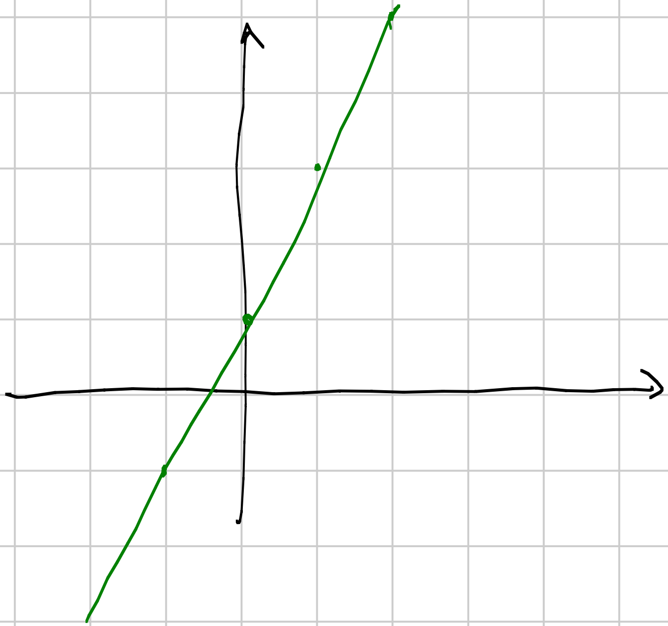
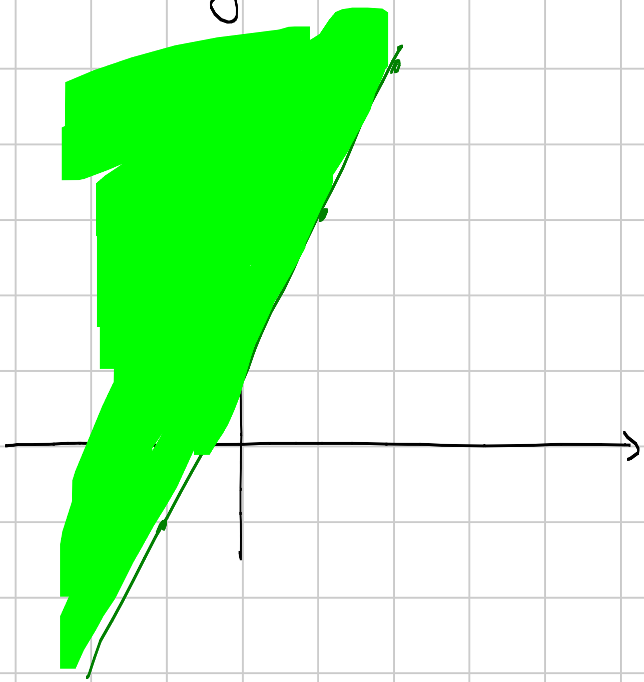
Titolo nota 28/09/2007

INSIEMI DEL PIANO

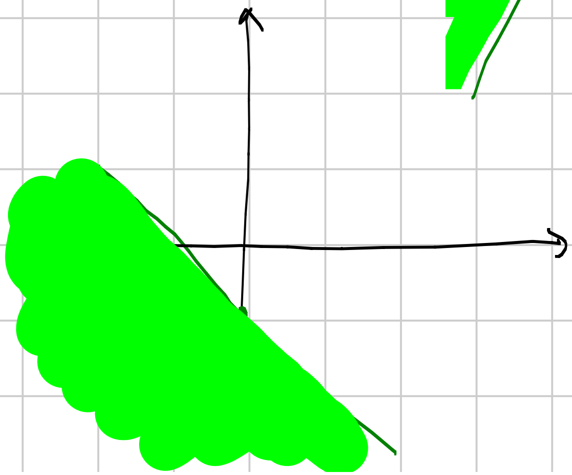
$y = 2x + 1$



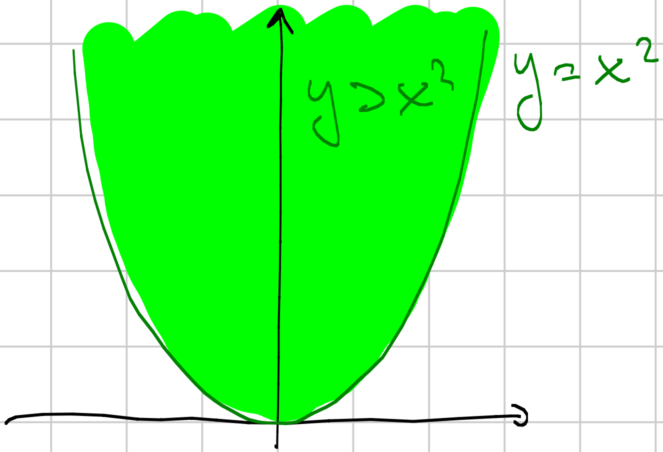
$y > 2x + 1$



$y < -x - 1$



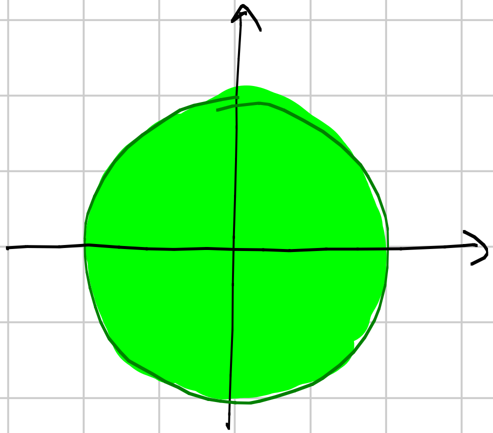
$y \geq x^2$



$x^2 + y^2 \leq 1$

↑

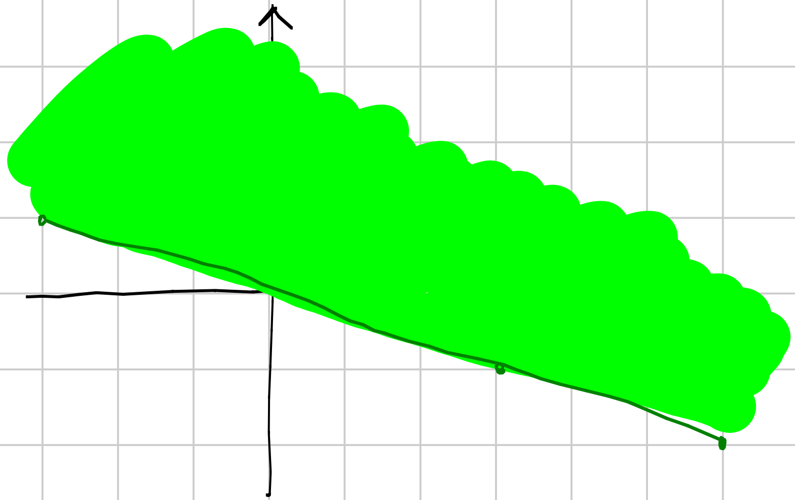
Distanza dal centro ≤ 1



$x + 3y > 0$

$3y > -x$

$y > -\frac{x}{3}$



Modo più preciso di scrivere

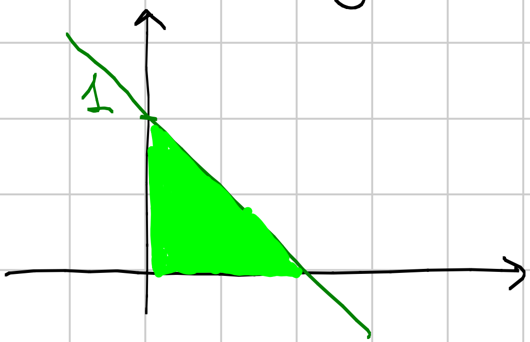
$$\{ \underbrace{(x,y) \in \mathbb{R}^2 ;}_{\text{parte comune}} \ \underbrace{3y + x > 0}_{\text{tutte le relazioni che devono soddisfare}} \}$$

$$\{ (x,y) \in \mathbb{R}^2 ; \ \underbrace{x \geq 0, \ y \geq 0, \ x+y \leq 1}_{\text{3 relazioni}} \}$$

(devono essere soddisfatte tutte)

$x \geq 0, \ y \geq 0 \rightsquigarrow$ I quadrante

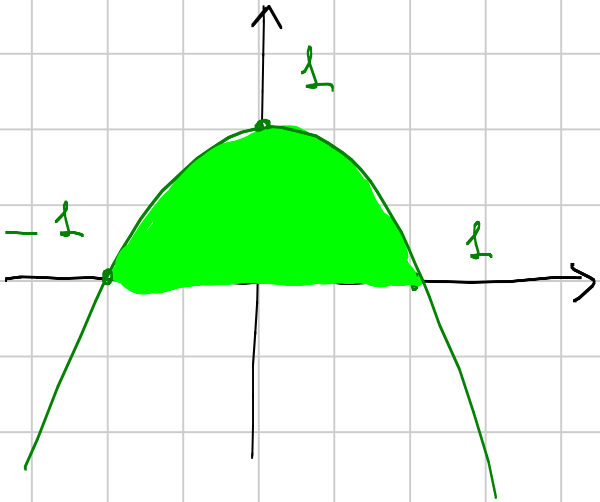
$x+y \leq 1 \rightsquigarrow y \leq 1-x \rightsquigarrow$ sotto la retta $y = 1-x$



$$\{(x,y)\in\mathbb{R}^2;\ 0\le y\le 1-x^2\}$$

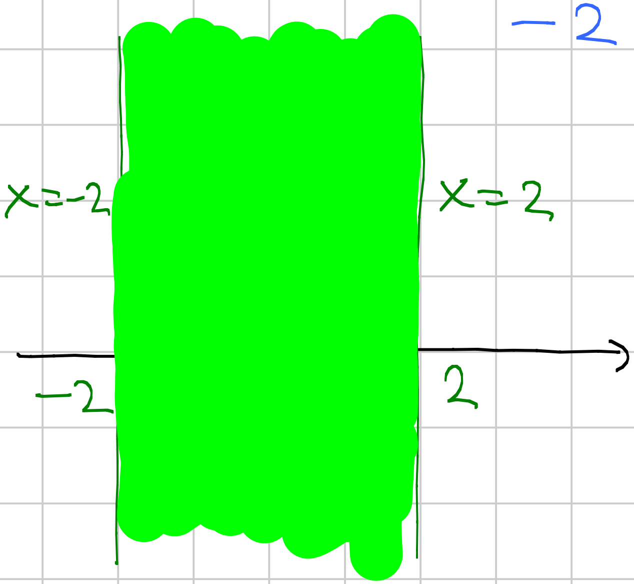
$y\ge 0 \rightsquigarrow$ I e II quadrante

$y\le 1-x^2 \rightsquigarrow$ sotto la parabola $y=1-x^2$



$$\{(x,y)\in\mathbb{R}^2;\ \underbrace{x^2\le 4}_{-2\le x\le 2}\}$$

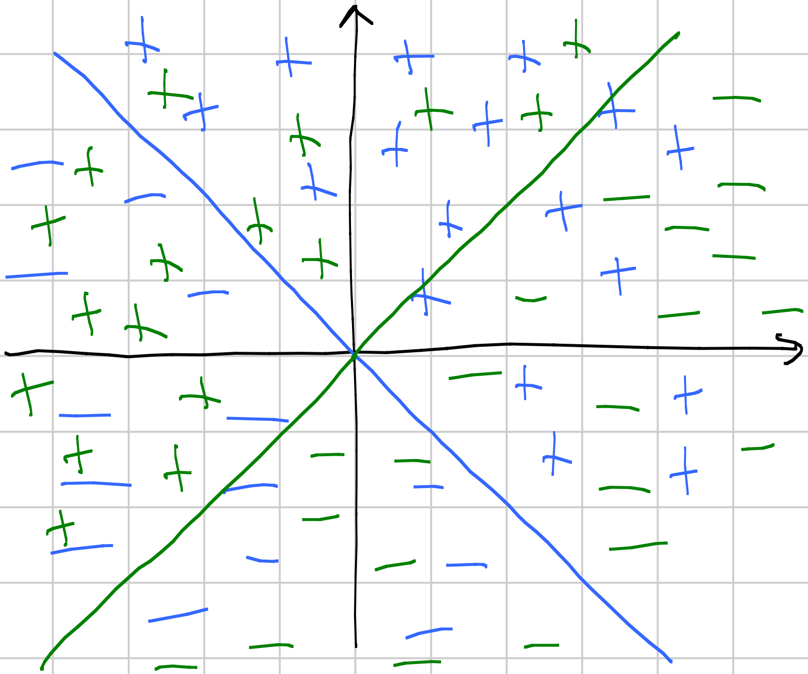
$-2\le x\le 2$

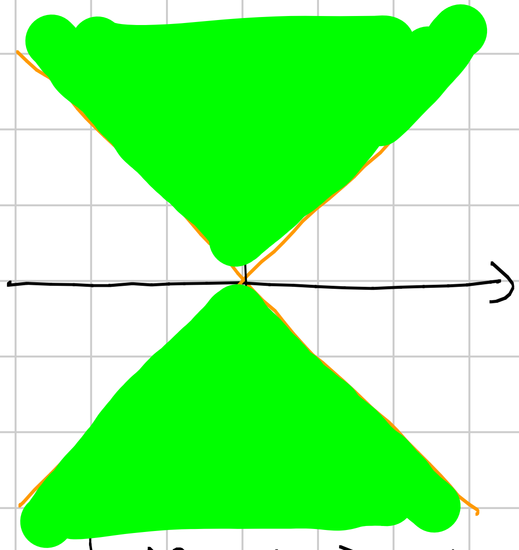


$$\{(x,y)\in\mathbb{R}^2 : y^2 - x^2 \geq 0\}$$

$$(y+x)(y-x) \geq 0$$

1° FATT. 2° FATT





1° FATT.

$y+x>0$ dove $y>-x$

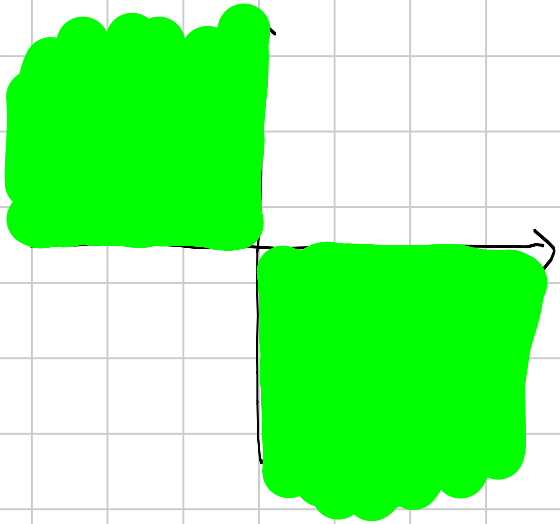
$y+x<0$ dove $y<-x$

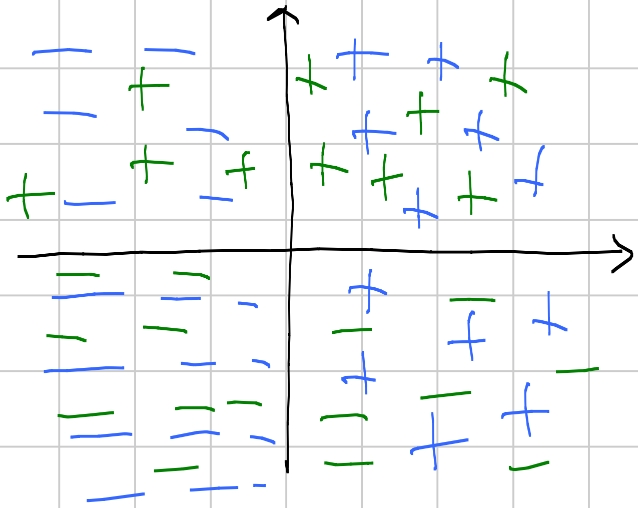
2° FATT.

$y-x>0$ dove $y>x$

$y-x<0$ dove $y<x$

$$\{(x,y) \in \mathbb{R}^2 : \underbrace{x}_{1° \text{ FATT}}\,\underbrace{y}_{2° \text{ FATT}} \le 0\}$$





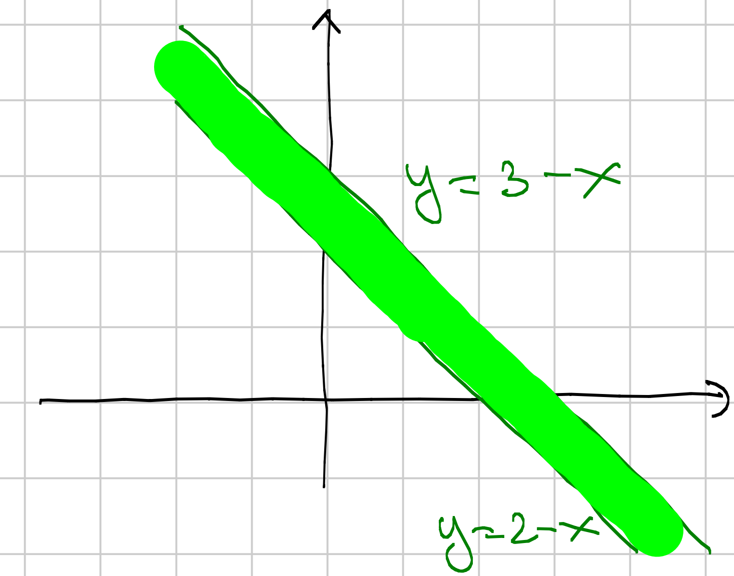
Altro modo: affinché sia $xy < 0$ ci sono 2 possibilità

- $x > 0,\ y < 0 \rightsquigarrow$ IV quadrante
- $x < 0,\ y > 0 \rightsquigarrow$ II quadrante

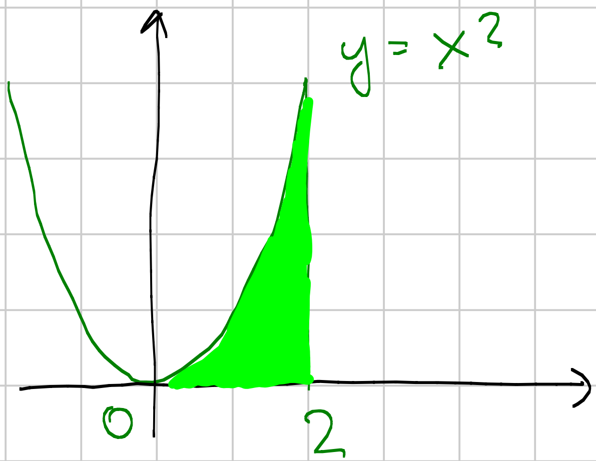
$\{(x,y) \in \mathbb{R}^2 ; \quad 2 \leq x+y \leq 3\}$

$x+y \geq 2 \rightsquigarrow y \geq 2-x \rightsquigarrow$ sopra retta $y=2-x$

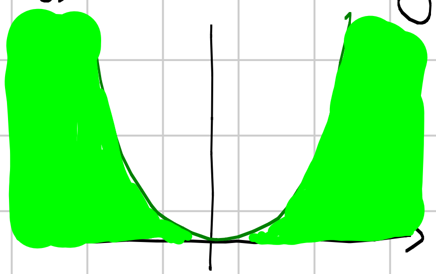
$x+y \leq 3 \rightsquigarrow y \leq 3-x \rightsquigarrow$ sotto retta $y=3-x$



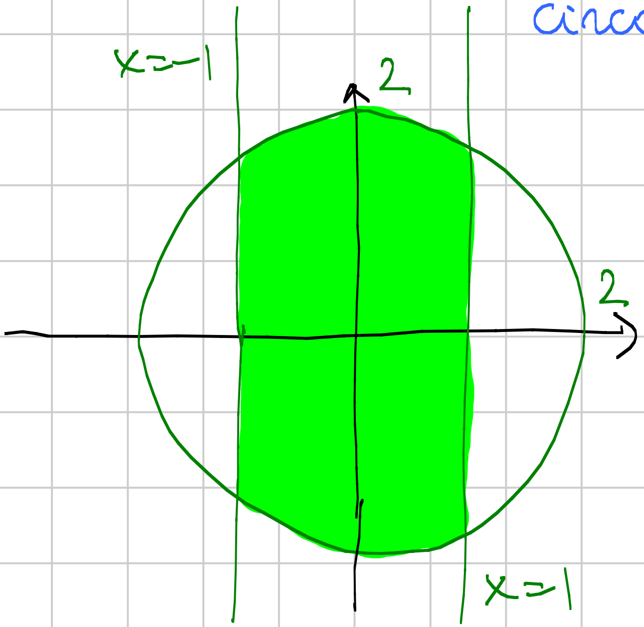
$\{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 2, \ 0 \leq y \leq x^2\}$



$\{(x,y) \in \mathbb{R}^2 : 0 \leq y \leq x^2\}$



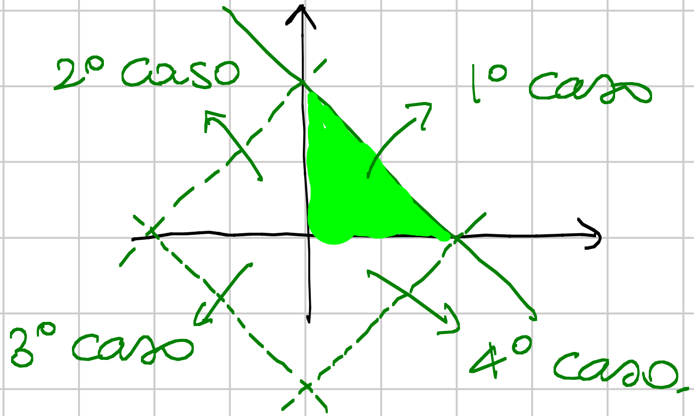
$$\left\{(x,y)\in\mathbb{R}^2 : \underbrace{x^2+y^2\le 4}_{\text{Dentro la circonferenza}},\ \underbrace{|x|\le 1}_{-1\le x\le 1}\right\}$$



$$\{(x,y)\in\mathbb{R}^2 : |x|+|y|\le 1\}$$

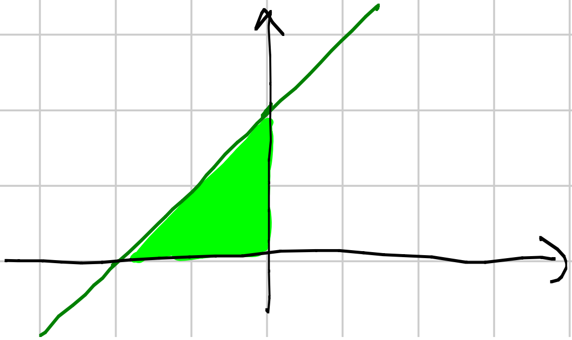
4 casi a seconda del segno di x e y

1° caso: $x\ge 0,\ y\ge 0,\ x+y\le 1$



2° caso: $\overbrace{x<0,\ y\ge 0}^{\text{II quadr.}},\ -x+y\le 1$

$y\le x+1$

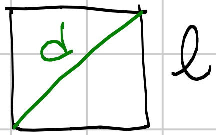


3° caso: $x<0, y<0, -x-y\leq 1$

4° caso: $x\geq 0, y<0, x-y\leq 1$

— o — o —

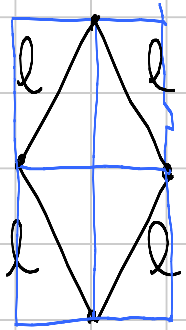
$\boxed{\text{Geometria euclidea classica}}$



Quadrato: 4 lati uguali e 4 angoli retti

$$d^2 = l^2 + l^2$$
$$= 2l^2$$
$$d = \sqrt{2}\, l$$

Rombo: 4 lati uguali

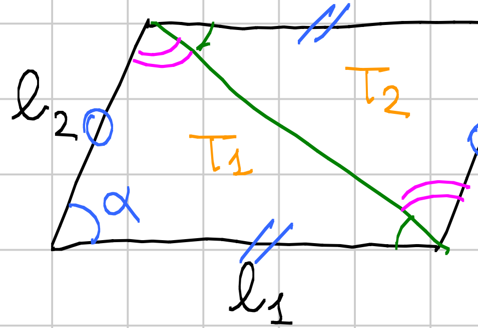


$$\text{Area} = \text{diagonale} \times \text{diagonale} \cdot \frac{1}{2}$$

Rettangolo: 4 angoli retti

Parallelogrammo

Lati paralleli e uguali a 2 a 2.



alterni interni

$$\text{Area} = l_1 \cdot l_2 \cdot \sin\alpha$$

Perché?

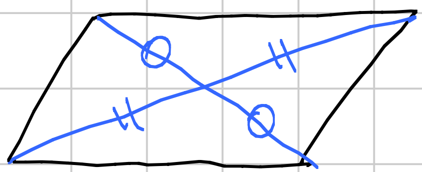
$$\text{Area}(T_1) = \frac{1}{2} l_1 \cdot l_2 \cdot \sin\alpha$$

Formula trigonometrica

T_1 e T_2 sono uguali

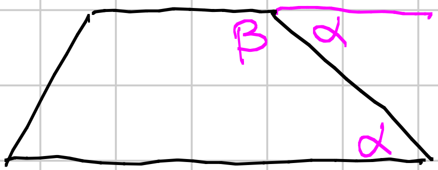
$$\text{Area}(T_2) = \text{Area}(T_1)$$

$$\text{Area (parall.)} = \text{Area}(T_1) + \text{Area}(T_2) = 2\,\text{Area}(T_1)$$

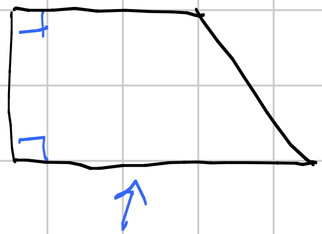


Per calcolare la lunghezza delle diagonali basta usare CARNOT.

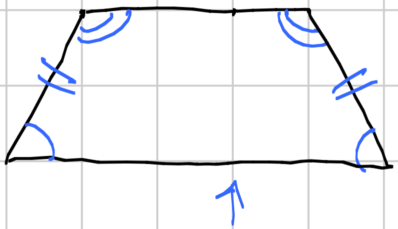
Trapezio : una coppia di lati paralleli



$\alpha + \beta = 180°$

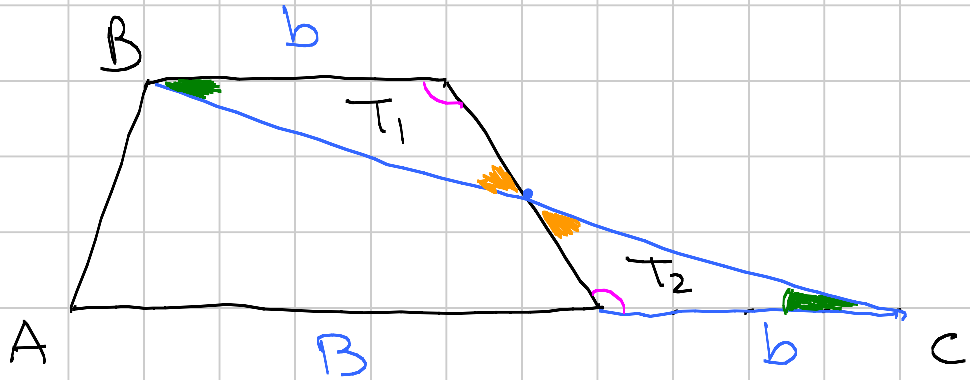


trapezio rettangolo



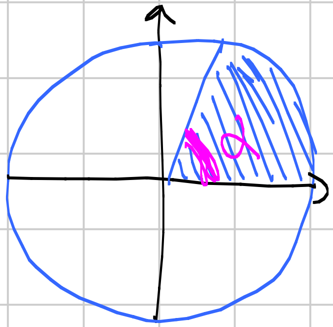
trapezio isoscele

$$\text{Area trapezio} = \frac{1}{2}(\underbrace{B}_{\text{base magg}} + \underbrace{b}_{\text{base minore}})\underbrace{h}_{\text{altezza}}$$



T_1 e T_2 sono uguali, dunque area trapezio = area (ABC)

Cerchio : Area $= \pi R^2$ lungh. circ. $= 2\pi R$



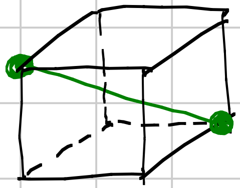
Area settore : Area cerchio $= \alpha : 2\pi$

— o — o — o —

<u>Cubo</u> : 1 parametro (il lato ℓ)

Volume : ℓ^3, 6 facce, 8 vertici, 12 spigoli

Sup. totale : $6\,\ell^2$

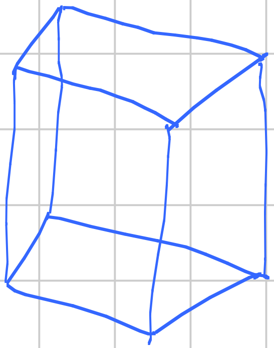


Diagonale $= \ell\sqrt{3}$

<u>Sfera</u> : 1 parametro (il raggio R). Volume $= \frac{4}{3}\pi R^3$

Sup $= 4\pi R^2$

Prisma



Volume = Area base × altezza

Sup. laterale = altezza × perimetro base

Piramide

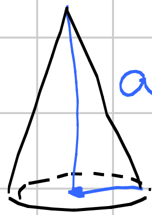
Volume = Area base · altezza · $\frac{1}{3}$

Se la base è un poligono regolare la piramide si dice retta se il piede dell'altezza è il centro della base

Cono

2 parametri : altezza R
raggio base R

$$\text{Volume} = \frac{1}{3} \overbrace{\pi R^2}^{\text{Area base}} \cdot R$$



apotema a

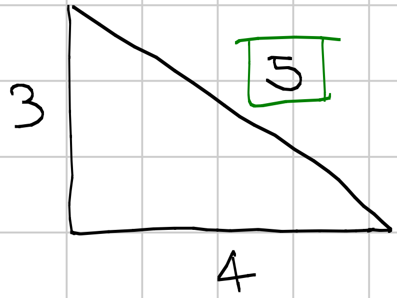
$$a^2 = R^2 + R^3$$

① Triangolo rett. Un lato lungo 3 } non sappiamo quali

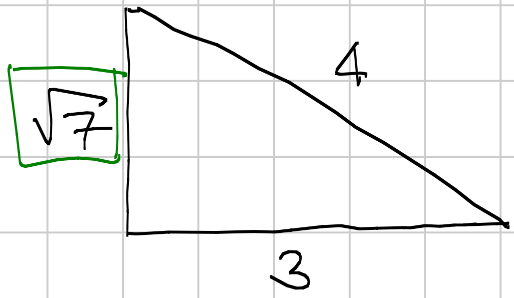
" " " 4

Quali le lungh. possibili per il 3° lato.

1° caso

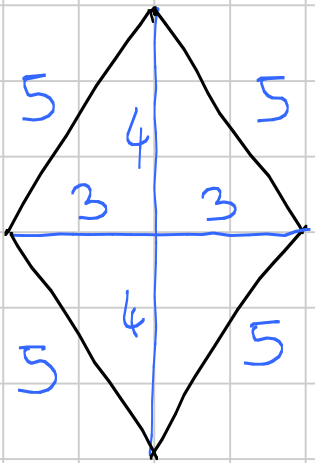


2° caso



$$\sqrt{16-9} = \sqrt{7}$$

②



Perimetro = 20

③ Volume cilindro =

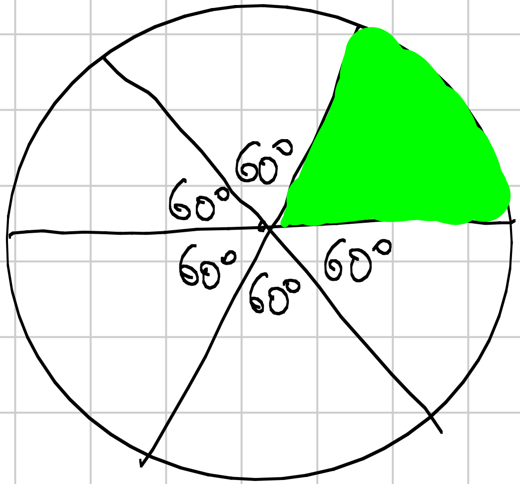
= Area base · alt

= $\pi R^2 \cdot h$

Volume cono = $\frac{1}{3}\pi R^2 \cdot h$

$$\frac{V(\text{cil.})}{V(\text{cono})} = 3$$

(4)



Perimetro fetta $= 2R + \frac{1}{6}$ lungh. circ.

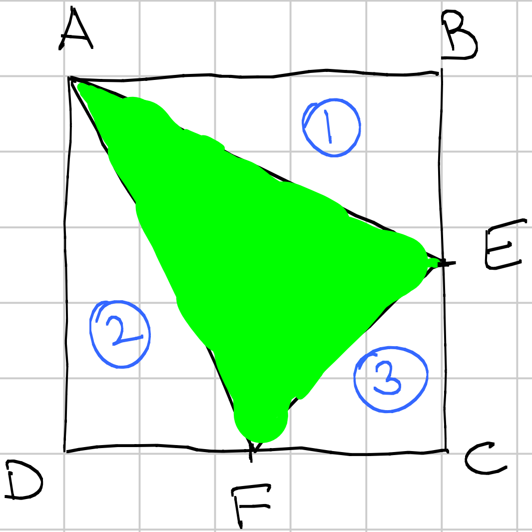
$= 2R + \frac{1}{6} 2\pi R$

$= R\left(2 + \frac{\pi}{3}\right)$

$= 30 \left(2 + \frac{\pi}{3}\right)$

$= 60 + 10\pi$

(5)

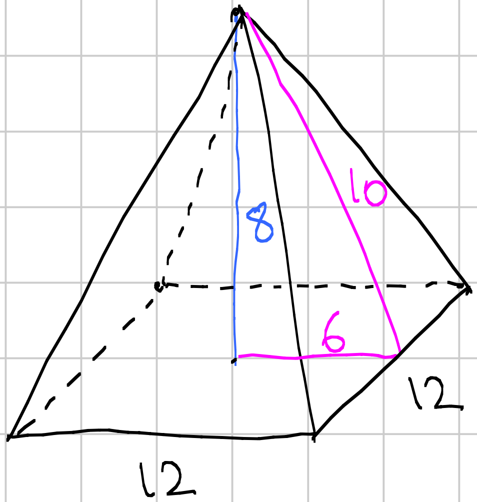


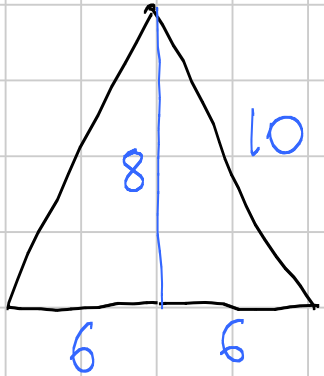
Area (AEF) = Area (quadrato) − (1) − (2) − (3)

$= 1 - \frac{1}{4} - \frac{1}{4} - \frac{1}{8} = \frac{3}{8}$

Area (1) = Area (2) $= \frac{1}{2} \cdot 1 \cdot \frac{1}{2} = \frac{1}{4}$; Area (3) $= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

(6)





L'altezza di ogni faccia è 10

Area di 1 faccia: $\frac{1}{2}$ base faccia · Altezza faccia

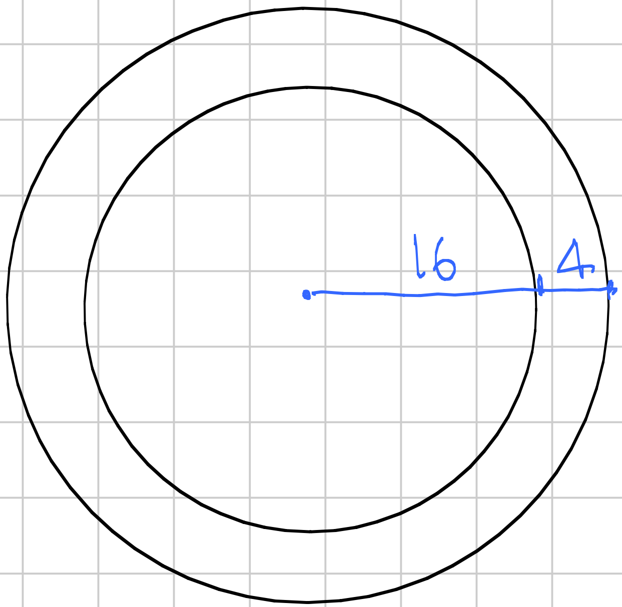
$= \frac{1}{2} \cdot 12 \cdot 10 = 60$

Sup. laterale $= 4 \cdot 60 = 240$

Se raddoppio tutte le dimensioni

* la sup. laterale si moltiplica per $4 = 2^2$
* il vol. " " " $8 = 2^3$

7 Cocomero con raggio esterno 20 cm
Buccia spessa 4 cm.
Quale percentuale del volume occupa la buccia?



Volume totale $= \frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi \cdot 20^3$

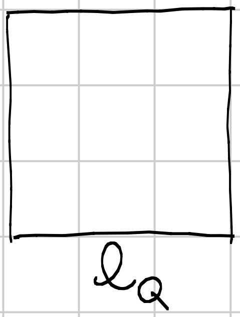
Volume parte buona $=$

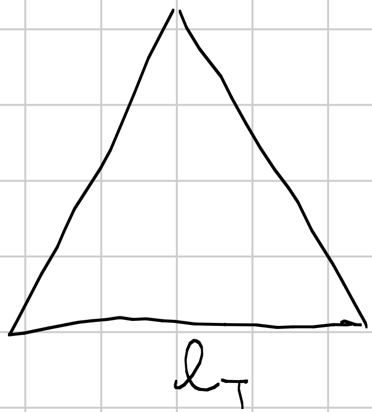
$= \frac{4}{3}\pi\, 16^3$

Percentuale buona $= \dfrac{\text{Volume "buono"}}{\text{Volume totale}} = \dfrac{\cancel{\frac{4}{3}}\cancel{\pi}\, 16^3}{\cancel{\frac{4}{3}}\cancel{\pi}\, 20^3} = \left(\frac{16}{20}\right)^3 =$

$= \left(\frac{4}{5}\right)^3 = \frac{64}{125} > \frac{64}{128} = \frac{1}{2}$ = poco + di $\frac{1}{2}$

⑧





Area quadrato =

Area $\triangle$ equilatero

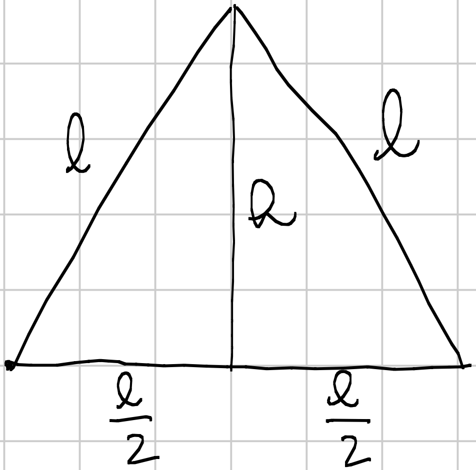
$$\frac{l_T}{l_Q} = ?$$

$$\text{Area quadrato} = (l_Q)^2$$

$$\text{Area triangolo eq.} = \frac{(l_T)^2 \sqrt{3}}{4}$$

$$1 = \frac{\text{Area }(Q)}{\text{Area }(T)} = \frac{(l_Q)^2}{\frac{(l_T)^2\sqrt{3}}{4}} \Rightarrow \left(\frac{l_Q}{l_T}\right)^2 = \frac{4}{\sqrt{3}} \Rightarrow \frac{l_Q}{l_T} = \frac{2}{\sqrt[4]{3}}$$

Area triangolo equilatero di lato l



$$h^2 = l^2 - \left(\frac{l}{2}\right)^2$$

$$= l^2 - \frac{l^2}{4} = \frac{3}{4} l^2$$

$$\Longrightarrow h = \sqrt{\frac{3}{4} l^2} = \frac{\sqrt{3}}{2} l$$

$$\boxed{h = \frac{\sqrt{3}}{2} l}$$

$$\text{Area} = \frac{1}{2} \text{base} \cdot \text{alt} = \frac{1}{2} l \cdot \frac{\sqrt{3}}{2} l = \boxed{\frac{\sqrt{3}}{4} l^2}$$

In alternativa: $\text{Area} = \frac{1}{2} l \cdot l \cdot \sin 60°$

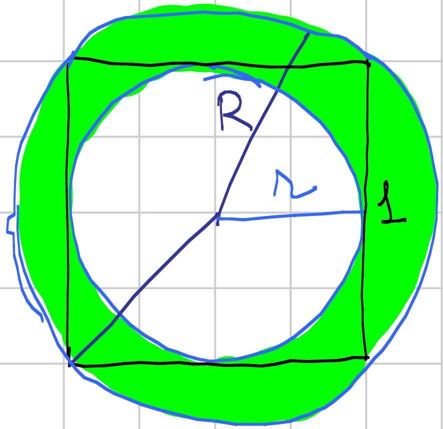
$$= \frac{1}{2} l^2 \cdot \frac{\sqrt{3}}{2} = l^2 \frac{\sqrt{3}}{4}$$

(9) I lati di T_1 sono 10, 12, 14
I lati di T_2 sono 5, 6, 7

$$\frac{\text{Area}(T_1)}{\text{Area}(T_2)} = ? \quad 4$$

(se moltiplico le dimensioni per k, l'area viene moltiplicata per k^2) eventuali volumi per k^3.

(10)



Area Corona circ =

= Area cerchio fuori − Area cerchio dentro

$$= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$$

$r = \frac{1}{2}$ $\quad R = \frac{\text{diagonale}}{2} = \frac{\sqrt{2}}{2}$

$$= \pi\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \frac{\pi}{4}$$