ESERCIZI SULLE POTENZE

Titolo nota 16/09/2008

A^x — x intero positivo, A qualunque

x intero ≤ 0, $A \neq 0$

x non intero, $A > 0$

$$A^{\frac{m}{n}} = \sqrt[n]{A^m}$$

Proprietà

1. $A^m \cdot A^n = A^{m+n}$
2. $\dfrac{A^m}{A^n} = A^{m-n}$
3. $(A^m)^n = A^{mn}$
4. $A^m \cdot B^m = (AB)^m$

$A^m \pm A^n$

$A^m \pm B^m$

Nulla di Furbo

Esercizi

$\boxed{1}$ $\sqrt{2} \cdot \sqrt{5} = \sqrt{a}$ $\quad 2^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \underset{\text{propr. 4}}{=} 10^{\frac{1}{2}} = a^{\frac{1}{2}} \quad a = 10$

$\boxed{2}$ $\sqrt{2} \cdot \sqrt{2} = \sqrt{a}$

$\sqrt{2} \cdot \sqrt{2} = 2 = \sqrt{4} \Rightarrow a = 4$

$\boxed{3}$ $\sqrt{2} \cdot \sqrt{3} = \sqrt[a]{6}$ $\quad 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \overset{4}{=} 6^{\frac{1}{2}} \quad \Rightarrow a = 2$

$\boxed{4}$ $\sqrt{2} \cdot \sqrt{3} = \sqrt[4]{a}$ $\quad \left(2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}\right)^4 = \left(a^{\frac{1}{\cancel{4}}}\right)^{\cancel{4}} = a$

$$a = \left(2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}\right)^4 \underset{4}{=} \left(6^{\frac{1}{2}}\right)^4 \underset{3}{=} 6^2 = 36$$

5. $\sqrt{\sqrt{2}} = \sqrt{2^{\frac{1}{2}}} = \left(2^{\frac{1}{2}}\right)^{\frac{1}{2}} \underset{3}{=} 2^{\frac{1}{4}} = \sqrt[4]{2}$

$\boxed{6}$ $\sqrt{\sqrt[3]{2}} = \left(2^{\frac{1}{3}}\right)^{\frac{1}{2}} \underset{3}{=} 2^{\frac{1}{6}} = \sqrt[6]{2}$

7 $\sqrt[3]{\sqrt{27}} = \left((27)^{\frac{1}{2}}\right)^{\frac{1}{3}} = 27^{\frac{1}{6}} = (3^3)^{\frac{1}{6}} = 3^{\frac{1}{2}} = \sqrt{3}$

8 $\sqrt{\sqrt{2}} = \sqrt[8]{a}$; $\left(2^{\frac{1}{2}}\right)^{\frac{1}{2}} = a^{\frac{1}{8}}$, $2^{\frac{1}{4}} = a^{\frac{1}{8}}$

elevo alla 8ª: $\left(2^{\frac{1}{4}}\right)^8 = a$, $2^2 = a = 4$.

9 $\sqrt{2\sqrt{2}} = (2\sqrt{2})^{\frac{1}{2}} = \left(2 \cdot 2^{\frac{1}{2}}\right)^{\frac{1}{2}} \overset{1}{=} \left(2^{1+\frac{1}{2}}\right)^{\frac{1}{2}} =$

$= \left(2^{\frac{3}{2}}\right)^{\frac{1}{2}} = 2^{\frac{3}{4}}$

$2^{\frac{3}{4}} = 8^{\frac{1}{a}}$, $2^{\frac{3}{4}} = (2^3)^{\frac{1}{a}} = 2^{\frac{3}{a}} \Rightarrow a = 4$

10 $\sqrt{4\sqrt{2}} = \sqrt{2^2 \cdot 2^{\frac{1}{2}}} = \left(2^{2+\frac{1}{2}}\right)^{\frac{1}{2}} = \left(2^{\frac{5}{2}}\right)^{\frac{1}{2}} = 2^{\frac{5}{4}}$

$2^{\boxed{\frac{5}{4}}} = \sqrt{2^a} = 2^{\boxed{\frac{a}{2}}} \Rightarrow \frac{5}{4} = \frac{a}{2} \Rightarrow a = \frac{5}{2}$

In alternativa: $4\sqrt{2} = \sqrt{16} \cdot \sqrt{2} = \sqrt{32}$ $\left(16^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}\right)$

11 $3\sqrt{7} = \sqrt{9} \cdot \sqrt{7} = \sqrt{63}$ $9^{\frac{1}{2}} \cdot 7^{\frac{1}{2}} \overset{4}{=} (9 \cdot 7)^{\frac{1}{2}}$

12 $\sqrt{2} \cdot \sqrt[3]{2} \cdot \sqrt[4]{2} = 2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{1}{4}} \overset{1}{=} 2^{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = 2^{\frac{6+4+3}{12}} = 2^{\frac{13}{12}}$

13 $\sqrt[6]{8} = \sqrt[a]{4}$, $(2^3)^{\frac{1}{6}} = (2^2)^{\frac{1}{a}}$, $2^{\frac{1}{2}} = 2^{\frac{2}{a}}$, $\frac{1}{2} = \frac{2}{a}$, $a = 4$

N.B. $\sqrt[6]{8} = \sqrt{2}$

14 $\sqrt[3]{2} : \sqrt[6]{a} = 2^{-1}$, $2^{\frac{1}{3}} : a^{\frac{1}{6}} = 2^{-1}$, $\frac{2^{1/3}}{a^{1/6}} = \frac{1}{2}$

$2 \cdot 2^{\frac{1}{3}} = a^{\frac{1}{6}}$, $2^{\frac{4}{3}} = a^{\frac{1}{6}}$ elevo alla sesta:

$\left(2^{\frac{4}{3}}\right)^6 = a \Rightarrow a = 2^8 = 256$

15 $\sqrt{100^{100}} = \left(100^{100}\right)^{\frac{1}{2}} = \boxed{100^{50}}$; $\left(100^{100}\right)^{\frac{1}{2}} = \left(100^{\frac{1}{2}}\right)^{100} = \boxed{10^{100}}$

STESSA COSA

$\boxed{16}$ $\sqrt{3\sqrt{2\sqrt{3}}}$ $\qquad 2\sqrt{3} = \sqrt{4}\cdot\sqrt{3} = \sqrt{12}$; $\sqrt{2\sqrt{3}} = \sqrt{\sqrt{12}}$

$$\sqrt{3\sqrt{\sqrt{12}}} = \sqrt{3\sqrt[4]{12}} = \sqrt{\sqrt[4]{81}\cdot\sqrt[4]{12}} = \sqrt{\sqrt[4]{81\cdot 12}} = \sqrt[8]{81\cdot 12}$$
$$= \sqrt[8]{972}$$

$\boxed{17}$ $\sqrt{2}\cdot\sqrt[a]{4} = 2$, $\quad 2^{\frac{1}{2}}\cdot 2^{\frac{2}{a}} = 2^1$, $\quad 2^{\frac{1}{2}+\frac{2}{a}} = 2^1$

$$\frac{2}{a}+\frac{1}{2} = 1 \Rightarrow \frac{2}{a} = \frac{1}{2} \Rightarrow a = 4$$

$\boxed{18}$ $\sqrt{2}+\sqrt{2} = \sqrt{a}$, $\quad \sqrt{2}+\sqrt{2} = 2\sqrt{2} = \sqrt{4}\cdot\sqrt{2} = \sqrt{8}$

$\boxed{19}$ $\sqrt{8}\cdot\sqrt[3]{2^a}\cdot\sqrt[4]{8} = 8$, $\quad 2^{\frac{3}{2}}\cdot 2^{\frac{a}{3}}\cdot 2^{\frac{3}{4}} = 2^3$

$\Rightarrow \frac{3}{2}+\frac{a}{3}+\frac{3}{4} = 3$ e da qui si ricava a

$\boxed{20}$ $\sqrt{32}-\sqrt{2} = \sqrt{a}$ $\qquad$ (Non pensare nemmeno $\sqrt{30}$!!!)

$$\sqrt{32} = \sqrt{16\cdot 2} \overset{4}{=} \sqrt{16}\cdot\sqrt{2} = 4\sqrt{2}$$

$$\sqrt{32}-\sqrt{2} = 4\sqrt{2}-\sqrt{2} = 3\sqrt{2} = \sqrt{9}\cdot\sqrt{2} = \sqrt{18}$$

$\boxed{21}$ $\sqrt{2}\cdot\sqrt[3]{3} = \sqrt[6]{a}$, $\quad 2^{\frac{1}{2}}\cdot 3^{\frac{1}{3}} = a^{\frac{1}{6}}$ $\quad$ elevo alla sesta:

$$\left(2^{\frac{1}{2}}\cdot 3^{\frac{1}{3}}\right)^6 = a = \left(2^{\frac{1}{2}}\right)^6\cdot\left(3^{\frac{1}{3}}\right)^6 = 2^3\cdot 3^2 = 8\cdot 9 = 72$$

$$\sqrt{2}\cdot\sqrt[3]{3} = \sqrt[6]{72}$$

In alternativa: $\quad \sqrt{2} = \sqrt[6]{8} \qquad 2^{\frac{1}{2}} = 2^{\frac{3}{6}}$

$\sqrt[3]{3} = \sqrt[6]{9} \qquad 3^{\frac{1}{3}} = 3^{\frac{2}{6}}$ $\qquad$ stesso esponente

Quindi $\sqrt{2}\cdot\sqrt[3]{3} = \underbrace{\sqrt[6]{8}\cdot\sqrt[6]{9}} = \sqrt[6]{72}$ $\quad \left[\, 8^{\frac{1}{6}}\cdot 9^{\frac{1}{6}} \overset{4}{=} 72^{\frac{1}{6}} \,\right]$

POLINOMI E SCOMPOSIZIONI

$p(x) = x^4 + 3x^2 + 6x - 25$ grado 4 (massimo esponente della x)

$p(x)$ ha grado m
$q(x)$ ha grado n $\Rightarrow$ $p(x) \cdot q(x)$ ha grado $m+n$
$p(x) \pm q(x)$ ha grado $\leq$ del massimo fra m ed n

Grado della somma: • Se $m \neq n$, allora $p(x) \pm q(x)$ ha come grado il max fra i 2 gradi

• Se $m = n$ il grado di $p(x) + q(x)$ può anche scendere, il che accade se i due termini di grado max "si semplificano"

$p(x) = x^3 + x^2 - 2x + 1 \qquad q(x) = -x^3 + 6x^2 + 7$

$p(x) + q(x) = 7x^2 - 2x + 8$

Esempio $p(x) = 2x^2 - 3 \qquad q(x) = x + 2$

$p(x) \cdot q(x) = (2x^2 - 3)(x + 2) = 2x^3 + 4x^2 - 3x - 6$

Radice di un polinomio Sono quei valori a t.c. $p(a) = 0$

N.B. I polinomi non hanno nec. coeff interi

$$\left(2x + \frac{1}{3}\right)\left(x^2 + \frac{5}{2}x - 3\right) = 2x^3 + 5x^2 - 6x + \frac{x^2}{3} + \frac{5}{6}x - 1$$

Radici di un pol. di 2° grado $ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \Delta = b^2 - 4ac$$

$\Delta > 0 \Rightarrow$ 2 soluzioni reali e distinte

$\Delta = 0 \Rightarrow$ 1 soluzione reale di molteplicità 2

$\Delta < 0 \Rightarrow$ 0 soluzioni reali

SCOMPOSIZIONI E POTENZE

$a^2 - b^2 = (a-b)(a+b)$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $\quad [= a^3 + \cancel{a^2b} + \cancel{ab^2} - \cancel{ba^2} - \cancel{ab^2} - b^3]$

$a^4 - b^4 = (a-b)(a^3+a^2b+ab^2+b^3)$

$a^5 - b^5 = (a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$

$a^6 - b^6 = (a-b)(a^5+a^4b+a^3b^2+a^2b^3+ab^4+b^5)$

... e così via ...

Se c'è il segno $+$:

a^2+b^2 NULLA DI FURBO

$a^3+b^3 = (a+b)(a^2-ab+b^2)$ $\quad [= a^3 - \cancel{a^2b} + \cancel{ab^2} + \cancel{ba^2} - \cancel{ab^2} + b^3]$

a^4+b^4 NULLA

$a^5+b^5 = (a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$

a^6+b^6 NULLA

$a^7+b^7 = (a+b)(a^6-a^5b+a^4b^2-a^3b^3+a^2b^4-ab^5+b^6)$

... e così via ...

$(a+b)^2 = a^2+b^2+2ab \qquad (a-b)^2 = a^2+b^2-2ab$

$(a+b)^2 = (a+b)(a+b) = a^2 + \underbrace{ab+ba} + b^2 = a^2+b^2+2ab$

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$[\text{Mostro}]^2$ = somma di tutti i quadrati + somma di tutti i doppi prodotti

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3, \quad (a-b)^3 = a^3-3a^2b+3ab^2-b^3$

$(a+b+c)^3 = a^3+b^3+c^3+3a^2b+3ab^2+3a^2c+3ac^2+3b^2c+3bc^2+6abc$

[Per esercizio dimostrarla calcolando $(a+b+c)(a+b+c)(a+b+c)$]

$[\text{Mostro}]^3$ = somma di tutti i cubi + somma di tutti i tripli prodotti + 6 volte tutti i prodotti a 3 a 3

$(a+b)^4 = a^4 + \boxed{4}a^3b + \boxed{6}a^2b^2 + \boxed{4}ab^3 + b^4$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

TRIANGOLO DI TARTAGLIA:

ogni numero è la somma dei 2 che "stanno sopra"

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1 $(a+b)^6$

$(a+b)^1 = (a+b)$

$(a+b)^2 = a^2+2ab+b^2$

$(a+b)^3 =$

$(a+b)^4 =$

$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

Scomposizioni

$(a^3-1) = (a-1)(a^2+a+1)$ $\quad b=1$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$ $\quad b=2$

$a^3+1 = (a+1)(a^2-a+1)$

$a^3-8 = (a-2)(a^2+2a+4)$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$a^2-4 = (a+2)(a-2)$

$a^4-1 = (a-1)(a^3+a^2+a+1)$ $\quad b=1$

$a^4-b^4 = (a-b)(a^3+a^2b+ab^2+b^3)$

$a^4-b^4 = (a^2-b^2)(a^2+b^2) = (a-b)\boxed{(a+b)(a^2+b^2)}$

$= (a-b)\boxed{(a^3+a^2b+ab^2+b^3)}$ STESSA COSA

$a^6-b^6 = (a-b)(a^5+a^4b+a^3b^2+a^2b^3+ab^4+b^5)$

$= (a^3-b^3)(a^3+b^3) = (a-b)(a^2+ab+b^2)(a+b)(a^2-ab+b^2)$

$= (a^2-b^2)(a^4+a^2b^2+b^4)$

$a^6-b^6 = (a^2)^3-(b^2)^3$