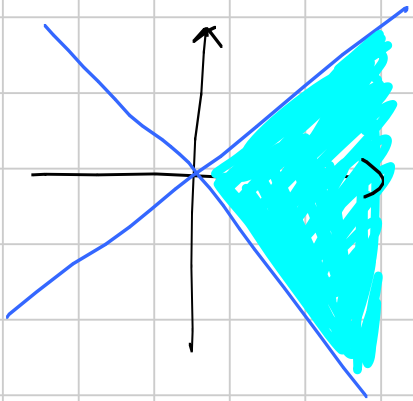
$$(x+y)(x-y) \geq 0$$

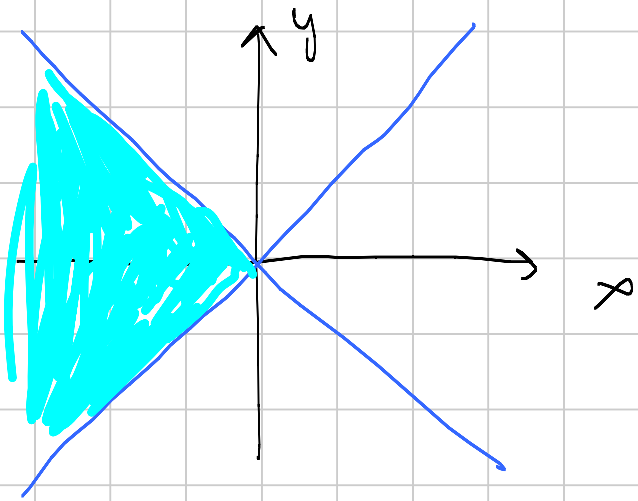
$$\begin{array}{cc} + & + \\ - & - \end{array}$$



$(++)\quad \begin{cases} x+y \geq 0 \\ x-y \geq 0 \end{cases}$ $\qquad$ $(--)\quad \begin{cases} x+y \leq 0 \\ x-y \leq 0 \end{cases}$

$\begin{cases} y \geq -x \\ y \leq x \end{cases}$ $\qquad$ $\begin{cases} y \leq -x \\ y \geq x \end{cases}$





$$\iint_A |x^2-y^2|\,dx\,dy = \iint_{A^+} (x^2-y^2)\,dx\,dy + \iint_{A^-} (y^2-x^2)\,dx\,dy$$

$$= \int_0^2 dx \int_{-x}^{x} (x^2-y^2)\,dy + \int_0^2 dx \int_{x}^{2x} (y^2-x^2)\,dy$$

$f(x,y) = 4e^{xy} + x^2 + y^2$ Trovare punti stazionari e precisare cosa sono

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \qquad \begin{cases} 4ye^{xy} + 2x = 0 \\ 4xe^{xy} + 2y = 0 \end{cases} \qquad \begin{cases} 2ye^{xy} + x = 0 & (\cdot x) \\ 2xe^{xy} + y = 0 & (\cdot y) \end{cases}$$

$$\begin{cases} 2xy\,e^{xy} + x^2 = 0 \\ 2xy\,e^{xy} + y^2 = 0 \end{cases}$$

Sottraggo: $x^2 - y^2 = 0$, $x^2 = y^2$, $y = \pm x$

Sostituisco nella 1ª: $\boxed{y = x}$ $2xe^{x^2} + x = 0$ $x\underbrace{(2e^{x^2} + 1)}_{\neq 0} = 0$

$\Rightarrow x = 0 \Rightarrow$ punto $(0,0)$

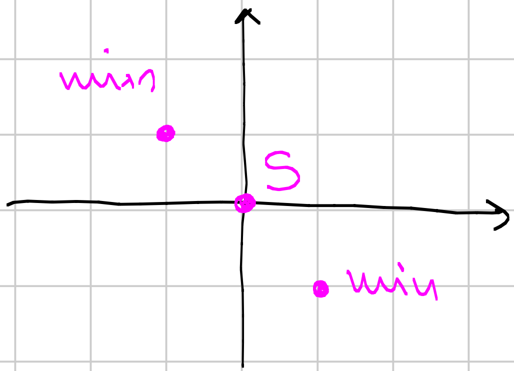
$\boxed{y = -x}$ $-2xe^{-x^2} + x = 0$, $x(1 - 2e^{-x^2}) = 0$

$x=0 \rightsquigarrow$ ritrovo $(0,0)$

$1-2e^{-x^2}=0 \Rightarrow 2e^{-x^2}=1 \qquad e^{-x^2}=\frac{1}{2}$

$$-x^2=\log\frac{1}{2}=-\log 2$$

$$x=\pm\sqrt{\log 2}\,,\quad y=\mp\sqrt{\log 2}$$



Volendo si può fare l'Hessiano

$$\begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}$$

Det <0

$\Downarrow$

SELLA

$$f(x,y) \sim 4(1+xy)+x^2+y^2 = 4+\boxed{4xy+x^2+y^2}$$

Dico che esiste $\min\{f(x,y): (x,y)\in\mathbb{R}^2\}$

$$f(x,y) = 4e^{xy} + x^2 + y^2 \geq x^2 + y^2$$

$$\lim_{x^2+y^2 \to +\infty} f(x,y) = +\infty$$

W. generalizzato $\Rightarrow \exists$ minimo $\Rightarrow$ da qualche parte c'è un p.to di minimo

D'altra parte $f(-x,-y) = f(x,y) \Rightarrow f$ ha lo stesso valore nei 2 restanti punti stazionari.

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$$\int_0^{\pi} dx \int_0^{\pi} dy \; e^{3x} \cos(x+y) = \int_0^{\pi} e^{3x} dx \int_0^{\pi} \cos(x+y)\, dy$$

$$= \int_0^{\pi} e^{3x} dx \left[\sin(x+y) \right]_{y=0}^{y=\pi}$$

$$= \int_0^{\pi} e^{3x} \left\{ \sin(x+\pi) - \sin x \right\} dx$$

$$= -2 \int_0^{\pi} e^{3x} \sin x \, dx$$

$$\int \underset{f}{e^{3x}} \, \underset{G}{\sin x} \, dx = \underset{F}{\frac{e^{3x}}{3}} \, \underset{G}{\sin x} - \int \underset{F}{\frac{e^{3x}}{3}} \, \underset{g}{\cos x} \, dx$$

$$= \frac{e^{3x}}{3} \sin x - \frac{1}{3} \int \underset{f}{e^{3x}} \cdot \underset{G}{\cos x} \, dx = \cdots$$

$$\int_0^1 dx \int_0^1 dy \frac{x}{1+x^2y^2} = \int_0^1 dx \Big[\arctan(xy)\Big]_{y=0}^{y=1}$$

$$\int \frac{a}{1+a^2y^2} dy = \arctan(ay)$$

$$= \int_0^1 \arctan x \, dx = \dots$$