RICEVIMENTO MAT I TLC

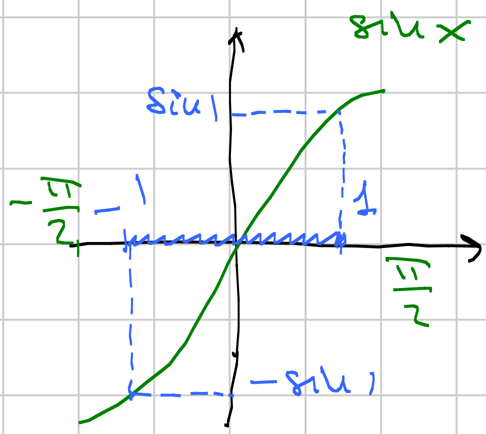
Titolo nota 16/11/2006

$$\lim_{n\to+\infty} \left\{\arctan\left(\frac{n+20}{n}\right)\right\}^n = 0.$$

Radice $\sqrt[n]{\cdots} = \arctan\left(\frac{n+20}{n}\right) \to \arctan 1 = \frac{\pi}{4} < 1$

— o — o —

$$\lim_{n\to+\infty} \left[\sin\left(\cos\left(\text{Mostro}\right)\right)\right]^n$$



$$-1 \le \cos(\text{Mostro}) \le 1$$

$$-\sin 1 = \sin(-1) \le \sin(\cos(\text{Mostro})) \le \sin 1$$

Ma allora

$$0 \leq \boxed{\left|\sin(\cos(\text{Mostro}))\right|^{n}} \leq (\sin 1)^{n}$$

$0 \to 0$, $\left|\sin(\cos(\text{Mostro}))\right|^{n} \to 0$, $(\sin 1)^n \to 0$ base < 1

— o — o —

$$\lim_{x \to 0} \left\{ \tan\left(\frac{\arccos x}{2}\right) \right\}^{\frac{1}{x}}$$

$$\arccos 0 = \frac{\pi}{2}$$

$$\boxed{1^{\infty}}$$

$$\tan\left(\frac{\pi}{4}\right) = 1$$

$$e^{\frac{1}{x} \log\left(\tan\left(\frac{\arccos x}{2}\right)\right)} \to e^{-1}.$$

$$f(x) = f(0) + x f'(0) + o(x)$$

$$\arccos x = \frac{\pi}{2} - x + o(x)$$

$$\frac{1}{x}\log\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}+o(x)\right)\right)$$

$$\tan\left(\frac{\pi}{4}+t\right)=1+2t+o(t)$$

$$=\frac{1}{x}\log\left(1-x+o(x)\right)$$

$$f\left(\frac{\pi}{4}+t\right)=f\left(\frac{\pi}{4}\right)+f'\left(\frac{\pi}{4}\right)t+o(t)$$

$$=\frac{1}{x}\left(-x+o(x)\right)\to-1$$

$$\log(1+t)=t+o(t)$$

$$\log\left(1-x+o(x)\right)=-x+o(x)$$

$$t=-x+o(x)$$

— o — o —

$$\sum\left(\sqrt{n^2+1}-n\right)$$ DIVERGE

$$\left(\sqrt{n^2+1}-n\right)\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n}=\frac{\cancel{n^2}+1-\cancel{n^2}}{\sqrt{n^2+1}+n}=\frac{1}{\sqrt{n^2+1}+n}\sim\frac{1}{2n}$$

$$\sum \left(\sqrt[n]{5} - \sqrt[n]{4}\right)^{\pi} n^{e}$$ DIVERGE

$$\sqrt[n]{5} - \sqrt[n]{4} = e^{\frac{1}{n}\log 5} - e^{\frac{1}{n}\log 4}$$

$$\sim \cancel{1} + \frac{1}{n}\log 5 - \cancel{1} - \frac{1}{n}\log 4 = \frac{\log 5 - \log 4}{n}$$

$$\left(\sqrt[n]{5} - \sqrt[n]{4}\right)^{\pi} n^{e} = \frac{(\log 5 - \log 4)^{\pi}}{n^{\pi}} n^{e} = \frac{\text{Numero}}{n^{\boxed{\pi - e}}} < 1$$

— o — o —

$$\sum \left(\sqrt[n]{n^2 + 2^n} - 2\right)$$

$$\sqrt[n]{n^2+2^n} - 2 = \sqrt[n]{2^n\left(1+\frac{n^2}{2^n}\right)} - 2 =$$

$$= 2\sqrt[n]{1+\frac{n^2}{2^n}} - 2 = 2\left(\sqrt[n]{1+\frac{n^2}{2^n}} - 1\right) =$$

$$= 2\left(e^{\frac{1}{n}\log\left(1+\frac{n^2}{2^n}\right)} - 1\right) \sim 2\left(\cancel{1+} \frac{1}{n}\log\left(1+\frac{n^2}{2^n}\right) \cancel{-1}\right)$$

$$\sim \frac{2}{n}\log\left(1+\frac{n^2}{2^n}\right) \sim \frac{2}{\cancel{n}}\,\frac{n^{\cancel{2}}}{2^n} = \frac{n}{2^{n-1}}$$

$\sum \frac{n}{2^{n-1}}$ converge $\Rightarrow$ iniziale converge

— o —

$$\sum \frac{\log(1+2^n)}{n^2}$$

$$\boxed{\frac{\log(1+2^n)}{n^2} \sim \frac{2^n}{n^2}}$$ $\Rightarrow$ la serie diverge

NOOOOO

$\log(1+t) \sim t$ vale per $t \to 0$

$$\frac{\log(1+2^n)}{n^2} \sim \frac{\log 2^n}{n^2} = \frac{n\log 2}{n^2} = \frac{\log 2}{n}$$

$$\sum \frac{2^{n^2}}{n!}$$

Radice $\sqrt[n]{\ldots} = \frac{2^n}{\sqrt[n]{n!}} = \underbrace{\boxed{\frac{2^n}{n}}}_{\to +\infty} \underbrace{\boxed{\frac{n}{\sqrt[n]{n!}}}}_{\to e} \Rightarrow +\infty$

DIVERGE

—o—o—

$$\sum (n!)\, \alpha^{n^2}$$

Caso $\alpha > 0$

Radice $\sqrt[n]{\ldots} = \sqrt[n]{n!}\ \alpha^n = \underbrace{\boxed{\frac{\sqrt[n]{n!}}{n}}}_{\to \frac{1}{e}} \cdot \boxed{n\alpha^n}$

$\alpha \geq 1 \Rightarrow +\infty$

$\Rightarrow$ serie diverge

$0 \leq \alpha < 1 \Rightarrow 0$

$\Rightarrow$ serie converge

$$\sum (-1)^n \frac{3\sqrt{n}+\cos(\pi n)}{n} = \sum (-1)^n \frac{3\sqrt{n}+(-1)^n}{n} =$$

$$= \underbrace{\sum (-1)^n \frac{3}{\sqrt{n}}}_{\substack{\text{CONV x} \\ \text{Leibnitz}}} + \underbrace{\sum \frac{1}{n}}_{\substack{\text{DIV. a} \\ +\infty}} = +\infty$$

$$\sum (-1)^n \arctan\left(\frac{n+\sin n!}{n^2+\sin(n!)^2}\right)$$

$$\arctan(\ ) \sim \arctan\frac{1}{n} \sim \frac{1}{n}$$

$$= \sum (-1)^n \left[\arctan(\ ) - \frac{1}{n} + \frac{1}{n}\right] = \underbrace{\sum (-1)^n \left[\arctan(\ ) - \frac{1}{n}\right]}_{\text{CONV. ASSOL.}}$$

$$+ \underbrace{\sum (-1)^n \frac{1}{n}}_{\text{LEIBNITZ.}}$$

$$\sum \frac{n^4}{\alpha^n + n^\alpha} \quad (\alpha > 0)$$ chi comanda al denominatore?

per $\alpha > 1$ comanda α^n $\sim \sum \frac{n^4}{\alpha^n}$ conv. (radice)

per $0 \le \alpha \le 1$ comanda n^α, che perde da n^4 $\Rightarrow$ DIV.

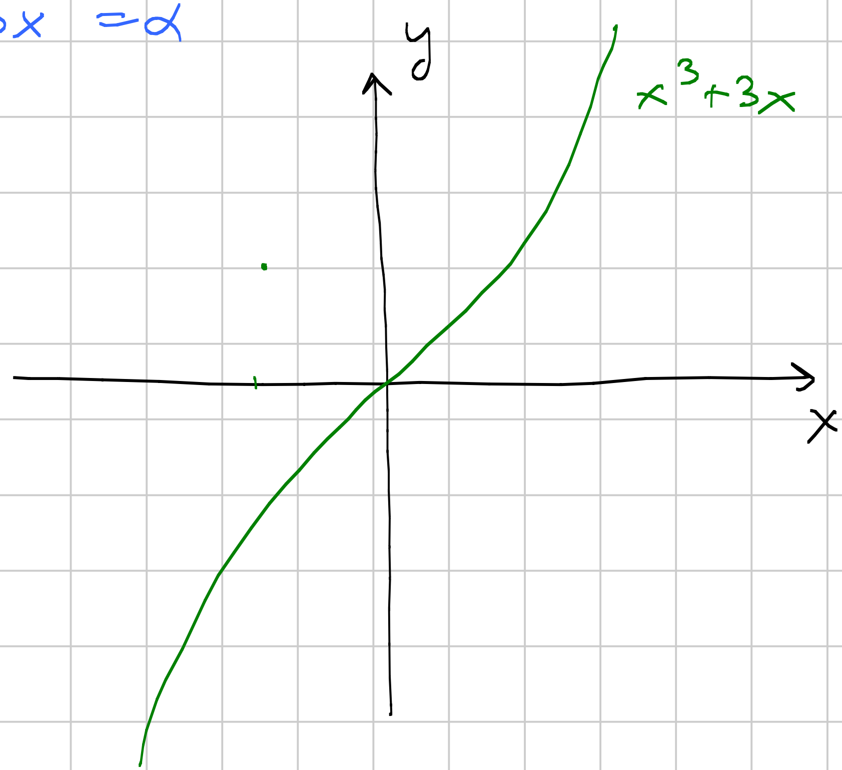
$$\sqrt[n]{\frac{n^4}{\alpha^n + n^\alpha}} = \frac{\sqrt[n]{n^4} \to 1}{\underbrace{\sqrt[n]{\alpha^n}}_{\alpha}\left(1 + \frac{n^\alpha}{\alpha^n}\right)} =$$

$1 + \frac{n^\alpha}{\alpha^n} \to 1$

vero solo se $\alpha > 1$

$$x^3+3x = \underbrace{\boxed{\lambda^3+3\lambda}}_{\alpha}$$

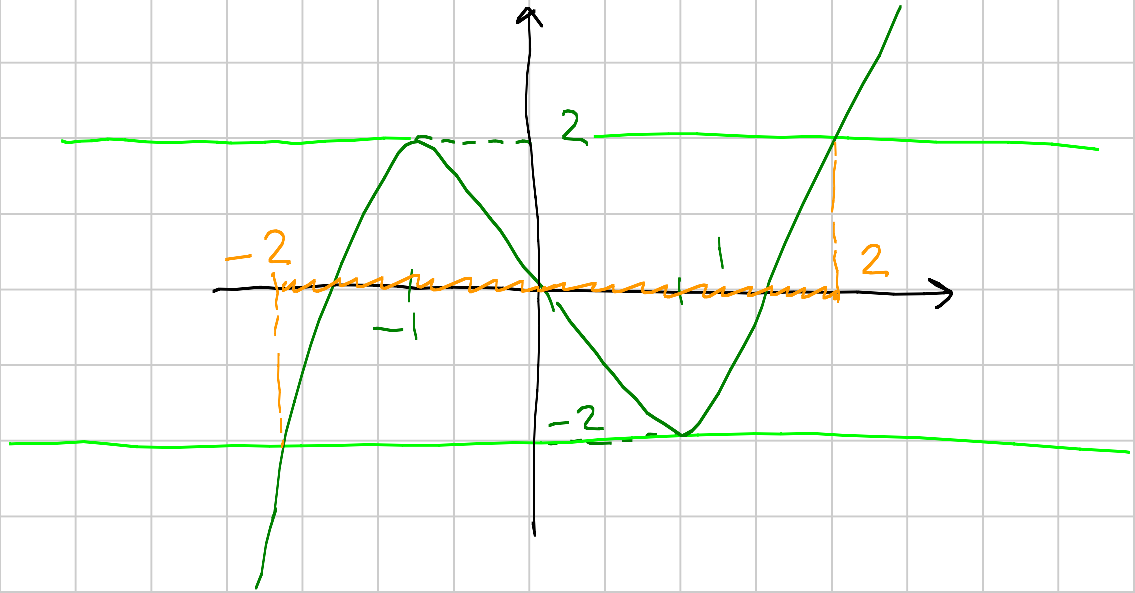
$$x^3+3x = \alpha$$



Qualunque sia il valore di α, l'eq. ha un'unica soluzione

Dato λ, $\lambda^3+3\lambda$ è comunque un numero fisso, per cui c'è comunque 1 soluz.

$$x^3 - 3x = \underbrace{\boxed{\lambda^3 - 3\lambda}}_{\alpha}$$



$$x^3 - 3x = \alpha$$

1 soluz se $\alpha < -2$

oppure $\alpha > 2$

2 soluz. se $\alpha = \pm 2$

3 soluz. se $-2 < \alpha < 2$

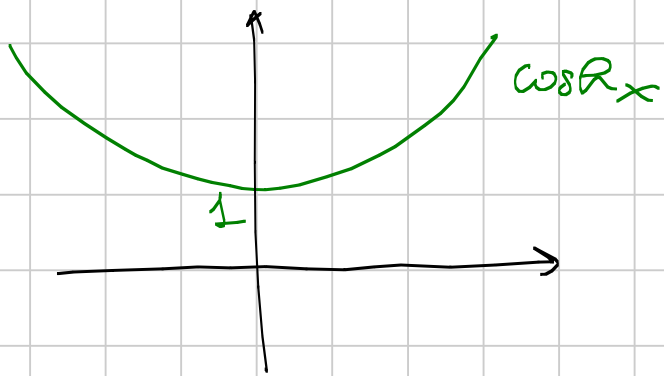
Quindi l'eq. iniziale ha 3 soluz.

quando $-2 < \lambda^3 - 3\lambda < 2$

$$\Updownarrow$$

$$-2 < \lambda < 2$$

$\cosh(x - \sin\lambda) = \lambda$



$\cosh x = \lambda$

$\lambda > 1 \Rightarrow 2$ sol.

$\lambda = 1 \Rightarrow 1$ sol.

$\lambda < 1 \Rightarrow 0$ sol.

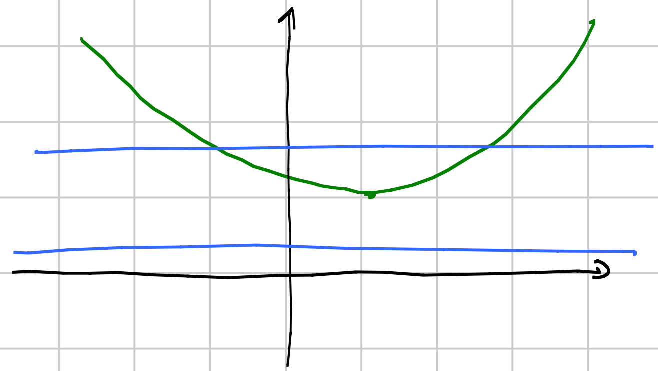
$\cosh(x - \sin\lambda)$

spostamento

dx − sx

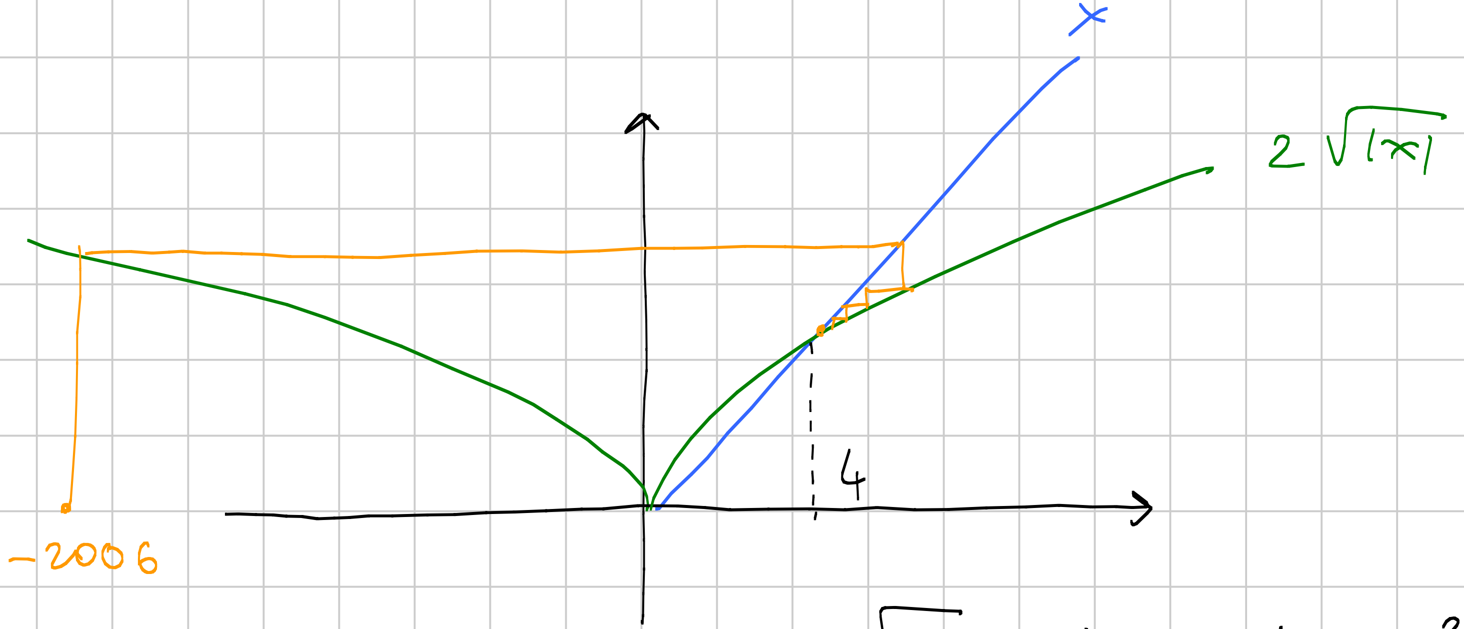
che non altera

il numero di soluz.



$\cosh(x - \text{Mostro pieno di } \lambda) = \lambda \Rightarrow$ stessa cosa

$$a_{n+1} = 2\sqrt{|a_n|} \qquad a_0 = -2006$$



$$2\sqrt{x} = x \qquad 4x = x^2$$

$\boxed{\text{PIANO}}$

(i) $a_n \geq 4 \quad \forall n \geq 1$

(ii) $a_{n+1} \leq a_n \quad \forall n \geq 1$

(iii) $a_n \to \ell \in \mathbb{R}$

(iv) $\ell = 4$