RICEVIMENTO

Titolo nota 23/11/2006

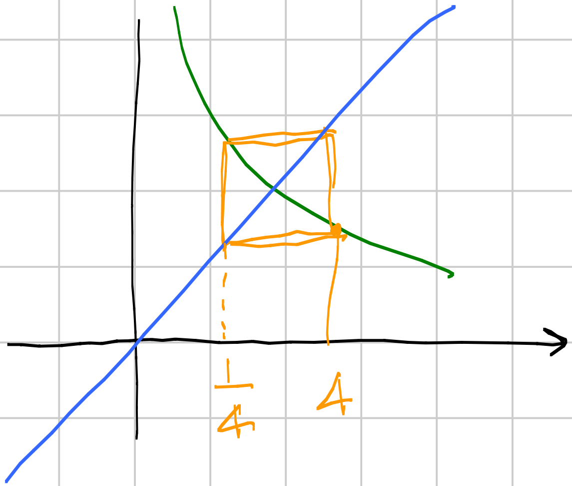
$$\begin{cases} a_{n+1} = \dfrac{1}{a_n} \\ a_0 = 4 \end{cases}$$

$a_0 = 4 \quad a_1 = \dfrac{1}{4} \quad a_2 = 4 \quad a_3 = \dfrac{1}{4}$

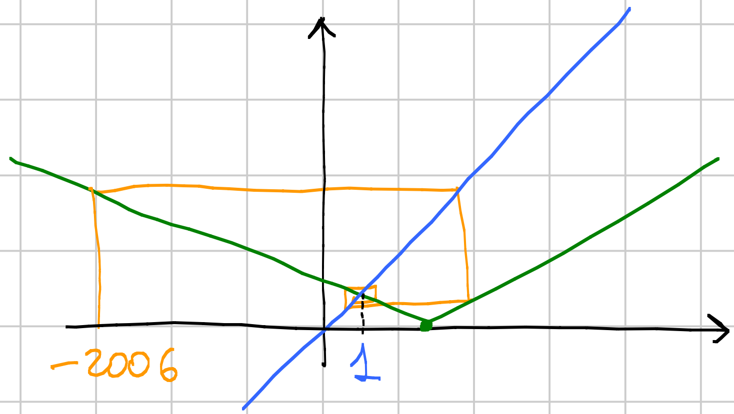
$$a_n = \begin{cases} 4 & \text{se } n \text{ pari} \\ \dfrac{1}{4} & \text{se } n \text{ dispari} \end{cases}$$

PER INDUZIONE

$\lim_{n \to +\infty} a_n$ N.E.



$$a_{n+1} = \frac{|a_n - 3|}{2} \qquad a_0 = -2006$$



$$f(x) = \frac{|x-3|}{2}$$

$f(x)$ è lipschitziana con costante $\frac{1}{2}$

$$|f(x) - f(y)| \leq \frac{1}{2}|x-y| \qquad \text{Perché ?}$$

$$|f(x) - f(y)| = \left| \frac{|x-3|}{2} - \frac{|y-3|}{2} \right| = \frac{1}{2} \Big| |x-3| - |y-3| \Big| \leq \frac{1}{2}|x-y|$$

$$\big| |A| - |B| \big| \leq |A-B|$$

Formula con i valori assoluti

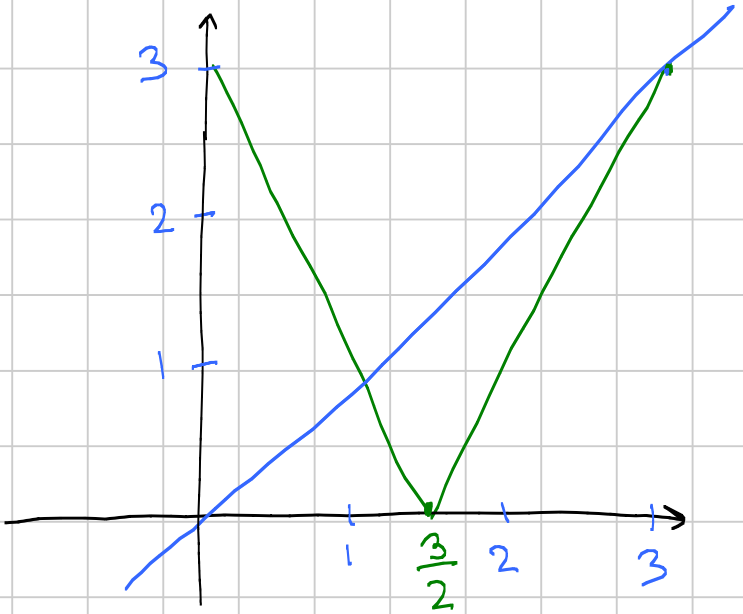
$\boxed{\text{PIANO CON DISTANZA}}$

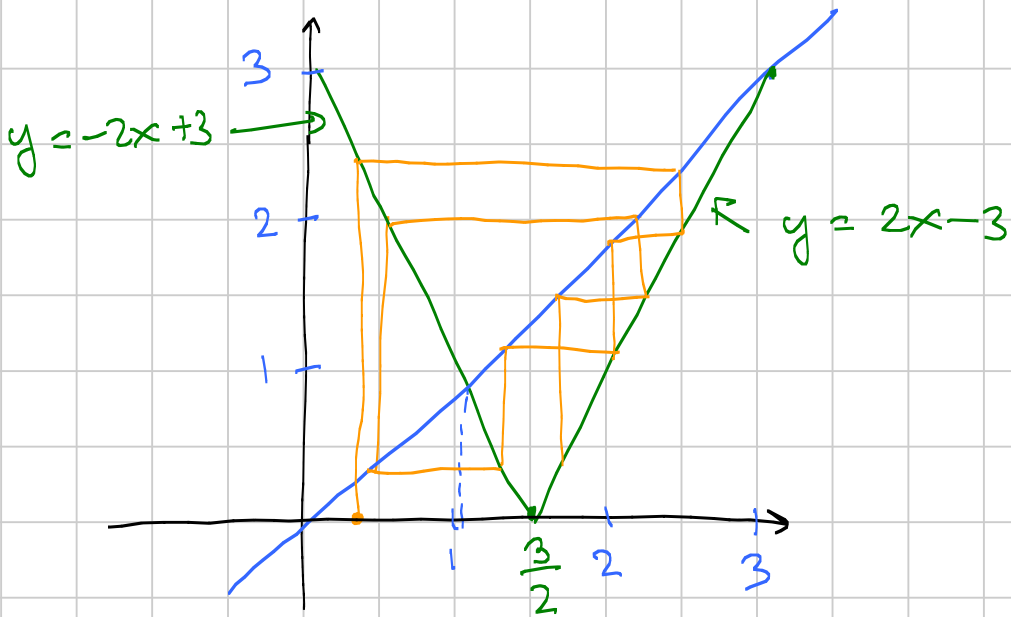
$$d_n = |a_n - 1|$$

$$d_{n+1} = |a_{n+1} - 1| = |f(a_n) - f(1)| \leq \frac{1}{2}|a_n - 1| = \frac{1}{2} d_n$$

$$d_{n+1} \leq \frac{1}{2} d_n \implies \dots \quad d_n \to 0 \implies a_n \to 1$$

$a_{n+1} = |2a_n - 3| \qquad a_0 = \sqrt{2}$





In questo caso a_n non ha limite. IDEA: $0 \le a_n \le 3 \quad \forall n \in \mathbb{N}$

(si dim. per INDUZIONE)

$a_n \nearrow \ell \in \mathbb{R} \implies \ell = 1$ opp. $\ell = 3$

$a_n \searrow$ NON HA LIMITE

Suppoiamo $\ell = 3$. Allora definitiv. $a_n \ge \frac{3}{2}$. Fino a quando sta in questa zona si dim per induz. che la distanza da 3 raddoppia a ogni passaggio

Supponiamo $l=1$. Allora defin. $a_n \leq \frac{3}{2}$ e ancora come prima si dimostra che in questo caso la distanza tra a_n e 1 raddoppia ad ogni passaggio.

Resta da escludere che a_n possa valere 1 o 3 per qualche valore di n. Ma essendo all'inizio $a_n = \sqrt{2}$, si vede (per induzione) che a_n sarà sempre del tipo

$$a_n = \alpha_n + \beta_n \sqrt{2}$$ con α_n e β_n interi e $\beta_n \neq 0$

Quindi "il radice di 2" non va mai via.

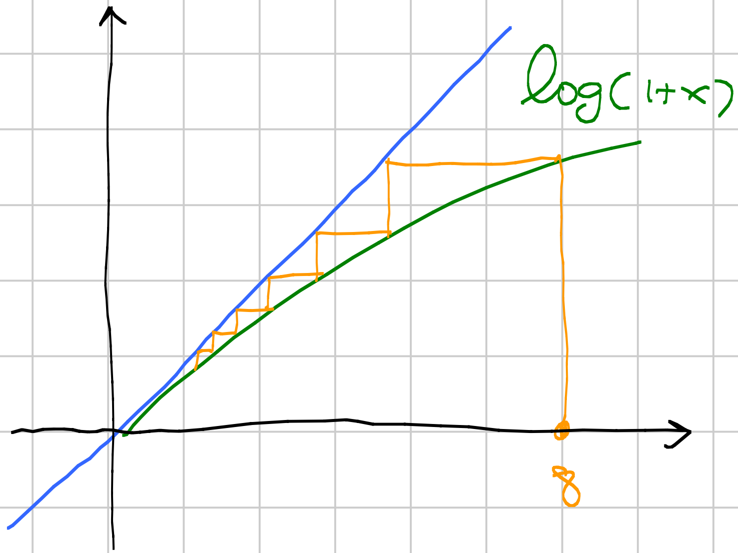
$a_{n+1} = \log(1+a_n) \qquad a_1 = 8$

$f(x) = \log(1+x)$

Importante $\nearrow f(0)=0$

$\searrow f(x) \leq x$ per ogni $x \geq 0$

$<$ " $>$



Si dimostra con le disug. classiche oppure studiando la funzione $x - \log(1+x)$

$\boxed{\text{PIANO}}$ (i) $a_n \geq 0 \quad \forall n \in \mathbb{N}$

(ii) $a_{n+1} \leq a_n \quad \forall n \in \mathbb{N}$ $\to$ $a_{n+1} \overset{?}{\leq} a_n$, $f(a_n) \overset{?}{\leq} a_n$

ma noi già sappiamo che $f(x) \leq x$ per ogni $x \geq 0$, quindi la disug. segue dalla (i)

(iii) $a_n \to \ell \in \mathbb{R}$

(iv) $\ell = 0$

SOLITO

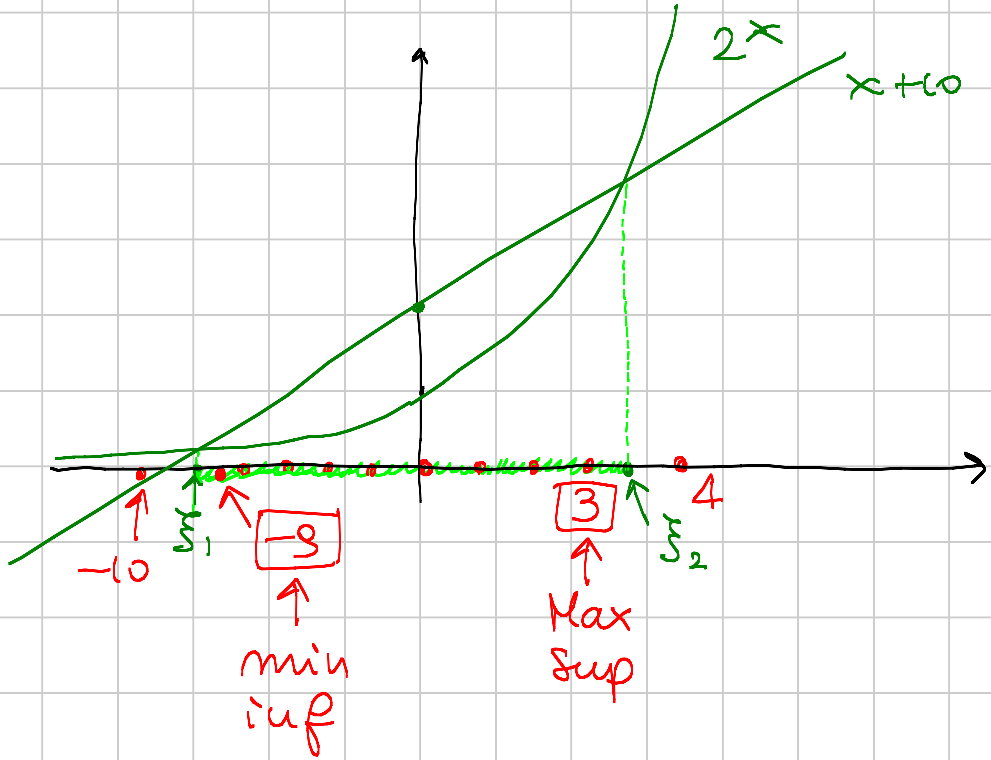
INDUZIONE! $a_0 \geq 0$ ok.

f è CRESCENTE

$\boxed{\text{P.I.}}$ se $a_n \geq 0$, allora $a_{n+1} = f(a_n) \geq f(0) = 0$

$\{n \in \mathbb{Z} : 2^n \le n+10\}$ sup / inf / max / min

$\{x \in \mathbb{R} : 2^x \le x+10\}$



$n = 3 \quad 2^3 \le 13 \quad$ ok

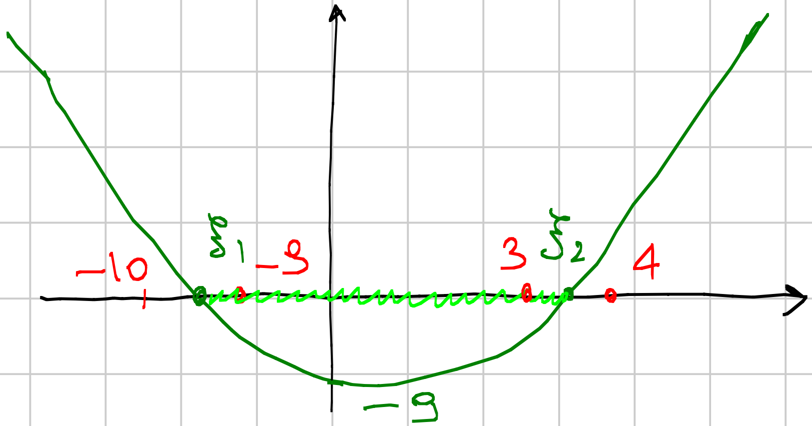
$n = 4 \quad 2^4 \le 14 \quad$ NO

$n = -10 \quad 2^{-10} \le 0 \quad$ NO

$n = -9 \quad 2^{-9} \le 1 \quad$ SI

$f(x) = 2^x - x - 10$

convessa



$$\sum_{n=1}^{\infty} \underbrace{\frac{2^{n^2}}{(n!)^{\sqrt{n}}}}_{a_n}$$

$$\sqrt[n]{A^B} = \left(A^B\right)^{\frac{1}{n}} = A^{\frac{B}{n}}$$

$$\left((n!)^{\sqrt{n}}\right)^{\frac{1}{n}} = (n!)^{\frac{\sqrt{n}}{n}} = (n!)^{\frac{1}{\sqrt{n}}}$$

Radice : $\sqrt[n]{a_n} = \underbrace{\boxed{\frac{2^n}{(n!)^{\frac{1}{\sqrt{n}}}}}}_{b_n}$

Per fare il lim di b_n, facciamo un' altra volta la radice

$$\sqrt[n]{b_n} = \frac{2}{(n!)^{\frac{1}{n\sqrt{n}}}} \to 2$$

$$\left(\sqrt[n]{n!}\right)^{\frac{1}{\sqrt{n}}} = \left(\frac{\sqrt[n]{n!}}{n} \cdot n\right)^{\frac{1}{\sqrt{n}}} = \underbrace{\overbrace{\boxed{\left(\frac{\sqrt[n]{n!}}{n}\right)^{\frac{1}{\sqrt{n}}}}}^{\to 1}}_{\to \left(\frac{1}{e}\right)^0} \cdot \overbrace{\boxed{n^{\frac{1}{\sqrt{n}}}}}^{\to 1}$$

$$n^{\frac{1}{\sqrt{n}}} = e^{\frac{\log n}{\sqrt{n}}} \to e^0$$

$\sqrt[n]{b_n} \to 2 \Rightarrow b_n \to +\infty \Rightarrow a_n \to +\infty$

$\Rightarrow$ la serie diverge (NO COND. NEC.)

$$\underbrace{\sum_{n=1}^{\infty} \left(\sqrt[n]{5} - \sqrt[n]{4}\right)^{\pi} \cdot n^{e}}_{a_n}$$

$$\sqrt[n]{5} = e^{\frac{1}{n}\log 5} \sim 1 + \frac{\log 5}{n} \; ; \; \sqrt[n]{4} = e^{\frac{1}{n}\log 4} \sim 1 + \frac{\log 4}{n}$$

$$e^{x} \sim 1+x$$

$$\sqrt[n]{5} - \sqrt[n]{4} \sim \cancel{1} + \frac{\log 5}{n} - \cancel{1} - \frac{\log 4}{n} = \frac{\log 5 - \log 4}{n}$$

$$a_n \sim \frac{(\log 5 - \log 4)^{\pi}}{n^{\pi}} \cdot n^{e} \sim \frac{\text{Numero}}{n^{\boxed{\pi - e}}}$$

esponente < 1

$\Downarrow$

Serie DIVERGE

C.A. con a_n = testo

$$b_n = \frac{1}{n^{\pi - e}}$$

volendo nel limite si può porre $\frac{1}{n} = x$

$$\sum_{n=1}^{\infty} \underbrace{\left(\sum_{k=n}^{3n} \frac{1}{k^2} \right)}_{a_n}$$

$$a_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(3n)^2}$$

$$\geq \frac{1}{(3n)^2} + \frac{1}{(3n)^2} + \dots + \frac{1}{(3n)^2}$$ ← i termini sono $2n+1$

$$= \frac{2n+1}{9n^2}$$

Abbiamo dim. che $\qquad a_n \geq \frac{2n+1}{9n^2}$

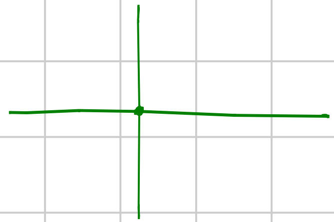
$\sum \frac{2n+1}{9n^2}$ DIVERGE per C.A. con $\sum \frac{1}{n}$ $\xRightarrow{\text{CONFR.}}$ $\sum a_n$ DIVERGE

$$\begin{cases}\phi_x=0\\ \phi_y=0\\ \phi_z=0\end{cases}$$

Prendiamo $(x_0,y_0)\in V=\{\phi=0\}$

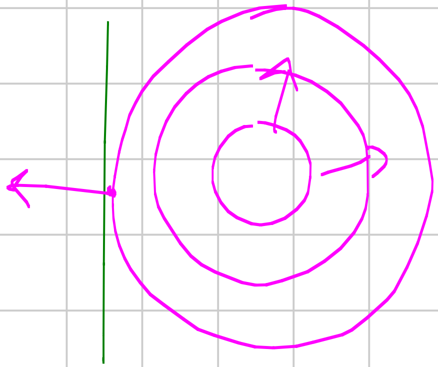
$\nabla\phi\neq 0$ se non ci sono soluz.

$\phi(x,y)=xy$ $\quad V=\{xy=0\}$



$\nabla\phi(x,y)=(y,x)$ si annulla in $(0,0)$

$V=\{\underbrace{x^2}_{\phi}=0\}$



$\nabla f=\lambda\underbrace{\nabla\phi}_{=0}$

$\nabla\phi=(2x,0)$ si annulla su tutto V

$$\lim_{x\to 0} \frac{2\cos x - 2 + x^2}{x^5}$$

$$\lim_{x\to 0^+} = +\infty$$

$$\lim_{x\to 0^-} = -\infty$$

$$\frac{2\cos x - 2 + x^2}{x^5} = \frac{2\left(1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right) - 2 + x^2}{x^5}$$

$$= \frac{\cancel{2} - \cancel{x^2} + \frac{x^4}{12} - \cancel{2} + \cancel{x^2} + o(x^5)}{x^5}$$

$$= \frac{\frac{x^4}{12} + o(x^5)}{x^5} = \boxed{\frac{1}{12x}} + \frac{o(x^5)}{x^5}$$

$$\frac{o(x^5)}{x^5} \to 0$$

$+\infty$ se $x \to 0^+$

$-\infty$ se $x \to 0^-$

$$\frac{2\cos x - 2 + x^2}{x^5} = 2\,\frac{\cos x - 1}{x^5} + \frac{x^2}{x^5} = 2\,\frac{\cos x - 1}{x^5} + \boxed{\frac{1}{x^3}}$$

$$\frac{1}{x^3} \downarrow \text{N.E.}$$

$+\infty$ se $x \to 0^+$

$-\infty$ se $x \to 0^-$

$$2\,\underbrace{\boxed{\frac{\cos x - 1}{x^2}}}_{-\frac{1}{2}} \cdot \frac{1}{x^3} + \frac{1}{x^3}$$

NO

$$-\frac{1}{x^3} + \frac{1}{x^3} \to 0$$

$$\ldots \quad \frac{\frac{x^3}{6} + o(x^3)}{\frac{x^3}{4} + o(x^3)} = \frac{\cancel{x^3}\left(\frac{1}{6} + \underbrace{\frac{o(x^3)}{x^3}}_{\to 0}\right)}{\cancel{x^3}\left(\frac{1}{4} + \underbrace{\frac{o(x^3)}{x^3}}_{\to 0}\right)} \longrightarrow \frac{1}{6} : \frac{1}{4} = \ldots$$

$$\left(2-\cos\frac{1}{n+n^2}\right)^{n^4} \sim \left(2-\cos\frac{1}{n^2}\right)^{n^4} = (2-\cos x)^{\frac{1}{x^2}}$$

$$\sim \left(2-1+\frac{x^2}{2}\right)^{\frac{1}{x^2}} = \left(1+\frac{x^2}{2}\right)^{\frac{1}{x^2}} \to e^{\frac{1}{2}}$$

$$n^4 \log(\quad)$$

$$\to e^{1/2}$$

$$e^{\boxed{n^4}\,\underset{\downarrow\,1}{\boxed{\dfrac{\log\left(1+\left(1-\cos\frac{1}{n+n^2}\right)\right)}{1-\cos\frac{1}{n+n^2}}}}\;\underset{\downarrow\,1/2}{\boxed{\dfrac{1-\cos\frac{1}{n+n^2}}{\frac{1}{(n+n^2)^2}}}}\;\boxed{\dfrac{1}{(n+n^2)^2}}} \to \frac{1}{2}$$

$$\downarrow$$

$$1$$