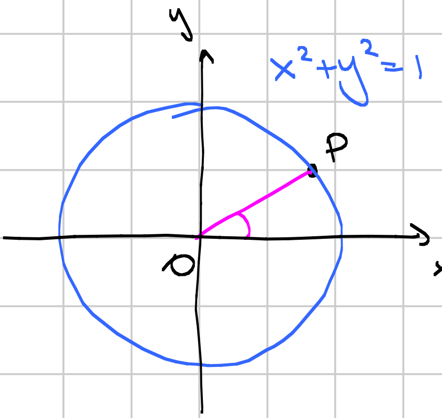
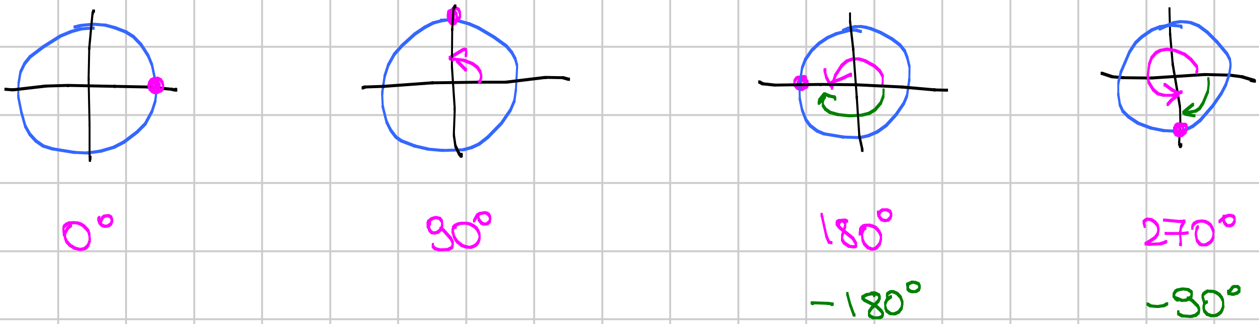
TRIGONOMETRIA

CIRCONFERENZA TRIGONOMETRICA = centro in (0,0) e raggio 1.



Come individuare un p.to P sulla circonferenza?

GRADI SESSAGESIMALI (misura dell'angolo)

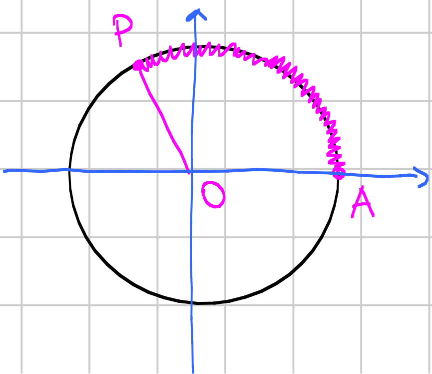


Angolo: quello formato dal semiasse positivo delle x e il segmento OP misurato in senso antiorario.

Giri successivi: gli angoli sono definiti a meno di multipli di 360°:

- angoli $\geq 360°$ = avere fatto più di una volta il giro
- angoli $< 0°$ = girare in senso orario (al contrario)

$\boxed{\text{RADIANTI}}$ Lunghezza del percorso che parte dalla linea di partenza (il p.to (1,0) della circ. trig.) e segue la circ. trigonometrica fino al p.to P



Tutto il giro = 360° = lunghezza circ.
= 2π

- Archi $> 2\pi$ vuol dire fare più di un giro
- Archi < 0 " " girare al contrario

Conversione gradi $\leftrightarrow$ radianti

$$\boxed{\text{GRADI} : \text{RADIANTI} = 360° : 2\pi}$$

$0° \leftrightarrow 0$

$90° \leftrightarrow \frac{\pi}{2}$

$180° \leftrightarrow \pi$

$360° \leftrightarrow 2\pi$

$270° \leftrightarrow \frac{3\pi}{2}$

$45° \leftrightarrow \frac{\pi}{4}$

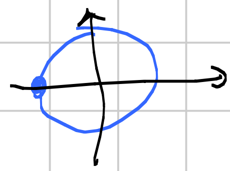
$30° \leftrightarrow \frac{\pi}{6}$

$60° \leftrightarrow \frac{\pi}{3}$

$120° \leftrightarrow \frac{2\pi}{3}$

$150° \leftrightarrow \frac{5\pi}{6}$

$540° \leftrightarrow 3\pi$

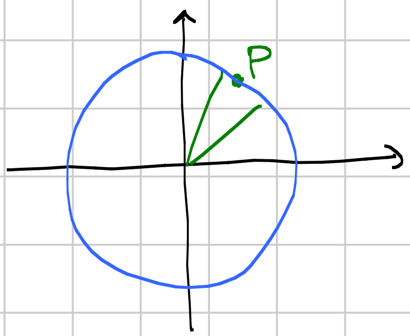


$10°$

$$\text{GRADI} : \text{RAD} = 360° : 2\pi$$

$$10° : \text{RAD} = 360° : 2\pi \qquad \text{RAD} = \frac{2\pi \cdot 10°}{360°} = \frac{\pi}{18}$$

Dove si trova il p.to corr. ad 1 RADIANTE



$$\text{GRADI} : \text{RAD} = 360° : 2\pi$$

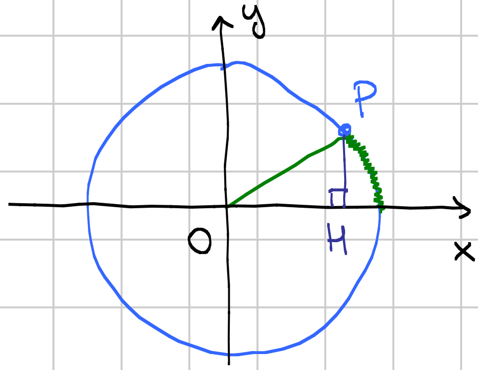
$$\text{GRADI} : 1 = 360° : 2\pi$$

$$\text{GRADI} = \frac{1 \cdot 360°}{2\pi} = \frac{180°}{\pi} = \frac{180°}{3,\ldots}$$

$=$ un po' meno di $60°$

—o—o—

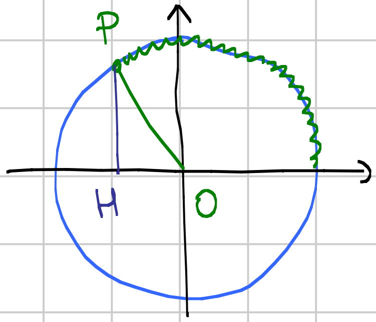
Definizione di sen, cos, tan.



Sia P un p.to della circ. trig. corrispondente ad un arco α. Le coordinate di P sono

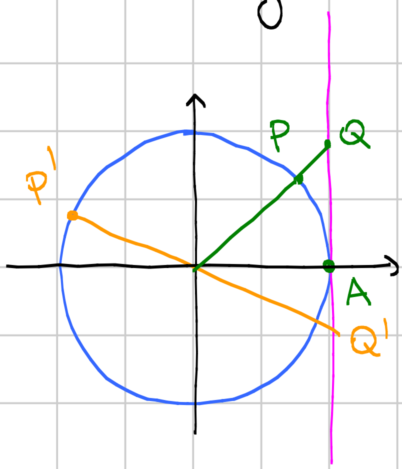
$$P = (\underset{\overline{OH}}{\cos\alpha}, \underset{\overline{PH}}{\sin\alpha})$$

OH e PH vanno pensati con segno

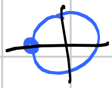
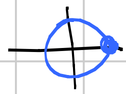


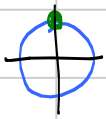
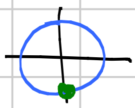
PH è positivo
OH è negativo

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = AQ$$



La tangente si misura sempre sulla retta tangente in A ed è >0 se il p.to Q si trova sopra A, negativa se si trova sotto A

$\tan\alpha > 0$ nel 1° e nel 3° quadrante
$\tan\alpha < 0$ nel 2° e nel 4° quadrante
$\tan\alpha = 0$ per P corrispondenti a  

tan α non è definita  

Geometricamente: OP non incontra la retta tangente in A
Algebricamente: $\frac{\sin\alpha}{\cos\alpha}$ richiede $\cos\alpha \neq 0$, ma $\cos\alpha = 0$

quando la coord. x di P è $=0$, cioè nei 2 p.ti incriminati

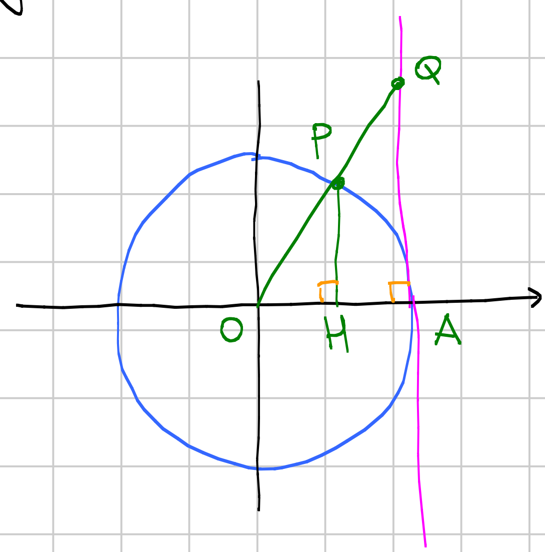
$\boxed{\text{RELAZIONE FONDAMENTALE}}$: $\boxed{\sin^2\alpha + \cos^2\alpha = 1}$

Geometricamente è il teo. di Pitagora: $OH^2 + PH^2 = OP^2 = 1$

Perché $\tan\alpha = AQ$?
I triangoli OHP e OAQ sono simili (stessi angoli), quindi hanno i lati in proporzione

$$\frac{PH}{OH} = \frac{AQ}{OA}$$, cioè

$$\frac{\sin\alpha}{\cos\alpha} = \frac{AQ}{1}$$ ← Raggio



Angoli e archi notevoli

$0° \leftrightarrow 0$ rad. $\quad \cos 0 = 1, \ \sin 0 = 0, \ \tan 0 = 0$

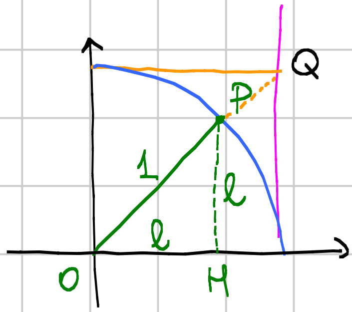
$90° \leftrightarrow \pi/2$ rad. $\quad \cos \frac{\pi}{2} = 0, \ \sin \frac{\pi}{2} = 1, \ \tan \frac{\pi}{2}$ non def.

$180° \leftrightarrow \pi$ rad. $\quad \cos \pi = -1, \ \sin \pi = 0, \ \tan \pi = 0$

$270° \leftrightarrow \frac{3\pi}{2}$ rad $\quad \cos \frac{3\pi}{2} = 0, \ \sin \frac{3\pi}{2} = -1, \ \tan$ N.D.

$360° \leftrightarrow 2\pi$ rad $\quad \cos(2\pi) = 1, \ \sin(2\pi) = 0, \ \tan(2\pi) = 0$

$45° \leftrightarrow \frac{\pi}{4}$



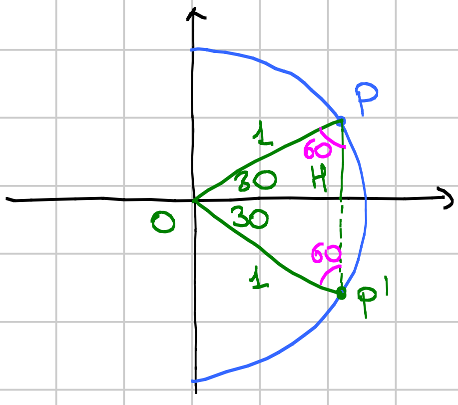
$OH = PH = l$

Pitagora: $l^2 + l^2 = 1$

$2l^2 = 1 \qquad l^2 = \frac{1}{2} \qquad l = \pm\sqrt{\frac{1}{2}} \Rightarrow l = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\Rightarrow \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \ ; \quad \tan \frac{\pi}{4} = 1$

$30° \leftrightarrow \frac{\pi}{6}$ radianti



il triangolo OPP' è equilatero, quindi $PP' = 1$ e di conseguenza $PH = \frac{1}{2}$

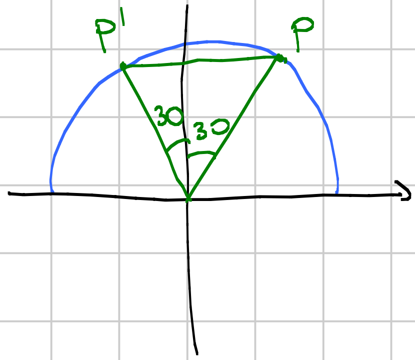
Quindi $\sin 30° = \sin \frac{\pi}{6} = \frac{1}{2}$. Per calcolare il cos uso

Pitagora: $OH^2 = OP^2 - PH^2 = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow OH = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan \frac{\pi}{6} = \frac{\sin \frac{\pi}{6}}{\cos \frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

$60°$ $\frac{\pi}{3}$ Esattamente come prima, ma ruotato di $90°$

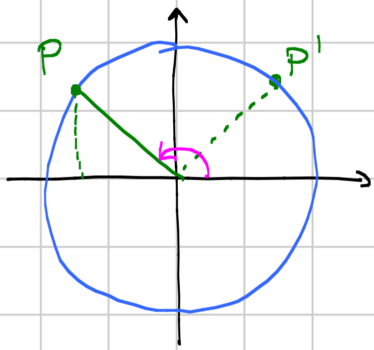


$\Rightarrow \cos\frac{\pi}{3} = \frac{1}{2}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\tan\frac{\pi}{3} = \sqrt{3}$

— o — o —

$\frac{3\pi}{4}$ $135°$



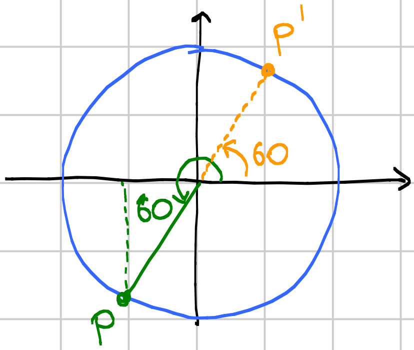
$\sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\tan\frac{3\pi}{4} = -1$

— o — o —

$240°$ $\frac{4\pi}{3}$



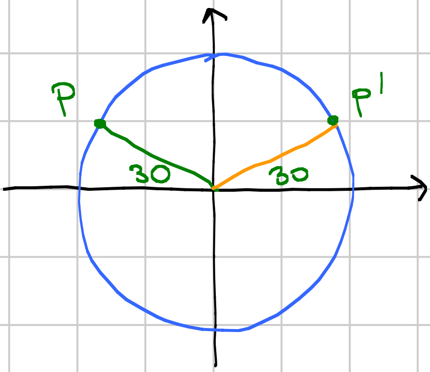
$\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$\tan\frac{4\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$ (individuano lo stesso p.to sulla retta tangente per A)

— o — o —

$\frac{5\pi}{6}$ $150°$



$\sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$

$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$\tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$

ARCHI ASSOCIATI

α

$\pi-\alpha$

$\pi+\alpha$

$\frac{\pi}{2}-\alpha$

$\frac{\pi}{2}+\alpha$

$-\alpha$

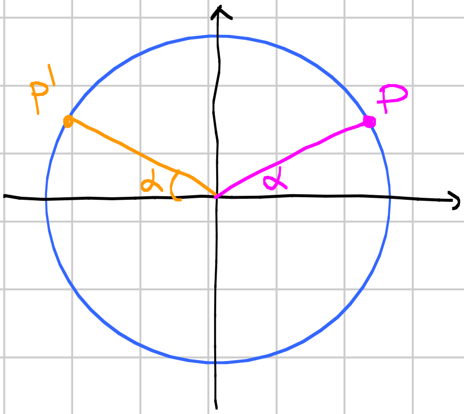
$\frac{3\pi}{2}\pm\alpha$

Domanda: se conosco $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, come trovo le funzioni trigonometriche degli archi associati?

Pensare al disegno !!!!

— o —

$\boxed{\pi-\alpha}$

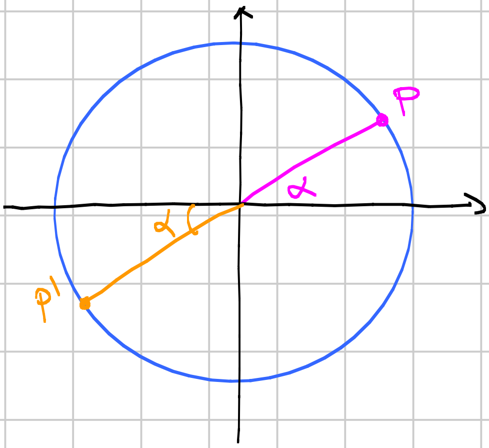


P' corrisponde a $\pi-\alpha$
P " ad α

$$\sin(\pi-\alpha) = \sin\alpha$$
$$\cos(\pi-\alpha) = -\cos\alpha$$
$$\tan(\pi-\alpha) = -\tan\alpha$$

— o — o —

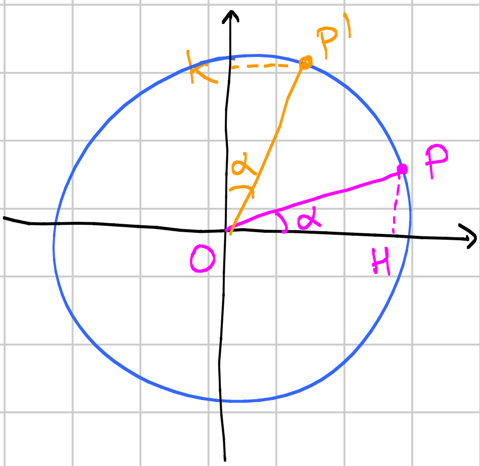
$\boxed{\pi+\alpha}$



$$\sin(\pi+\alpha) = -\sin\alpha$$
$$\cos(\pi+\alpha) = -\cos\alpha$$
$$\tan(\pi+\alpha) = \tan\alpha$$

— o — o —

$\boxed{\frac{\pi}{2}-\alpha}$



sin e cos si scambiano

$$\underset{P'K}{\underset{\shortparallel}{\cos\left(\frac{\pi}{2}-\alpha\right)}} = \underset{PH}{\underset{\shortparallel}{\sin\alpha}}$$

$$\underset{OK}{\underset{\shortparallel}{\sin\left(\frac{\pi}{2}-\alpha\right)}} = \underset{OH}{\underset{\shortparallel}{\cos\alpha}}$$

$$\tan\left(\frac{\pi}{2}-\alpha\right) = \frac{1}{\tan\alpha}$$

$\boxed{-\alpha}$

$$\cos(-\alpha) = \cos\alpha$$
$$\sin(-\alpha) = -\sin\alpha$$
$$\tan(-\alpha) = -\tan\alpha$$

$\boxed{\frac{\pi}{2}+\alpha}$

$$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha \qquad \tan\left(\frac{\pi}{2}+\alpha\right) = -\frac{1}{\tan\alpha}$$

$$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$$

Per esercizio ricavare (con disegno) $\frac{3\pi}{2} \pm \alpha$

$2\pi \pm \alpha \rightsquigarrow$ esattamente come $\pm\alpha$

$21\pi + \alpha \rightsquigarrow \pi + \alpha$