Sistemi di disequazioni di primo grado

$$\begin{cases} x + 3 > 0 & (1) \\ x - 1 \leq 0 & (2) \end{cases}$$

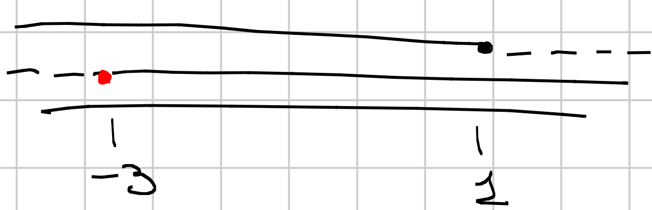
cerco gli x che verificano tutte e 2 le disequazioni

(1) $x + 3 > 0 \rightarrow x > -3$

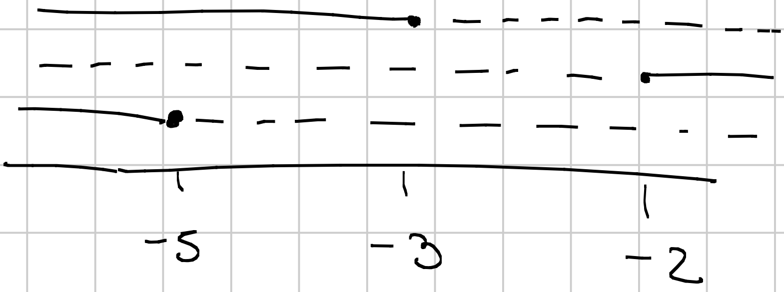
(2) $x - 1 \leq 0 \rightarrow x \leq 1$

$]-3, 1]$

(1)



$$\begin{cases} x+5 \leq 0 & (1) \\ -x \leq 2 & (2) \\ -x \geq 3 & (3) \end{cases}$$



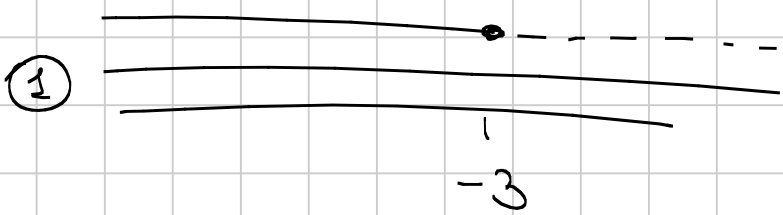
(1) $x \leq -5$

(2) $x \geq -2$

(3) $x \leq -3$

Soluzione $\emptyset$

$$\begin{cases} x^2+1 \geq 1 & (1) \\ x-3 \geq 2x & (2) \end{cases}$$

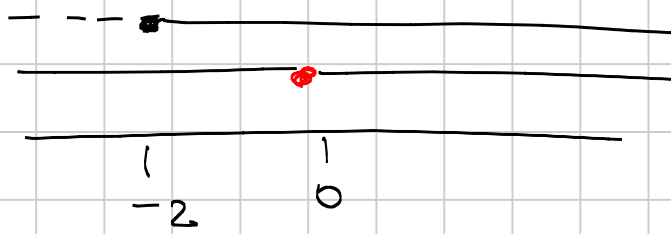


(1) $x^2 \geq 0$ sempre

(2) $x \leq -3$

Soluzione $]-\infty, -3]$

$$\begin{cases} x^2+1 > 1 & (1) \\ x-2 \leq 2x & (2) \end{cases}$$



(1) $x^2 > 0$ $\quad x \neq 0$

(2) $x \geq -2$

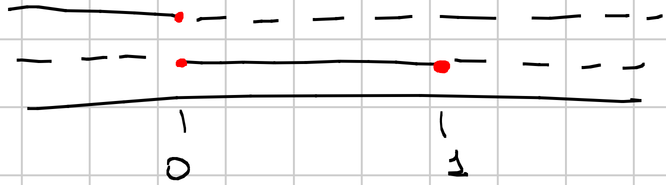
$[-2, 0[\cup]0, +\infty[$

$$\begin{cases} x^2 < x & (1) \\ x < 0 & (2) \end{cases}$$

(1) $\underbrace{x^2 - x}_{x(x-1)} < 0 \longrightarrow]0, 1[$ (radici sono $x=0$, $x=1$)

(2) $x < 0$

Soluzione $\emptyset$



$$\begin{cases} x^2 \le x \\ x \le 0 \end{cases}$$

nel disegno i punti rossi diventano neri, cioè sono compresi

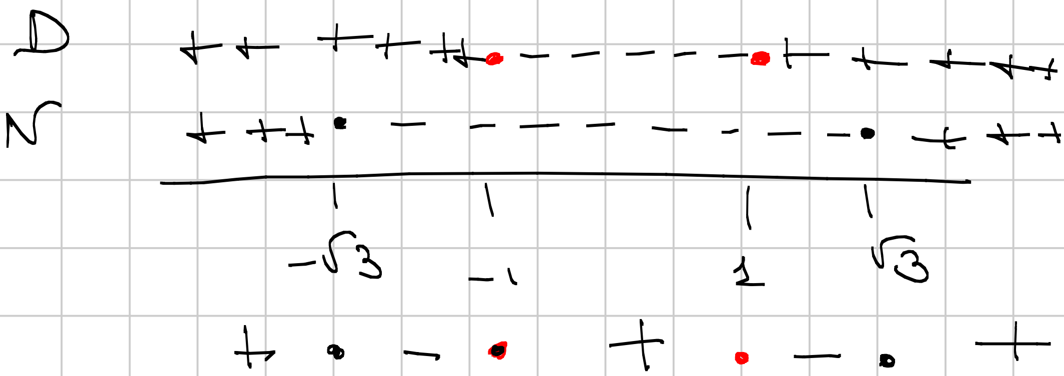
Soluzione $x = 0$

Studiamo il segno

$$\begin{array}{l} N \\ D \end{array} \quad \frac{x^2-3}{x^2-1}$$

N $\quad x^2-3>0 \quad$ se $\quad x<-\sqrt{3} \cup x>\sqrt{3} \quad e =0 \quad x=\sqrt{3}, x=-\sqrt{3}$

D $\quad x^2-1>0 \quad$ se $\quad x<-1 \cup x>1 \quad e=0 \quad x=1, x=-1$



$$\frac{x^2-3}{x^2-1} \ge 0$$

$$]-\infty, -\sqrt{3}] \cup]-1, 1[\cup [\sqrt{3}, +\infty[$$

$$x^8 + 3x^4 \ge 0 \longrightarrow x^4(x^4+3) \ge 0$$

sempre $\qquad x^4 \geq 0$ sempre

$x^4 + 3 > 0$ sempre

— . —

$x^3 + x \geq 0 \longrightarrow x\,(x^2+1) \geq 0$

(1) $\qquad$ (2)

(2) $x^2 + 1 > 0$ sempre

(1) $x \geq 0$ se $x \geq 0 \longrightarrow x \geq 0$

— . —

$x^6 - 2x^3 + 1 > 0 \qquad x^3 = y$

$y^2 - 2y + 1 = (y-1)^2$

$(x^3 - 1)^2 > 0$ $\qquad$ Soluzioni : $x^3 \neq 1$

ma $x^3 = 1$ se e solo se $x = 1$

Soluzione ; $x \neq 1$, $]-\infty, 1[\ \cup\ ]1, +\infty[$

— . —

$(x+2)^3 \geq 0$ se e solo se $x + 2 \geq 0$

Soluzione $[-2, +\infty[$

$(x+2)^{2009} \geq 0$ se e solo se $x + 2 \geq 0$.

In generale $(f(x))^n \geq 0$ con n dispari se e solo se $f(x) \geq 0$

$(x+2)^3 \geq 8 \longrightarrow x+2 \geq \sqrt[3]{8} = 2$

— · —

$(x+4)^4 \geq 0$ sempre

$(f(x))^n \geq 0$ se n pari

sempre vero.

$(x+4)^{1000} > 0$ se $x \neq -4$

$]-\infty, -4[\cup]-4, +\infty[$

— · —

N $\dfrac{x(x-2)^3}{x+6} \geq 0$

D

$[0,2] \cup]6, +\infty[$

$]-\infty, -6[\cup [0,2]$

$]-6, 0] \cup [2, +\infty[$

D $x+6 > 0$ se $x > -6$ $=0$ se $x = -6$

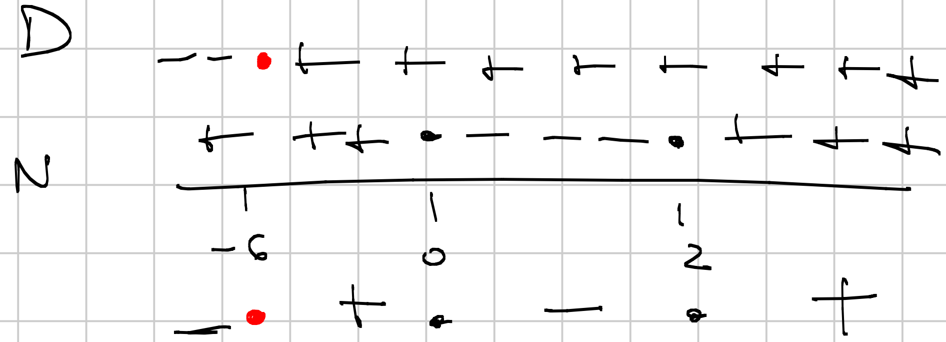
N $x(x-2)^3 \geq 0$

① $x > 0$ se $x > 0$ e $=0$ $x = 0$

② $(x-2)^3 > 0$ se $x-2 > 0$ e $=0$ se $x = 2$

		0		2	
②	−	−	−	•	+
①	−	•	+	+	+
	+	•	−	•	+

$N > 0$ se $x < 0 \cup x > 2$ e $= 0$ $x = 0$ opp. $x = 2$



$]-6, 0] \cup [2, +\infty[$

$\frac{1}{x} \geq 10$

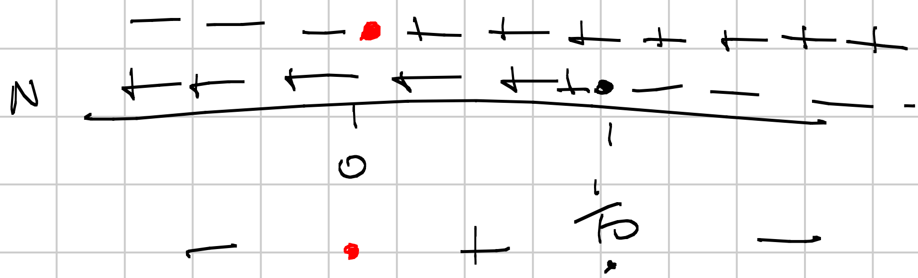
$x \leq \frac{1}{10}$, $x \leq -\frac{1}{10}$

$0 < x \leq \frac{1}{10}$)

$\frac{1}{x} - 10 \geq 0 \longrightarrow \frac{1 - 10x}{x} \geq 0$

$N = 1 - 10x > 0$ se $x < \frac{1}{10}$ e $= 0$ $x = \frac{1}{10}$

D $x > 0$ se $x > 0$ e $= 0$ $x \leq 0$



Soluzione $]0, \frac{1}{10}]$

$\frac{x}{x+1} > 2$

$x > -\frac{2}{3}$, $]-2, -1[$,

$[-2, -1[$, $]-1, 1[$,

$$\frac{x}{x+1} - 2 \geq 0 \longrightarrow \frac{x - 2(x+1)}{x+1} \geq 0 \longrightarrow$$

$$\frac{x - 2x - 2}{x+1} \geq 0 \qquad \frac{-x-2}{x+1} \geq 0$$

N $\quad -x - 2 \geq 0 \longrightarrow x < -2 \;,\; = 0$ se $x = -2$

D $\quad x+1 > 0$ se $x > -1 \quad = 0$ se $x = -1$

		-2		-1	
D	$-$		$-$	$\bullet$	$+$
N	$+$	$\bullet$	$-$		$-$
	$-$	$\bullet$	$+$	$\bullet$	$-$

Soluzione $\;]-2,-1[$

$$x^{2000} + x^{1000} - 2 < 0$$

$\mathbb{R}$, $\quad x \neq 1$, $\quad]-1,1[$, $\quad]-2,1[\quad]-\infty,1[$

$]1,\infty[$

$$x^{1000} = y \qquad y^2 + y - 2 = (y+2)(y-1)$$

$$(x^{1000}+2)(x^{1000}-1) < 0$$

① $\qquad$ ②

① $\quad x^{1000} + 2 > 0 \quad$ sempre

② $\quad (x^{1000} - 1) > 0 \longrightarrow x < -1 \;\cup\; x > 1$

Soluzioni

$]-1, 1[$

② + + + • − − − • + + +

① + + + + + + + + + + +

$-1 \qquad 1$

$+ \quad \bullet \quad - \quad \bullet \quad +$

$x^n < a \qquad (a > 0),$ n pari

vero se $x \in]-\sqrt[n]{a}, \sqrt[n]{a}[$

$x^n > a$ se $x < -\sqrt[n]{a}$ oppure $x > \sqrt[n]{a}$

$x^{10} < 2 \qquad]-\sqrt[10]{2}, \sqrt[10]{2}[$

$3^x \leq 3^2 \rightarrow x \leq 2$

$3^{2x} \leq 3^{x+1} \rightarrow 2x \leq x+1 \rightarrow x \leq 1$

$3^{x^2+x} \leq 1$

$1 = 3^0 \qquad 3^{x^2+x} \leq 3^0 \rightarrow x^2+x \leq 0$

$\rightarrow x(x+1) \leq 0 \rightarrow [-1, 0]$

$3^x \leq 2 = 3^{\log_3 2} \rightarrow x \leq \log_3 2$

$$4^x \geq 2 \qquad 4^x = 2^{2x}$$

$$\downarrow \qquad \qquad \| $$

$$\qquad \qquad (2^2)^x$$

$$2^{2x} \geq 2 \qquad 2x \geq 1 \longrightarrow x \geq \frac{1}{2}$$

$$\left(\frac{1}{3}\right)^x \geq 3$$

1° metodo in base 3

$$\left(\frac{1}{3}\right)^x = 3^{-x} \qquad 3^{-x} \geq 3^1$$

$$\longrightarrow \quad -x \geq 1 \longrightarrow x \leq -1$$

2° metodo in base $\frac{1}{3}$

$$\left(\frac{1}{3}\right)^x \geq \left(\frac{1}{3}\right)^{-1} \longrightarrow x \leq -1$$

in generale

$a^x \geq a^y$ e $\boxed{a > 1}$ $\xrightarrow{\text{se e solo se}}$ $\boxed{x \geq y}$

$a^x \geq a^y$ e $\boxed{a < 1}$ $\xrightarrow{\text{se e solo se}}$ $\boxed{x \leq y}$