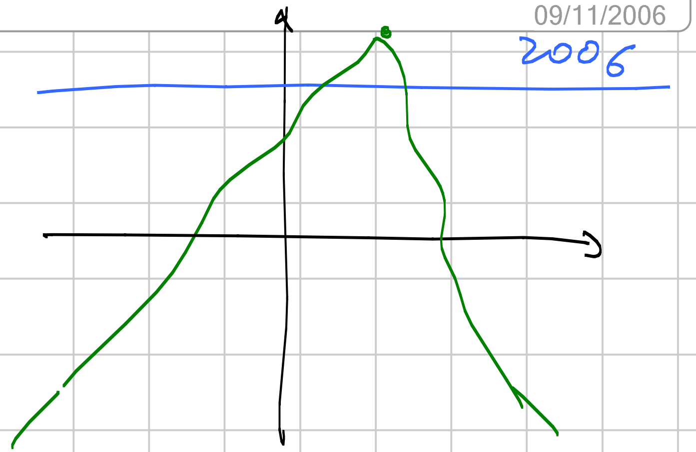
Titolo nota 09/11/2006

$f(x) = 2006$ quante soluz.

$$\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(x) = -\infty$$



Se trovi un valore di x per cui
$f(x) > 2006$

Per forza c'è un p.to di max. Se esiste f' il p.to di max sarà un p.to in $f'(x) = 0$.

$$\frac{\sin x^4 - x^4}{(\arctan x - x)^4} = \frac{\cancel{x^4} - \frac{x^{12}}{6} - \cancel{x^4} + o(x^{12})}{\left(\cancel{x} - \frac{x^3}{3} - \cancel{x} + o(x^3)\right)^4}$$

$$= \frac{-\frac{x^{12}}{6} + o(x^{12})}{\frac{x^{12}}{81} + o(x^{12})} \to -\frac{1}{6} \cdot 81 = -\frac{27}{2}$$

$$(\arctan x - x)^4 = \left(\cancel{x} - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots - \cancel{x}\right)^4$$

$$= \left[x^3\left(-\frac{1}{3} + \frac{x^2}{5} - \frac{x^4}{7} + \dots\right)\right]^4$$

$$= x^{12}\left(\frac{1}{81} + \text{roba con dentro } x\right)$$

$$= \frac{x^{12}}{81} + x^{12} \cdot \underbrace{\text{roba con dentro } x}_{\omega(x)} = \frac{x^{12}}{81} + o(x^{12})$$

$$\lim_{n\to+\infty}\left(1+\frac{2}{n}\right)^{n} = \lim_{n\to+\infty} \boxed{\left(1+\frac{2}{n}\right)^{\frac{n}{2}}}^{\cdot 2} = e^{2}$$

$$\downarrow$$

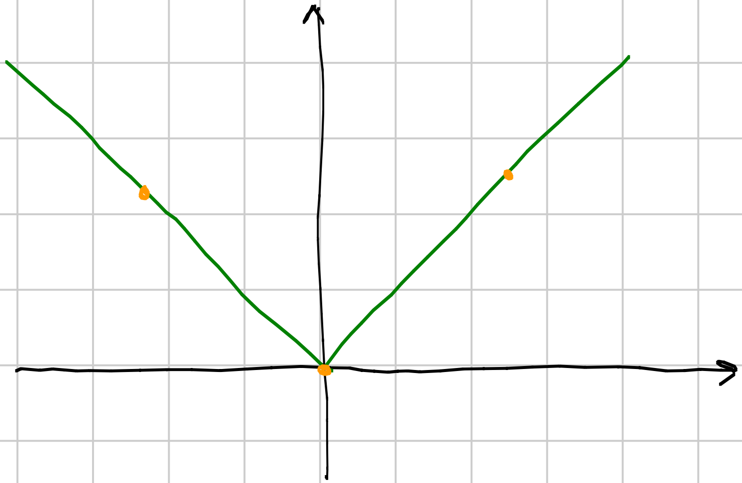
$$e$$

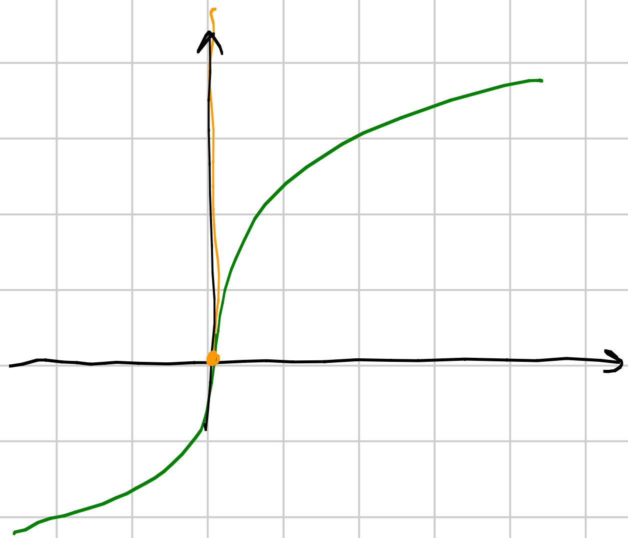
$$\lim_{n\to+\infty} e^{n\log\left(1+\frac{2}{n}\right)} = e^{2}$$

$$n\log\left(1+\frac{2}{n}\right) = \boxed{\frac{\log\left(1+\frac{2}{n}\right)}{\frac{1}{n}\cdot 2}}\cdot 2 \longrightarrow 2$$

$$\downarrow$$

$$1$$





— o —

$$\lim_{x \to 0} \frac{\log(1+\sin^2 x) - x^2}{\sin^2(\tan^2 x)} = \frac{\cancel{x^2} - \frac{2}{3}x^4 - \cancel{x^2} + o(x^4)}{x^4 + o(x^4)} \to -\frac{2}{3}$$

$$\sin^2(\tan^2 x) = x^4 + o(x^4) \; ; \qquad \sin^2 x = \left(x - \frac{x^3}{6} + o(x^4)\right)^2$$

$$= x^2 - \frac{x^4}{3} + o(x^4)$$

$$\log\Big(1+\underbrace{x^2-\frac{x^4}{3}+o(x^4)}_{t}\Big) = \underbrace{x^2-\frac{x^4}{3}}_{t} \underbrace{-\frac{x^4}{2}}_{-\frac{t^2}{2}} + o(x^4)$$

$$\frac{1}{3}+\frac{1}{2}=\frac{2+3}{6}=\frac{5}{6}$$

$$= x^2-\frac{5}{6}x^4+o(x^4)$$

— o — o —

$\sin(\sinh x)$ all'ordine 7

— o — o —

$$\left(\sin 1-\sin\frac{n+3}{n}\right)n = \left(\sin 1-\sin\left(1+\frac{3}{n}\right)\right)n$$

$$\sin(\alpha+\beta)=\dots$$

$$=\left[f(1)-f\left(1+\frac{3}{n}\right)\right]\cdot n = f'(\xi)\cdot\left[\cancel{1}-\cancel{1}-\frac{3}{\cancel{n}}\right]\cdot\cancel{n} \to -3f'(1)$$

$$f(a)-f(b) \qquad f'(\xi)\cdot(a-b) \qquad = -3\cos 1$$

$$\lim \frac{\sin 1 - \sin\left(1+\frac{3}{n}\right)}{\frac{1}{n}} \overset{x=\frac{1}{n}}{=} \lim_{x\to 0} \frac{\sin 1 - \sin(1+3x)}{x}$$

$$\overset{\text{Hop}}{=} \lim_{x\to 0} \frac{-\cos(1+3x)\cdot 3}{1} = -3\cos 1$$

— o — o —

$$\left\{ \sqrt{n+1} + \sqrt{4n+1} - \sqrt{9n+1} \right\}^{\frac{1}{\log n}}$$

$$= \left\{ \sqrt{n\left(1+\frac{1}{n}\right)} + \sqrt{4n\left(1+\frac{1}{4n}\right)} - \sqrt{9n\left(1+\frac{1}{9n}\right)} \right\}^{\frac{1}{\log n}}$$

$$= \left\{ \sqrt{n}\sqrt{1+\frac{1}{n}} + 2\sqrt{n}\sqrt{1+\frac{1}{4n}} - 3\sqrt{n}\sqrt{1+\frac{1}{9n}} \right\}^{\frac{1}{\log n}}$$

$$= \left(\sqrt{n}\right)^{\frac{1}{\log n}} \left\{ \sqrt{1+\frac{1}{n}} + 2\sqrt{1+\frac{1}{4n}} - 3\sqrt{1+\frac{1}{9n}} \right\}^{\frac{1}{\log n}}$$

$$= e^{\frac{1}{\log m}\log\sqrt{m}} \left\{ \cancel{1} + \frac{1}{2m} + \cancel{2} + \frac{1}{4m} - \cancel{3} - \frac{1}{6m} + o\left(\frac{1}{m^2}\right) \right\}^{\frac{1}{\log m}} \qquad \sqrt{1+x} = 1 + \frac{x}{2} + o(m)$$

$$= e^{\frac{1}{\cancel{\log m}}\cdot\frac{1}{2}\cancel{\log m}} \left\{ \frac{7}{12m} + o\left(\frac{1}{m^2}\right) \right\}^{\frac{1}{\log m}} \qquad \frac{1}{2} + \frac{1}{4} - \frac{1}{6} = \frac{6+3-2}{12}$$

$$= \sqrt{e} \left\{ \frac{1}{m}\left(\frac{7}{12} + o\left(\frac{1}{m}\right)\right) \right\}^{1/\log m} =$$

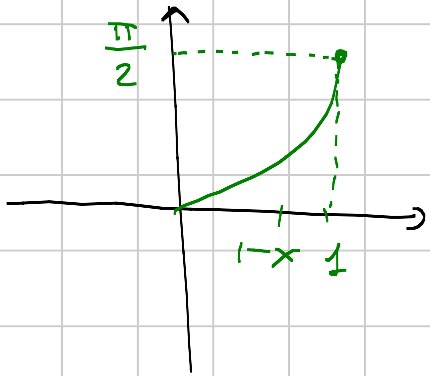
$$m^{\frac{1}{\log m}} = e^{\frac{1}{\cancel{\log m}}\cdot\cancel{\log m}} = e$$

$$= \boxed{\sqrt{e}} \; \frac{1}{m^{1/\log m}} \; \boxed{\left(\frac{7}{12} + o\left(\frac{1}{m}\right)\right)^{\frac{1}{\log m}}}$$

$$\sqrt{e} \qquad \frac{1}{e} \qquad \left(\frac{7}{12}\right)^0 = 1$$

$$\to \frac{\sqrt{e}}{e} = \boxed{\frac{1}{\sqrt{e}}}$$

$$n\left\{\arcsin\left(\frac{n-1}{n}\right) - \arcsin\left(\frac{n-2}{n}\right)\right\} =$$



$$= \frac{\left\{\arcsin\left(1-\frac{1}{n}\right) - \arcsin\left(1-\frac{2}{n}\right)\right\}}{\frac{1}{n}}$$

$$= \lim_{x\to 0^+} \frac{\arcsin(1-x) - \arcsin(1-2x)}{x} = \text{Hôpital.}$$

— o —

Per $x \to 0$ si ha che $\boxed{\arcsin(1-x) = \frac{\pi}{2} - \sqrt{2}\sqrt{x} + o(\sqrt{x})}$

$$\lim_{x\to 0} \frac{\arcsin(1-x) - \frac{\pi}{2} + \sqrt{2}\sqrt{x}}{\sqrt{x}} = 0$$

o equivalentemente

$$\lim_{x\to 0} \frac{\arcsin(1-x) - \frac{\pi}{2}}{\sqrt{x}} = -\sqrt{2} \quad \text{(si fa con Hôpital)}$$

$$= \lim_{x\to 0} \frac{\frac{-1}{\sqrt{1-(1-x)^2}}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to 0} \frac{-1}{\sqrt{\cancel{1}-\cancel{1}+2x-x^2}} \cdot 2\sqrt{x}$$

$$= \lim_{x\to 0} \frac{-1}{\cancel{\sqrt{x}}\,\sqrt{2-x}} \cdot 2\cancel{\sqrt{x}} = \frac{-2}{\sqrt{2}} = -\sqrt{2}$$

$$\frac{\left\{\arcsin\left(1-\frac{1}{n}\right) - \arcsin\left(1-\frac{2}{n}\right)\right\}}{\frac{1}{n}} = \frac{\cancel{\frac{\pi}{2}} - \sqrt{2}\,\frac{1}{\sqrt{n}} - \cancel{\frac{\pi}{2}} + \sqrt{2}\cdot\sqrt{\frac{2}{n}}}{\frac{1}{n}}$$

$$= \left(-\frac{\sqrt{2}}{\sqrt{n}} + \frac{2}{\sqrt{n}}\right)\cdot n = \frac{2-\sqrt{2}}{\sqrt{n}}\cdot n = (2-\sqrt{2})\sqrt{n} \to +\infty$$

$$\sqrt{n}\left(\sqrt{\pi}-\sqrt{\arccos\left(\frac{1-n}{n}\right)}\right) = \frac{\sqrt{\pi}-\sqrt{\arccos\left(-1+\frac{1}{n}\right)}}{\frac{1}{\sqrt{n}}} =$$

$$= \lim_{x\to 0^+} \frac{\sqrt{\pi}-\sqrt{\arccos(-1+x)}}{\sqrt{x}} = \text{Hôpital} \ldots$$

$$\arccos(-1+x) = \pi - \sqrt{2}\,\sqrt{x} + o(\sqrt{x})$$

$$\sim \frac{\sqrt{\pi}-\sqrt{\pi-\sqrt{2}\,\frac{1}{\sqrt{n}}}}{\frac{1}{\sqrt{n}}} = \frac{\sqrt{\pi}\left\{1-\sqrt{1-\frac{\sqrt{2}}{\pi}\,\frac{1}{\sqrt{n}}}\right\}}{\frac{1}{\sqrt{n}}} =$$

$$= \sqrt{\pi}\,\frac{\cancel{1}-\cancel{1}+\frac{\sqrt{2}}{2\pi}\,\cancel{\frac{1}{\sqrt{n}}}}{1/\cancel{\sqrt{n}}} \longrightarrow \sqrt{\pi}\,\frac{\sqrt{2}}{2\pi} = \frac{1}{\sqrt{2\pi}}$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{3\sqrt{n} + \cos(\pi n)}{n}$$

$\cos(\pi n) = (-1)^n$

$$\sum (-1)^n \frac{3\sqrt{n} + (-1)^n}{n} = \sum \left\{ (-1)^n \frac{3}{\sqrt{n}} + \boxed{\overset{1}{\overset{\shortparallel}{(-1)^{2n}}}}\, \frac{1}{n} \right\}$$

$$= 3\underbrace{\boxed{\sum (-1)^n \frac{1}{\sqrt{n}}}}_{\text{conv. per leibnitz}} + \underbrace{\boxed{\sum \frac{1}{n}}}_{\text{DIV a } +\infty}$$

DIV. A $+\infty$

$$\sum (-1)^n \arctan\left(\frac{n+\sin n!}{n^2+\sin n}\right)$$

Brutalm

$$\sum (-1)^n \arctan \frac{1}{n}$$

$$\sim \sum (-1)^n \frac{1}{n} \quad \text{CONV.}$$

$$= \sum (-1)^n \left\{ \frac{1}{n} - \frac{1}{n} + \arctan(\ ) \right\}$$

$$= \boxed{\sum \frac{(-1)^n}{n}} + \sum (-1)^n \boxed{\left\{ \arctan(\ ) - \frac{1}{n} \right\}}$$

CONV.
LEIBNITZ

SI SPERA CHE
QUESTO SI COMPORTI
ALMENO COME $\frac{1}{n^2}$

FUNZIONA !!

$$\sum \frac{2+\cos n}{n}$$

$$\frac{2+\cos n}{n} \geq \frac{1}{n}$$

$$\sum \frac{1}{n} \text{ div.} \Rightarrow \sum \frac{2+\cos n}{n} \text{ div.}$$

— o — o —

$$\sum_{n=1}^{\infty} \underbrace{\boxed{\left(\sum_{k=n}^{2n} \frac{1}{k^3} \right)}}_{a_n}$$

$$a_n = \frac{1}{n^3} + \frac{1}{(n+1)^3} + \dots + \frac{1}{(2n)^3} \leq \frac{1}{n^3} + \frac{1}{n^3} + \dots + \frac{1}{n^3} = \frac{n+1}{n^3}$$

$$\sum \frac{n+1}{n^3} \text{ conv.} \Rightarrow \sum a_n \text{ conv.}$$

$$\lim_{n\to+\infty} \frac{\log(1+n)}{n} = 0; \qquad \lim_{x\to 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x\to 0} \frac{\log(1+\cos x)}{\cos x} = \log 2; \qquad \lim_{x\to+\infty} \frac{\log(1+x)}{x} = 0$$

$$\sum \sqrt{n}\arctan\frac{1}{n^2} \sim \sum \frac{\sqrt{n}}{n^2} = \sum \frac{1}{n^{3/2}} \quad \text{conv.}$$

Rigoroso: C.A. con $\frac{1}{n^{3/2}}$

$$\lim_{n\to+\infty} \frac{\sqrt{n}\arctan\frac{1}{n^2}}{\frac{1}{n^{3/2}}} = \lim_{n\to+\infty} \frac{\arctan\frac{1}{n^2}}{\frac{1}{n^2}} = 1$$

stesso comport.

$$\sum \frac{|2n^2-n^3| - |n^3-1000|}{n^4-8} \underset{\substack{\uparrow \\ \text{definit.}}}{=} \sum \frac{n^3-2n^2-n^3+1000}{n^4-8}$$

$$= \sum \frac{-2n^2+1000}{n^4-8} \qquad \text{CONV.}$$