FORMULE TRIGONOMETRICHE

Titolo nota 22/09/2008

FORMULA FONDAMENTALE : $\cos^2\alpha + \sin^2\alpha = 1$

ARCHI ASSOCIATI sin / cos / tan $\pi \pm \alpha$, $\frac{\pi}{2} \pm \alpha$, $\frac{3\pi}{2} \pm \alpha$, $2\pi \pm \alpha$

FORMULE ADDIZIONE sin / cos / tan $\alpha \pm \beta$

FORMULE DUPLICAZIONE " " " 2α

" PRODOTTO → SOMMA $\sin\alpha \cdot \sin\beta = \ldots + \ldots$

" SOMMA → PRODOTTO $\sin\alpha + \sin\beta = \ldots \cdot \ldots$

— o — o —

FORMULE DI ADDIZIONE

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

RICORDARE

Gli archi associati sono particolari esempi di formule di addizione

$$\cos(\pi-\alpha) = \underbrace{\cos(\pi)}_{=-1}\cos\alpha + \underbrace{\sin(\pi)}_{=0}\sin\alpha$$
$$= -\cos\alpha$$

$$\sin(\pi+\alpha) = \underbrace{\sin(\pi)}_{=0}\cos\alpha + \underbrace{\cos(\pi)}_{=-1}\sin\alpha = -\sin\alpha$$

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DUPLICAZIONE $\sin(2x) = 2\sin x\cos x$ ($\alpha=\beta=x$ nella $\sin(\alpha+\beta)$)

$$\cos(2x) = \cos^2x - \sin^2x = (1-\sin^2x) - \sin^2x$$
$$= 1 - 2\sin^2x$$

$$= \cos^2x - \sin^2x = \cos^2x - (1-\cos^2x)$$
$$= 2\cos^2x - 1$$

$$\boxed{\begin{aligned}\sin(2x) &= 2\sin x\cos x\\ \cos(2x) &= \cos^2 x-\sin^2 x\\ &= 1-2\sin^2 x\\ &= 2\cos^2 x-1\end{aligned}}$$

RICORDARE

Esercizio

$$\begin{aligned}\cos(3x) &= \cos(2x+x)\\ &= \cos(2x)\cos x-\sin(2x)\sin x\\ &= (2\cos^2 x-1)\cdot\cos x-2\sin x\cos x\sin x\\ &= 2\cos^3 x-\cos x-2\sin^2 x\cos x\\ &= 2\cos^3 x-\cos x-2(1-\cos^2 x)\cos x\\ &= 2\cos^3 x-\cos x-2\cos x+2\cos^3 x\\ &= 4\cos^3 x-3\cos x\end{aligned}$$

$$\begin{aligned}\sin(3x) &= \sin(2x+x)\\ &= \sin(2x)\cos x+\cos(2x)\sin x\\ &= 2\sin x\cos x\cdot\cos x+(1-2\sin^2 x)\cdot\sin x\\ &= 2\sin x\cdot\cos^2 x+\sin x-2\sin^3 x\\ &= 2\sin x(1-\sin^2 x)+\sin x-2\sin^3 x\\ &= 2\sin x-2\sin^3 x+\sin x-2\sin^3 x\\ &= 3\sin x-4\sin^3 x\end{aligned}$$

— o — o —

FORMULE PRODOTTO → SOMMA

$$\boxed{\begin{aligned}\cos(\alpha+\beta) &= \cos\alpha\cos\beta-\sin\alpha\sin\beta \quad &(1)\\ \cos(\alpha-\beta) &= \cos\alpha\cos\beta+\sin\alpha\sin\beta \quad &(2)\\ \sin(\alpha+\beta) &= \sin\alpha\cos\beta+\cos\alpha\sin\beta \quad &(3)\\ \sin(\alpha-\beta) &= \sin\alpha\cos\beta-\cos\alpha\sin\beta \quad &(4)\end{aligned}}$$

$$\boxed{\begin{aligned}\cos\alpha\cdot\cos\beta &= \frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right] \quad &(1)+(2)\\ \sin\alpha\cdot\sin\beta &= \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right] \quad &(2)-(1)\\ \sin\alpha\cdot\cos\beta &= \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right] \quad &(3)+(4)\end{aligned}}$$

FORMULE SOMMA → PRODOTTO

$$\boxed{\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}$$

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

$$\cos\underbrace{(\alpha+\beta)}_{x} + \cos\underbrace{(\alpha-\beta)}_{y} = 2\cos\alpha\cdot\cos\beta$$

(altro modo di usare ① + ②)

$\alpha+\beta = x$

$\alpha-\beta = y$

Come ricavo α e β? Sommo: $2\alpha = x+y$

$\rightarrow \alpha = \frac{x+y}{2}$

Sottraggo: $2\beta = x-y \rightarrow \beta = \frac{x-y}{2}$

Conclusione: $\underset{\substack{\uparrow\\ \alpha+\beta}}{\cos x} + \underset{\substack{\uparrow\\ \alpha-\beta}}{\cos y} = 2\cos\Big(\underset{\substack{\uparrow\\ \alpha}}{\frac{x+y}{2}}\Big)\cdot\cos\Big(\underset{\substack{\uparrow\\ \beta}}{\frac{x-y}{2}}\Big)$

—o—

Formula per $\sin x + \sin y$.

$$\sin\underbrace{(\alpha+\beta)}_{x} + \sin\underbrace{(\alpha-\beta)}_{y} = 2\sin\alpha\cos\beta$$

Come prima: $\alpha = \frac{x+y}{2}$ $\quad\beta = \frac{x-y}{2}$

$$\boxed{\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

Analogamente: $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

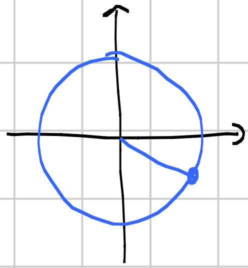
$\alpha+\beta$ $\quad$ $\alpha-\beta$ $\quad$ α $\quad$ β

$$\sin x - \sin y = -2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Esercizi

$\boxed{1}$ $\sin 330° = \sin(-30°) = -\frac{1}{2}$

$\cos 330° = \cos(30°) = \frac{\sqrt{3}}{2}$



330° in radianti $\quad 360° - 30° \rightsquigarrow 2\pi - \frac{\pi}{6}$

$= \frac{11\pi}{6}$

$\boxed{2}$ $15° \rightsquigarrow \frac{\pi}{12}$ $\quad \cos(15°) = \cos(45° - 30°)$

$= \cos(45°)\cos(30°) + \sin(45°)\sin(30°)$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin(15°) = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$

$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

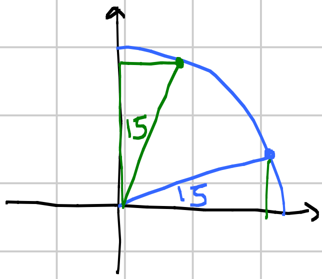
$\boxed{3}$ $75° \rightsquigarrow \frac{5\pi}{12}$ $\quad 75° = 60° + 15°$ esercizio

$75° = 45° + 30°$ "

$75° = 90° - 15°$

$\cos(75°) = \cos(90° - 15°) = \sin 15° = \ldots$

$\sin(75°) = \sin(90° - 15°) = \cos 15° = \ldots$



$\boxed{4}$ $2010° = 1800° + 210°$

$= 5 \cdot 360° + 210°$ $\quad$ (5 giri + 210°)

individua sulla circ. trigo. lo stesso punto individuato da 210°

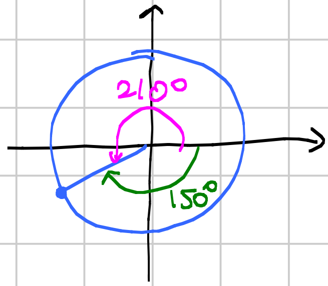
$210° \rightsquigarrow 180° + 30° \rightsquigarrow \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$2010° = 5 \cdot 360° + 210° \rightsquigarrow 10\pi + \frac{7\pi}{6} = \frac{67\pi}{6}$

$\cos(2010°) = \cos(210°) = \cos(180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\sin(2010°) = \sin(180° + 30°) = -\sin 30° = -\frac{1}{2}$

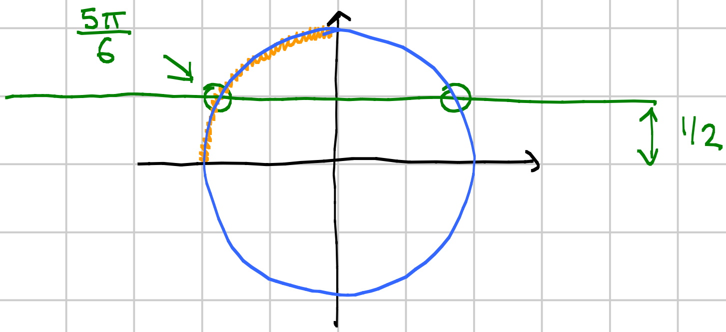
(5) Trovare un numero negativo $-n$ t.c. $210°$ individua lo stesso punto di $-n°$



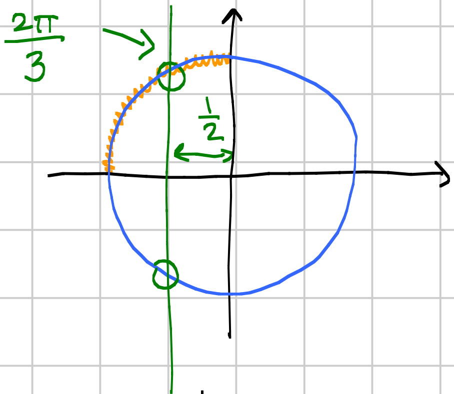
$-150°$, $-150° - 360°$, ...

[6] Se $\frac{\pi}{2} < x < \pi$ e $\sin x = \frac{1}{2}$, quanto vale x ?

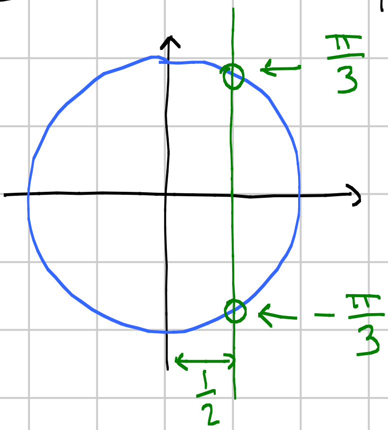
$x = \frac{5\pi}{6}$



[7] se $\frac{\pi}{2} < x < \pi$ e $\cos x = -\frac{1}{2}$, allora $x = \frac{2\pi}{3}$



[8] Risolvere l'equazione $\cos x = \frac{1}{2}$. Guardo il cerchio !!!



Se voglio le soluzioni contenute in $[0, 2\pi]$, allora ho 2 soluzioni

$x = \frac{\pi}{3}$ $\qquad$ $x = \frac{5\pi}{3}$

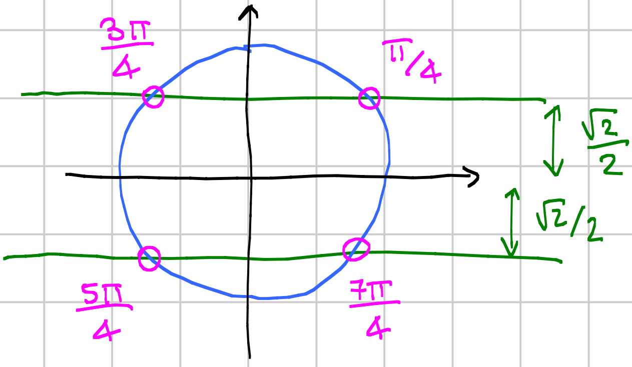
Se voglio TUTTE le soluzioni reali:

$x = \frac{\pi}{3} + 2k\pi$ con k intero

$x = -\frac{\pi}{3} + 2k\pi$ con k intero

9 $2\sin^2 x = 1 \qquad x \in [0, 2\pi]$

$\sin^2 x = \frac{1}{2} \qquad \sin x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$



4 soluzioni

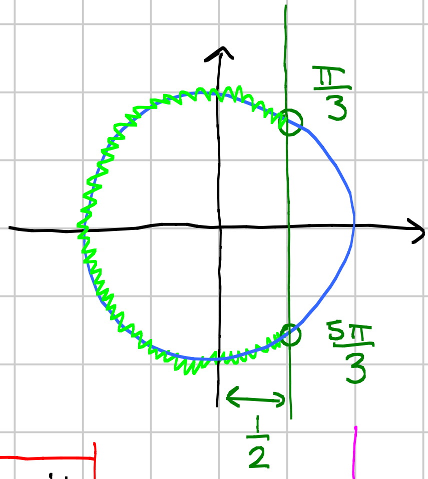
$x = \frac{\pi}{4} \qquad x = \frac{3\pi}{4}$ in cui $\sin = \frac{\sqrt{2}}{2}$

$x = \frac{5\pi}{4} \qquad x = \frac{7\pi}{4}$ in cui $\sin = -\frac{\sqrt{2}}{2}$

— o — o —

10 $\boxed{\cos x \le \frac{1}{2}} \qquad x \in [0, 2\pi]$

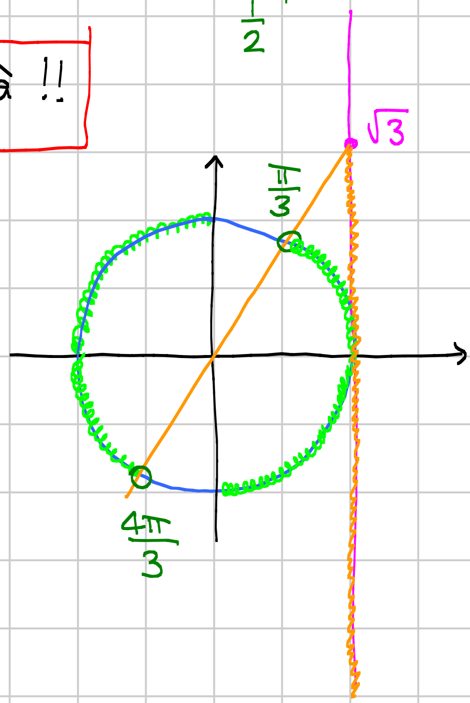
Cerco i p.ti della circ. trigon. con ASCISSA $\le \frac{1}{2}$



Soluzione: $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$

Occhio!! $\left[\frac{\pi}{3}, -\frac{\pi}{3}\right]$ è un' assurdità!!

11 $\tan x < \sqrt{3}$ in $[0, 2\pi]$



soluzione: $\left[0, \frac{\pi}{3}\right) \cup \left(\frac{\pi}{2}, \frac{4\pi}{3}\right) \cup$

$\cup \left(\frac{3\pi}{2}, 2\pi\right]$

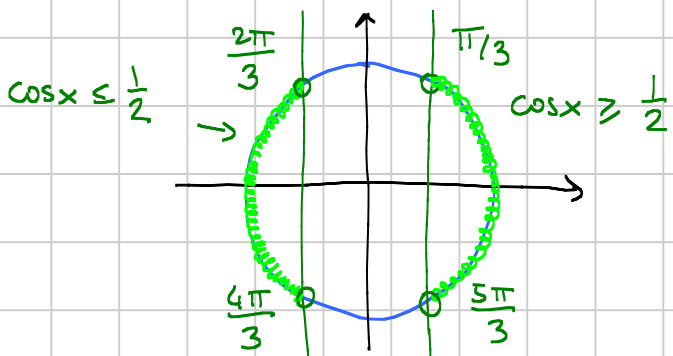
— o — o —

12 $4\cos^2 x \ge 1 \qquad x \in [0, 2\pi]$

$\cos^2 x \ge \frac{1}{4} \qquad t^2 \ge \frac{1}{4}$ VALORI ESTERNI $t \le -\frac{1}{2}$ e $t \ge \frac{1}{2}$

$\cos x \ge \frac{1}{2}$

$\cos x \le -\frac{1}{2}$



$\left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right]$