PRECORSO 2009 1.1

Titolo nota 21/09/2009

Esercizi su potenze - radici - logaritmi

① $x = 10^{100}$. Quanto fa $\frac{x}{100}$. Ridurre tutto in base 10

$$\frac{x}{100} = \frac{10^{100}}{10^2} = 10^{98}$$ (Applico $A^x : A^y = A^{x-y}$, $A^x \cdot A^y = A^{x+y}$)

Quanto fa $\sqrt{x} = x^{\frac{1}{2}} = (10^{100})^{\frac{1}{2}} = 10^{100\cdot\frac{1}{2}} = 10^{50}$

(Ho applicato la potenza di potenza: $(A^x)^y = A^{xy}$)

Attenzione: $2^{(3^2)} = 2^9$; $(2^3)^2 = 2^3 \cdot 2^3 = 2^6$

$x^5 = (10^{100})^5 = 10^{100\cdot 5} = 10^{500} = 100^a$ Quanto vale a ?

Mi riduco alla stessa base: $10^{500} = (10^2)^a = 10^{2a}$

$2a = 500$, quindi $a = 250$. $10^{500} = 100^a$

② $1000^{1000} = 100^a$. Vado in base 10: $(10^3)^{1000} = (10^2)^a$

$10^{3000} = 10^{2a}$, $2a = 3000$, $a = 1500$

③ $1000^a = \frac{1}{100}$, $(10^3)^a = 10^{-2}$, $10^{3a} = 10^{-2}$, $a = -\frac{2}{3}$

In generale $A^{\frac{p}{q}} = \sqrt[q]{a^p}$ $\qquad 10^{\frac{2}{3}} = \sqrt[3]{10^2} = \sqrt[3]{100}$

$$10^{-\frac{2}{3}} = \frac{1}{10^{2/3}} = \frac{1}{\sqrt[3]{100}}$$

④ $100^{100} + 100^{101} = 100^{201}$ NO!!! $(A^x + A^y \neq A^{x+y})$

$= 101 \cdot 100^{100}$

$100^{100} + 100^{101} = 100^{100} + 100 \cdot 100^{100} = 100^{100}(1+100) = 101 \cdot 100^{100}$

CANE + 100 · CANE $\qquad$ = 101 · CANE

(5) $a^{3/2} = 27$

$a = 27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}}$
$= 3^2 = 9$

$a = 9$

$9^{3/2} = 27$

$a^b = c$ se voglio a elevo dx e sx alla $\frac{1}{b}$

$(a^b)^{\frac{1}{b}} = c^{\frac{1}{b}}$, $a^{\cancel{b}\cdot\frac{1}{\cancel{b}}} = c^{\frac{1}{b}}$, $a = c^{\frac{1}{b}}$

Se invece voglio b, allora $\boxed{b = \log_a c}$

definizione di logaritmo

(6) $a^{-1/2} = \frac{1}{4}$ $\qquad a^{-\frac{1}{2}} = \frac{1}{\sqrt{a}}$ Quindi $\frac{1}{\sqrt{a}} = \frac{1}{4}$, $\sqrt{a} = 4$, $a = 16$

In alternativa; elevo tutto alla -2

$$\left(a^{-\frac{1}{2}}\right)^{-2} = \left(\frac{1}{4}\right)^{-2}, \qquad a = \left(\frac{1}{4}\right)^{-2} = \left(2^{-2}\right)^{-2} = 2^4 = 16$$

— o —

(7) $8^a = 4$ Vado in base 2: $(2^3)^a = 2^2$, $2^{3a} = 2^2$, $3a = 2$, $a = \frac{2}{3}$

In alternativa: faccio a dx e sx il log in base 2:

$\log_2(8^a) = \log_2 4$

$a \log_2 8 = \log_2 4$

$a \; 3 = 2 \quad \rightarrow a = \frac{2}{3}$

[Proprietà dei logaritmi: $\log_a B^c = c \log_a B$]

$\log_2 8 = \log_2 2^3 = 3 \log_2 2 = 3$

— o — o —

(8) $2^{20} - 2^{19} = 2^a$ $\qquad 2^{20} - 2^{19} = 2^{20-19} = 2^1$ NO!!!!

$2^{20} = 2 \cdot 2^{19}$, quindi $2^{20} - 2^{19} = 2 \cdot 2^{19} - 2^{19} = 2^{19}$ $\quad (a = 19)$

— o — o —

(9) $2^{20} - 2^{18} = 2^2 \cdot 2^{18} - 2^{18} = 4 \cdot 2^{18} - 2^{18} = 3 \cdot 2^{18}$

— o — o —

Proprietà potenze: $A^x \cdot B^x = (AB)^x$

(10) $\sqrt{2} \cdot \sqrt{3} = \sqrt{a}$ $\qquad \sqrt{2} \cdot \sqrt{3} = 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = (2 \cdot 3)^{\frac{1}{2}} = 6^{\frac{1}{2}} = \sqrt{6}$

(11) $\sqrt{2} \cdot \sqrt{3} = \sqrt[4]{a}$ $\qquad 1^\circ$ modo: $\sqrt{2} \cdot \sqrt{3} = \sqrt{6} = \sqrt[4]{a}$; $6^{\frac{1}{2}} = a^{\frac{1}{4}}$

Elevo alla 4^a:

$$\left(6^{\frac{1}{2}}\right)^4 = \left(a^{\frac{1}{4}}\right)^4 \Rightarrow a = 6^2 = 36$$

2° modo: scrivo direttamente le radici come potenze frazionarie:

$\sqrt{2}\cdot\sqrt{3}=\sqrt[4]{a}$; $2^{\frac{1}{2}}\cdot 3^{\frac{1}{2}}=a^{\frac{1}{4}}$, elevo alla quarta

$\left(2^{\frac{1}{2}}\cdot 3^{\frac{1}{2}}\right)^4=a$, $\left(2^{\frac{1}{2}}\right)^4\cdot\left(3^{\frac{1}{2}}\right)^4=a$, $2^2\cdot 3^2=a$, $36=a$

(12) $\sqrt[3]{\sqrt{27}}=\left(27^{\frac{1}{2}}\right)^{\frac{1}{3}}=27^{\frac{1}{2}\cdot\frac{1}{3}}=27^{\frac{1}{6}}=\left(3^3\right)^{\frac{1}{6}}=3^{3\cdot\frac{1}{6}}=3^{\frac{1}{2}}=\sqrt{3}$

(13) $\sqrt{\sqrt[3]{2}}=\sqrt[8]{a}$, $\left(2^{\frac{1}{3}}\right)^{\frac{1}{2}}=a^{\frac{1}{8}}$, $2^{\frac{1}{6}}=a^{\frac{1}{8}}$ elevo alla 8:

$\left(2^{\frac{1}{6}}\right)^8=\left(a^{\frac{1}{8}}\right)^8$, $2^{\frac{1}{6}\cdot 8}=a$, $a=2^{\frac{4}{3}}$, $a=\sqrt[3]{16}$

(14) $\sqrt{2}\cdot\sqrt[3]{2}\cdot\sqrt[4]{2}=2^a$, $2^{\frac{1}{2}}\cdot 2^{\frac{1}{3}}\cdot 2^{\frac{1}{4}}=2^a$, $2^{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}=2^a$

$a=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{6+4+3}{12}=\frac{13}{12}$

(15) $\sqrt[a]{2}\cdot\sqrt[3]{2}=2$ $\qquad 2^{\frac{1}{a}}\cdot 2^{\frac{1}{3}}=2^1$, $\frac{1}{a}+\frac{1}{3}=1$, $\frac{1}{a}=\frac{2}{3}$, $a=\frac{3}{2}$

(16) $\sqrt{2\sqrt{2}}$ 1° modo: $\sqrt{2^1\cdot 2^{\frac{1}{2}}}=\sqrt{2^{1+\frac{1}{2}}}=\sqrt{2^{3/2}}=\left(2^{\frac{3}{2}}\right)^{\frac{1}{2}}$

(stessa base)

$=2^{\frac{3}{4}}=\sqrt[4]{2^3}=\sqrt[4]{8}$

2° modo: (stesso esponente): $\sqrt{\boxed{2}\sqrt{2}}=\sqrt{\boxed{\sqrt{4}}\cdot\sqrt{2}}=\sqrt{\sqrt{8}}=\sqrt[4]{8}$

(17) $\sqrt{3\sqrt{2\sqrt{3}}}=\sqrt{3\sqrt{\sqrt{4}\cdot\sqrt{3}}}=\sqrt{3\sqrt{\sqrt{12}}}=$

$=\sqrt{3\sqrt[4]{12}}=\sqrt{\sqrt[4]{3^4}\sqrt[4]{12}}=\sqrt{\sqrt[4]{3^4\cdot 12}}=\sqrt[8]{3^4\cdot 12}$

$=\sqrt[8]{81\cdot 12}=\sqrt[8]{972}$

L'esercizio chiedeva: $\sqrt{3\sqrt{2\sqrt{3}}}=\sqrt[8]{a}$, elevo al quadrato:

$3\sqrt{2\sqrt{3}}=\left(a^{\frac{1}{8}}\right)^2=a^{\frac{1}{4}}$; elevo nuovamente al quadrato:

$\left(3\sqrt{2\sqrt{3}}\right)^2 = \left(a^{\frac{1}{4}}\right)^2$; $9 \cdot 2\sqrt{3} = a^{\frac{1}{2}}$; elevo ancora al $\square$:

$81 \cdot 4 \cdot 3 = a \Rightarrow a = 81 \cdot 12 = 972$

(18) $\sqrt{32} - \sqrt{2} = \sqrt{32-2} = \sqrt{30}$ No!!!!! $(A^x - B^x \overset{\text{NO}}{=} (A-B)^x)$

$\sqrt{32} = \sqrt{2^5} = \sqrt{2^4 \cdot 2} = \sqrt{2^4} \cdot \sqrt{2} = 4\sqrt{2}$
$= \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

$\sqrt{32} - \sqrt{2} = 4\sqrt{2} - \sqrt{2} = 3\sqrt{2} = \sqrt{18}$ $\left(3\sqrt{2} = \sqrt{9} \cdot \sqrt{2} = \sqrt{9 \cdot 2} = \sqrt{18}\right)$

— o — o —

Equazioni con potenze:

(1) $2^{x+5} = 4^{2x+1}$ [Idea fondamentale: stessa base. Si arriva a $A^{f(x)} = A^{g(x)}$ da cui $f(x) = g(x)$]

Andiamo in base 2:

$2^{x+5} = \left(2^2\right)^{2x+1}$

$2^{\overset{f(x)}{\boxed{x+5}}} = 2^{\overset{g(x)}{\boxed{4x+2}}}$ $x+5 = 4x+2 \Rightarrow 3x = 3 \Rightarrow x = 1.$

— o —

(2) $4 \cdot 2^{x^2} = 8^x$ Vado in base 2: $2^2 \cdot 2^{x^2} = \left(2^3\right)^x$

$2^{\boxed{2+x^2}} = 2^{\boxed{3x}}$, $x^2+2 = 3x$, $x^2-3x+2=0$, $(x-2)(x-1)=0$

quindi $x=2$ oppure $x=1$.

[$x^2 + Ax + B = 0$ (è importante che il coeff. di x^2 sia 1!!!)
Allora B = Prodotto delle radici, A = − Somma radici]

$x^2 - 3x + 2 = 0$ $P = 2$ $S = 3$ Radici $= 1, 2$

(3) $2^{x^2} = 4$
$2^{x^2} = 2^2$
$x^2 = 2$, $x = \pm\sqrt{2}$

$\left(2^x\right)^2 = 4$
$2^{2x} = 2^2$
$2x = 2$, $x = 1$

(4) $4^x - 2^{x+2} + 3 = 0 \qquad 2^{2x} - 2^2 \cdot 2^x + 3 = 0$

$(2^x)^2 - 4 \cdot 2^x + 3 = 0$ Pongo $y = 2^x$ e ottengo

$y^2 - 4y + 3 = 0$ (Prodotto : 3 Somma : 4 ⇒ Radici : 1, 3)

$y = \begin{cases} 1 \\ 3 \end{cases}$ Torno in x : $2^x = 1 \qquad 2^x = 2^0 \qquad \boxed{x = 0}$

$2^x = 3 \qquad \boxed{x = \log_2 3}$

(5) $x^4 - 4x^2 + 3 = 0$ Pongo $y = x^2$ e diventa $y^2 - 4y + 3 = 0$

Quindi $y = \begin{cases} 1 \\ 3 \end{cases}$ $x^2 = 1 \qquad x = \pm 1$

$x^2 = 3 \qquad x = \pm\sqrt{3}$ 4 soluzioni

(6) $x^4 - 2x^2 - 3 = 0$ Pongo $y = x^2$ e diventa $y^2 - 2y - 3 = 0$

($P = -3$, $S = 2$, Radici : 3, −1). Quindi

$y = \begin{cases} 3 \\ -1 \end{cases}$ $x^2 = 3 \qquad x = \pm\sqrt{3}$

$x^2 = -1$ NULLA 2 soluzioni (reali)

(7) $x^6 - 2x^3 - 3 = 0$ Pongo $y = x^3$ e diventa $y^2 - 2y - 3 = 0$

$y = \begin{cases} 3 \\ -1 \end{cases}$ $x^3 = 3 \qquad x = \sqrt[3]{3}$

$x^3 = -1 \qquad x = \sqrt[3]{-1} = -1$ 2 soluzioni

(8) $x^4 + 5x^2 + 4 = 0$ Pongo $y = x^2$ e diventa $y^2 + 5y + 4 = 0$

(Prodotto : 4, Somma : −5, Radici : −4, −1)

$y = \begin{cases} -4 \\ -1 \end{cases}$ $x^2 = -4$ NULLA

$x^2 = -1$ NULLA

Si poteva vedere subito !! $\underbrace{x^4}_{\geq 0} + \underbrace{5x^2}_{\geq 0} + 4 \geq 4$ quindi non sarà mai zero !