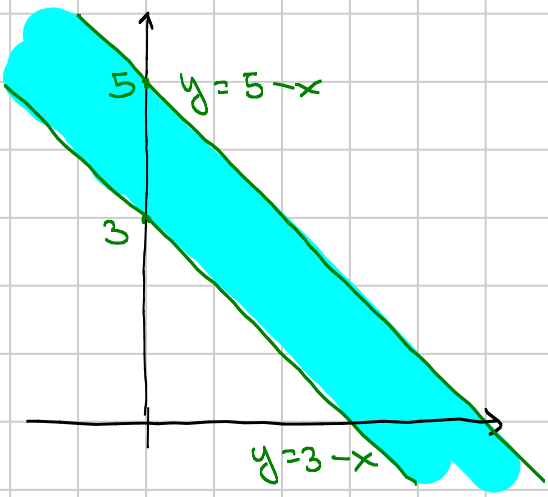
INSIEMI DEL PIANO

Es. 1 $3 \le x+y \le 5$. Sono 2 condizioni che devono essere verificate contemporaneamente

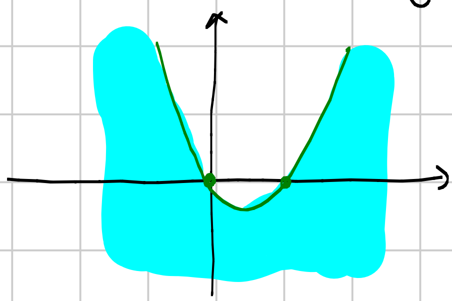
$x+y \ge 3 \iff y \ge 3-x$ "sopra la retta $y = 3-x$"

$x+y \le 5 \iff y \le 5-x$ "sotto la retta $y = 5-x$"



Es. 2 $y \le x^2 - x$

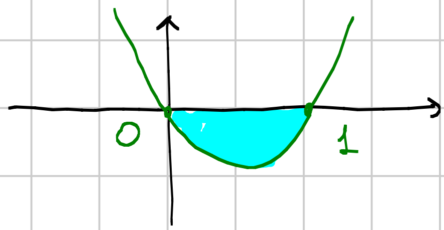
"sotto la parabola $y = x^2-x$"



Es. 3 $x^2 - x \le y \le 0$ Le 2 condizioni devono valere contemporaneamente

$y \le 0$ "sotto l'asse x"

$y \ge x^2 - x$ "sopra la parabola"



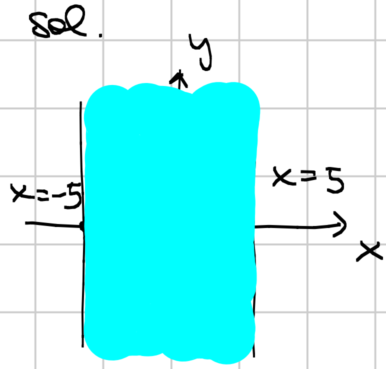
Es. 4 $|x| \le 5$ vuol dire $-5 \le x \le 5$

(in generale la disequazione $|x| \le A$ ← dato ha come sol:

- $\emptyset$ se $A < 0$
- $x = 0$ se $A = 0$
- $-A \le x \le A$ se $A > 0$.

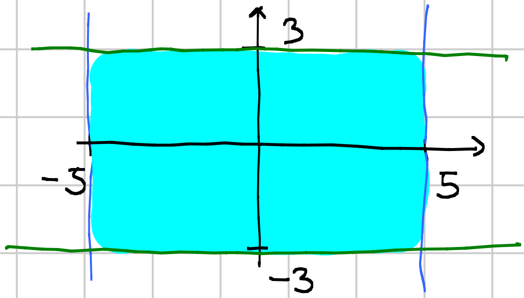
Similmente la diseq. $|x| \ge A$ ha come sol.

- tutto $\mathbb{R}$ se $A \le 0$
- $x \le -A$ e $x \ge A$ se $A > 0$)

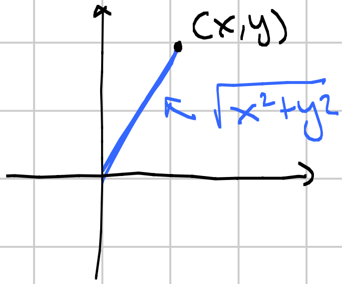


Es. 5 $\quad \underbrace{|x| \leq 5}_{\downarrow} \quad \underbrace{|y| \leq 3}_{\downarrow}$

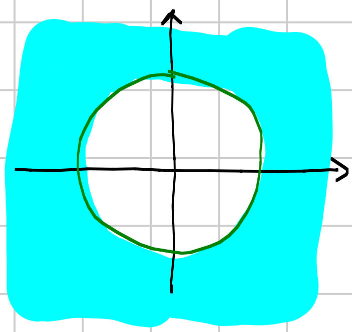
$-5 \leq x \leq 5, \quad -3 \leq y \leq 3$



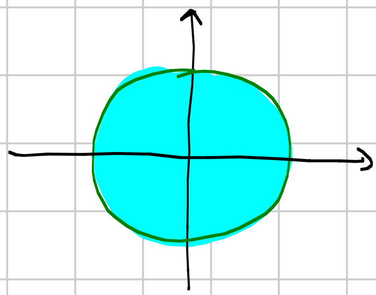
Es. 8 $\quad x^2+y^2 \geq 8$



x^2+y^2 è la distanza (al quadrato) di (x,y) dall'origine.
I p.ti (x,y) con $x^2+y^2 \geq 8$ sono i p.ti del piano con distanza dall'origine $\geq \sqrt{8} = 2\sqrt{2}$.
Quindi sono i p.ti esterni alla circ. con centro in $(0,0)$ e raggio $2\sqrt{2}$



$x^2+y^2 \geq 8$



$x^2+y^2 \leq 8$

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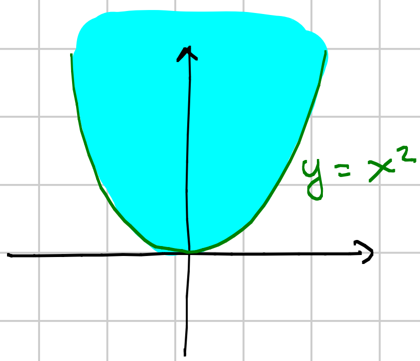
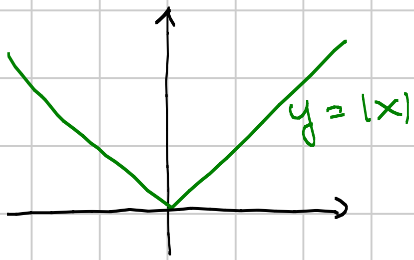
Es. 9 $\quad y - x^2 \geq 0$

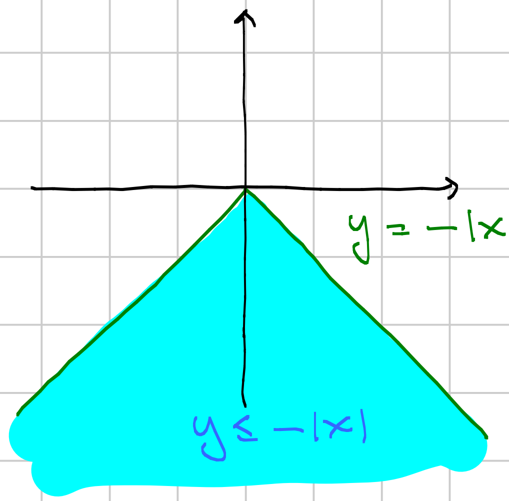
$y \geq x^2$

"Sopra la parabola"



Es. 10 $\quad y + |x| \leq 0 \qquad y \leq -|x|$



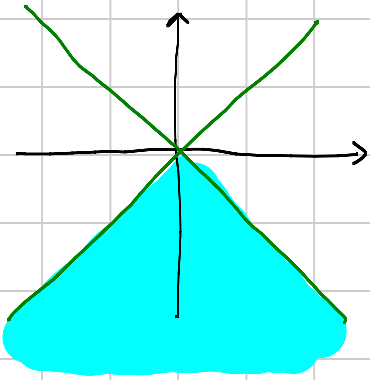


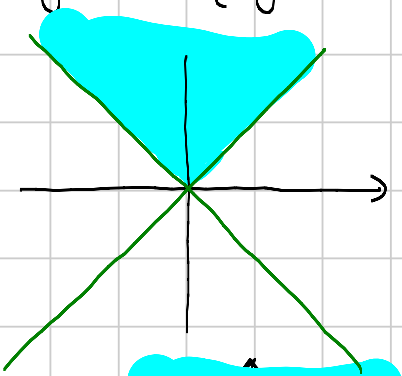
$y = -|x| \rightarrow \begin{cases} -x & \text{se } x \geq 0 \\ x & \text{se } x \leq 0 \end{cases}$

Es. 10 $x^2-y^2\le 0$ $(x-y)(x+y)\le 0$

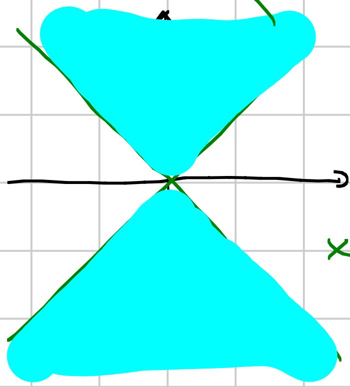
Ci sono 2 casi

$\begin{cases} x-y\ge 0 \\ x+y\le 0 \end{cases}$ $\begin{cases} y\le x \\ y\le -x \end{cases}$ UNIONE $\begin{cases} x-y\le 0 \\ x+y\ge 0 \end{cases}$ $\begin{cases} y\ge x \\ y\ge -x \end{cases}$





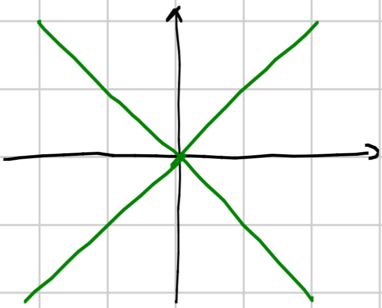
Facendo l'unione dei 2 casi ottengo



$x^2-y^2\le 0$

Metodo "rapido" : $x^2-y^2\le 0$

$x^2-y^2=0$; $y^2=x^2$; $y=\pm x$

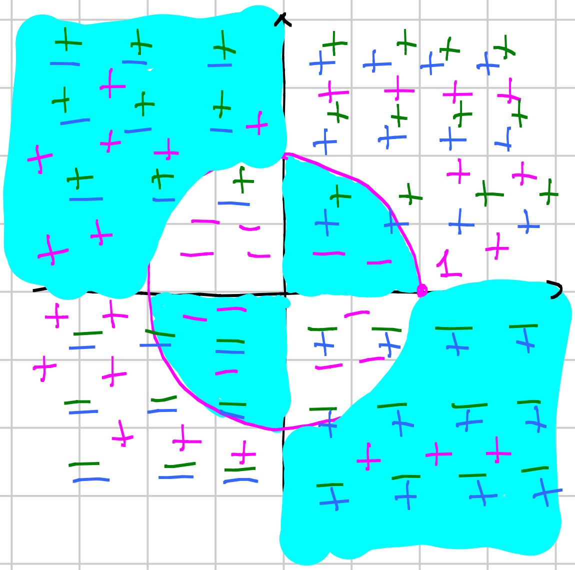


Es. 11 $xy(x^2+y^2-1)\le 0$

Prodotto di 3 fattori : 8 casi

PPP	$\boxed{\text{PPN}}$	$\boxed{\text{PNP}}$	$\boxed{\text{NPP}}$
NNP	NPN	PNN	$\boxed{\text{NNN}}$

Dovrei fare l'unione di 4 casi (esercizio)



1° FATTORE, cioè x

2° FATTORE, cioè y

3° FATTORE, cioè x^2+y^2-1. Cerco di capire dove è >0 e dove è <0 :

$x^2+y^2-1>0 \iff x^2+y^2>1$

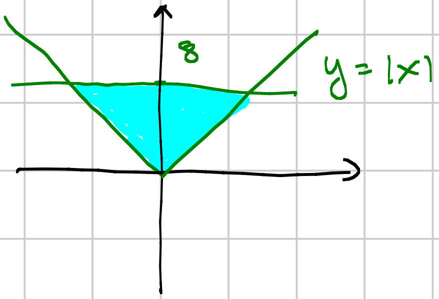
FUORI DALLA CIRC.

$x^2+y^2-1<0$ DENTRO LA CIRC.

Es. 12 $\quad |x| \leq y \leq 8$ $\quad$ Due condizioni

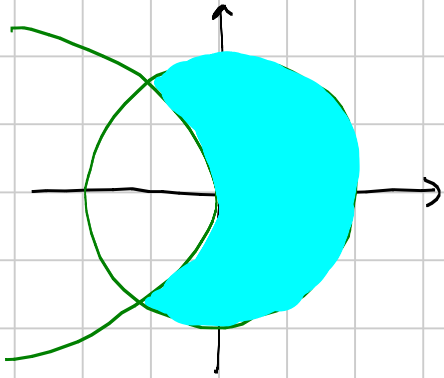
$y \geq |x|$ "sopra il grafico di $|x|$"

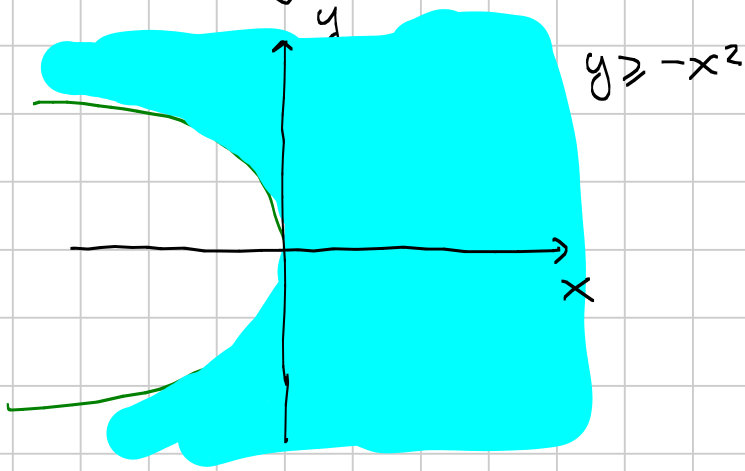
$y \leq 8$ "sotto la retta $y=8$"



Es. 13 $\quad \underbrace{x^2+y^2 \leq 1}_{\text{DENTRO IL CERCHIO}}, \quad \underbrace{x+y^2 \geq 0}_{x \geq -y^2}$ $\quad$ Due condizioni vere contemporaneamente

$x = -y^2$ è una parabola con asse lungo asse x





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$\boxed{\text{TEST 2006/2007}}$

$\boxed{1}$ $\log_3 8 - \log_3 7 = \log_3 \frac{8}{7}$ $\quad \left[\log \frac{a}{b} = \log a - \log b\right]$

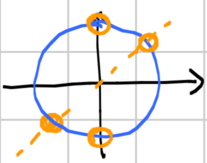
$\boxed{2}$ Risolvere $\quad 2\sin^2 x + \sin(2x) = 2 \quad$ in $[0, 2\pi]$

$$\sin(2x) = 2 - 2\sin^2 x$$

$$2\sin x \cdot \cos x = 2(1-\sin^2 x)$$

$$\cancel{2}\sin x \cos x = \cancel{2}\cos^2 x$$

$\cos^2 x - \sin x \cos x = 0 \qquad \cos x\,(\cos x - \sin x) = 0$ $\nearrow \cos x = 0$ $\searrow \cos x = \sin x$ cioè $\tan x = 1$



4 soluzioni:

$$\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$$

$\boxed{4}$ Se $a = \frac{1}{2}$ e $\frac{a}{a+b} = \frac{1}{7}$, quanto vale b?

2ª eq.: $7a = a + b \Rightarrow b = 6a = 3$

$\boxed{5}$ $9^{x+9} = 3^{x+3}$; $(3^2)^{x+9} = 3^{x+3}$; $3^{\boxed{2x+18}} = 3^{\boxed{x+3}}$

$2x + 18 = x + 3 \Rightarrow x = -15$

$\boxed{6}$ $\frac{a}{b} + \frac{a}{c} = a\left(\frac{1}{b} + \frac{1}{c}\right) = a\,\frac{b+c}{bc} = \frac{a(b+c)}{bc}$

$\boxed{7}$ $\underbrace{(x+1)}_{\boxed{x=-1}}\underbrace{(x^2+4)}_{\text{NULLA}}\underbrace{(x^3+3)}_{x^3=-3 \to \boxed{x=-\sqrt[3]{3}}} = 0$

$\boxed{7\text{bis}}$ $\underbrace{(x-1)}_{\boxed{x=1}}\underbrace{(x^2-4)}_{\boxed{x=\pm 2}}\underbrace{(x^3-3)}_{x^3=3 \;\; \boxed{x=\sqrt[3]{3}}} = 0$

4 soluzioni

$\boxed{7\text{ter}}$ $\underbrace{(x^2-2)}_{\boxed{x=\pm\sqrt{2}}}\underbrace{(x^3-3)}_{\boxed{x=\sqrt[3]{3}}}\underbrace{(x^4-4)}_{x^4=4} = 0$

$x^4 = 4 \nearrow x^2 = 2 \to x = \pm\sqrt{2}$ GIÀ PRESE

$x^4 = 4 \searrow x^2 = -2 \to$ Nulla

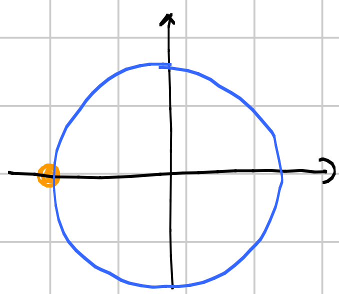
3 soluzioni !!!

$\boxed{8}$ $x^5 + 5x^2 + 1$ diviso $x^2 - 1$

$$
\begin{array}{rrrr|l}
x^5 & & +5x^2 & +1 & x^2-1 \\
\cline{5-5}
-x^5 & +x^3 & & & x^3+x+5 \\
\hline
// & x^3 & +5x^2 & +1 & \\
& -x^3 & & +x & \\
\hline
& // & 5x^2 & +x+1 & \\
& & -5x^2 & +5 & \\
\hline
& & // & x+6 & \\
\end{array}
$$

$\boxed{9}$ $540°$ individua lo stesso P sulla circ. di

$0°$ $40°$ $90°$ $-90°$ $\boxed{-180°}$



$540° = 360° + 180°$