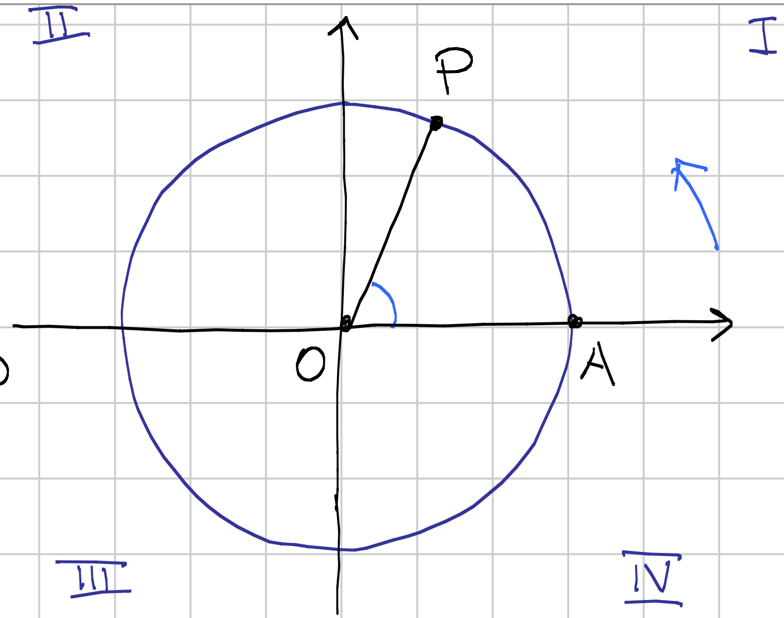
TRIGONOMETRIA

Titolo nota 25/09/2007

Angoli e archi



OA = 1

2 modi per individuare P

- mediante l'angolo $A\hat{O}P$

ANGOLI SESSAGESIMALI (GRADI)

- mediante la lungh. dell'arco AP

RADIANTI

PASSAGGIO GRADI ⟺ RADIANTI

$$\boxed{360° : 2\pi = \text{misura in gradi} : \text{misura in radianti}}$$

Esempi A quanti corrispondono $60°$?

$$360° : 2\pi = 60° : x \qquad x = \frac{2\pi \cdot \cancel{60°}}{\cancel{360°}_{6}} = \frac{\pi}{3}$$

A cosa corrispondono $200°$?

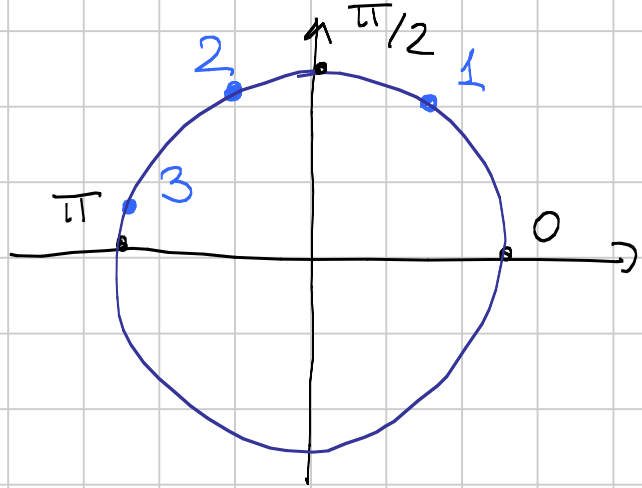
$$360° : 2\pi = 200° : x \qquad x = \frac{2\pi \cdot \cancelto{5}{200°}}{\cancelto{9}{360°}} = \frac{10}{9}\pi$$

A cosa corrispondono $\frac{3\pi}{4}$?

$$360° : 2\pi = x : \frac{3\pi}{4} \qquad x = \cancelto{45}{\cancel{360}} \cdot \frac{3\cancel{\pi}}{\cancel{4}} \cdot \frac{1}{\cancel{2\pi}}$$

$$= 135°$$

Disegnare (+ o -) il punto della circ. trigonometrica corrispondente a 2 radianti

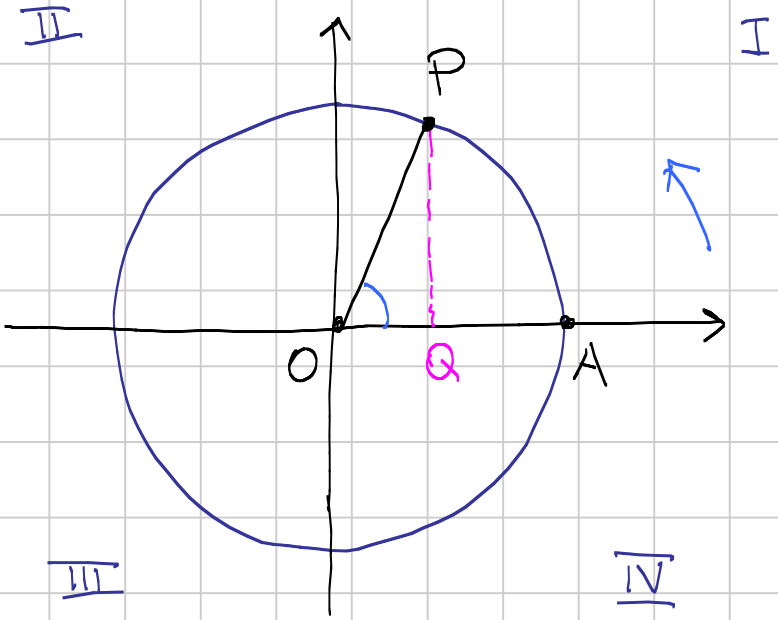


Quanti gradi corrispondono a 1 radiante?

$360° : 2\pi = x : 1$

$x = \dfrac{360°}{2\pi}$ = un po' meno di 60°

FUNZIONI TRIGONOMETRICHE



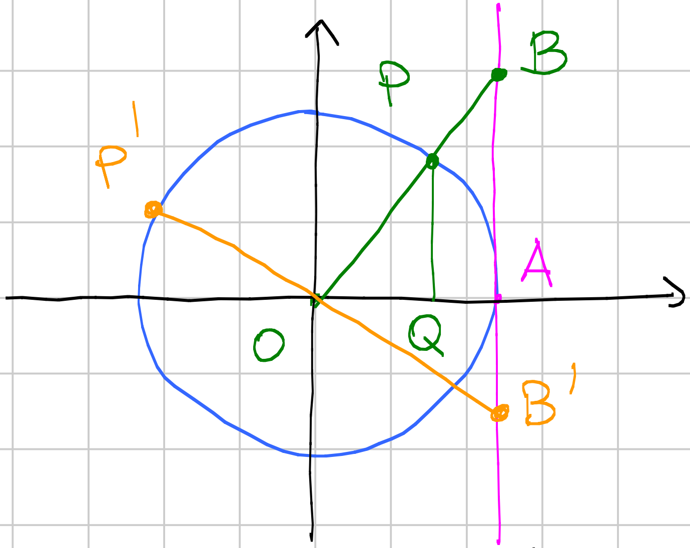
Se α indica l'angolo (o l'arco) che individua il punto P, allora le coordinate di P sono

$$(\cos\alpha, \sin\alpha)$$

$$\left.\begin{aligned} OQ &= \cos\alpha \\ PQ &= \sin\alpha \end{aligned}\right\} \text{intese con il segno}$$

I quadr.	$\cos > 0$	$\sin > 0$
II quadr.	$\cos < 0$	$\sin > 0$
III quadr.	$\cos < 0$	$\sin < 0$
IV quadr.	$\cos > 0$	$\sin < 0$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = AB$$



La tangente è definita per quei valori di α t.c.

$$\cos\alpha \neq 0$$

quindi per $\alpha \neq 90°, 270°$ e analoghi in gradi

$\alpha \neq \frac{\pi}{2}, \frac{3\pi}{2}$ e analoghi in radianti

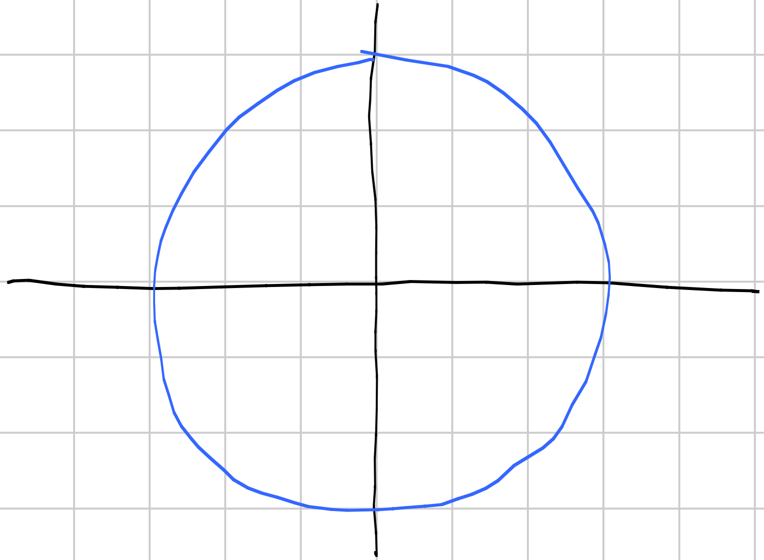
Geometricamente in questi casi la retta OP e la tg. in A alla circ. trigonometrica sono parallele

Il fatto che $\tan\alpha = AB$ si dimostra osservando la similitudine dei triangoli OPQ e OBA

$$\frac{PQ}{OQ} = \frac{AB}{OA} \Rightarrow \qquad AB = OA \cdot \frac{PQ}{OQ} = 1 \cdot \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

ARCHI NOTEVOLI

$0°$, 0 radianti, $\cos 0° = 1$
$\sin 0° = 0$
$\tan 0° = 0$

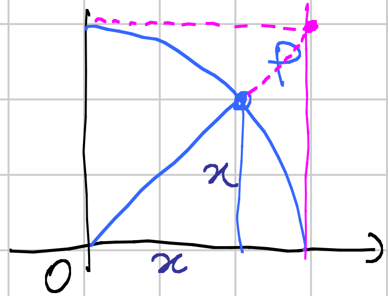


$90°$, $\frac{\pi}{2}$ radianti, $\cos 90° = 0$
$\sin 90° = 1$
$\tan 90°$ non definita

$180°$, π radianti, $\cos 180° = -1$, $\sin 180° = 0$, $\tan 180° = 0$

$45°$, $\frac{\pi}{4}$ radianti,

sin e cos sono uguali



Per il teo di Pitagora $x^2 + x^2 = OP^2 = 1$ $\quad 2x^2 = 1$

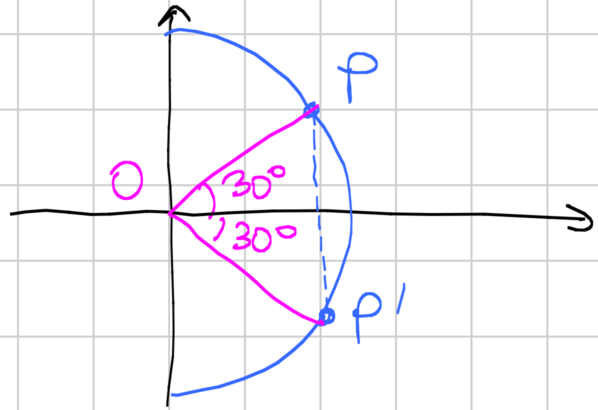
$x^2 = \frac{1}{2}$ $\quad x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ (Ho scelto + perché P è nel I quadrante)

$45°$, $\frac{\pi}{4}$ radianti, $\sin = \cos = \frac{\sqrt{2}}{2}$ $\tan = 1$

$30°$, $\frac{\pi}{6}$ radianti, $\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

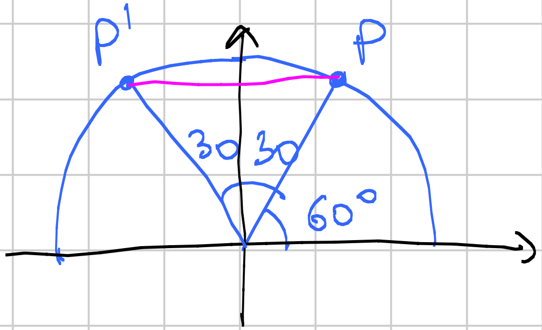


Il triangolo OPP' è isoscele sulla base PP' e ha l'angolo al vertice di 60°. Ne segue che tutti gli angoli sono di 60°, dunque è equilatero.

A questo p.to $\sin 30°$ = metà del lato PP' $= \frac{1}{2}$

Il $\cos 30°$ si ricava con teorema di Pitagora

$60°$, $\frac{\pi}{3}$ radianti, $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$



$120°$, $\frac{2\pi}{3}$ radianti, $\sin 120° = \frac{\sqrt{3}}{2}$, $\cos 60° = -\frac{1}{2}$,

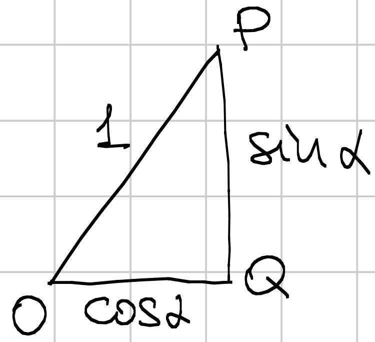
$\tan 120° = -\sqrt{3}$

— o — o —

RELAZIONE FONDAMENTALE

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$$

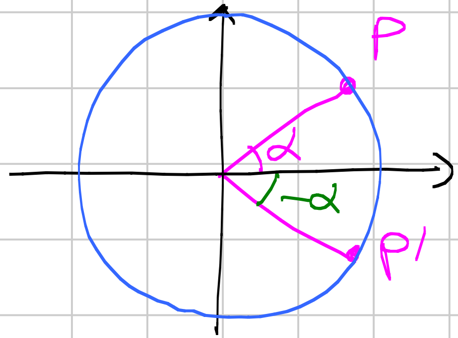
TEOREMA DI PITAGORA



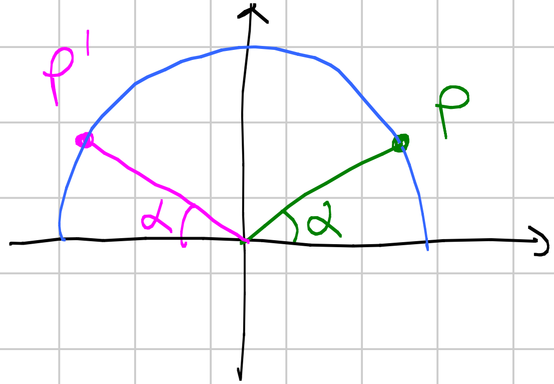
ARCHI ASSOCIATI

Dato un arco α, quali sono le funzioni trigonometriche di $\pi+\alpha$, $\pi-\alpha$, $-\alpha$, $\frac{\pi}{2}+\alpha$, $\frac{\pi}{2}-\alpha$, $\frac{3\pi}{2}\pm\alpha$

$\cos(-\alpha) = \cos\alpha$
$\sin(-\alpha) = -\sin\alpha$
$\tan(-\alpha) = -\tan\alpha$

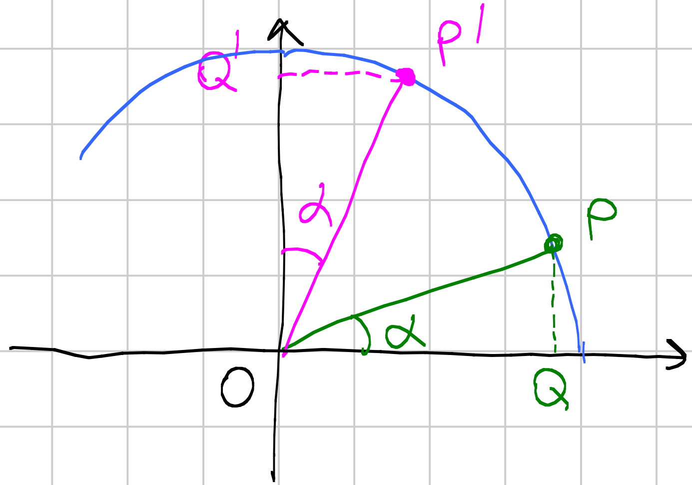


$\cos(\pi-\alpha) = -\cos\alpha$
$\sin(\pi-\alpha) = \sin\alpha$
$\tan(\pi-\alpha) = -\tan\alpha$



$$\cos(\pi+\alpha) = -\cos\alpha$$
$$\sin(\pi+\alpha) = -\sin\alpha$$
$$\tan(\pi+\alpha) = \tan\alpha$$

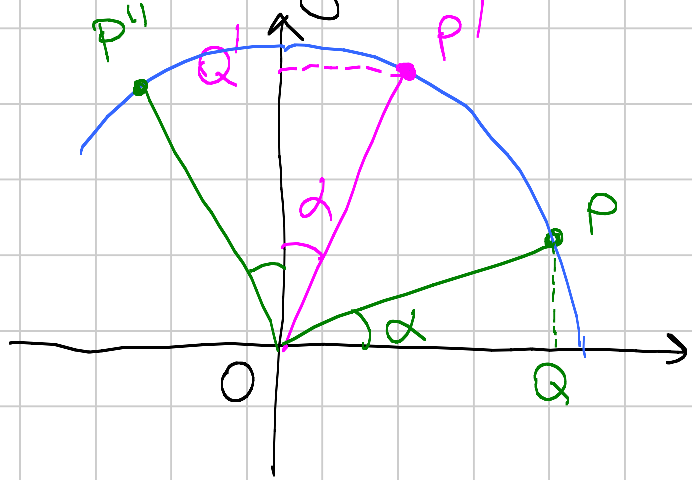
$$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$$
$$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$$
$$\tan\left(\frac{\pi}{2}-\alpha\right) = \frac{1}{\tan\alpha}$$



OPQ e OP'Q' sono uguali

$$\left.\begin{aligned}\cos\left(\frac{\pi}{2}+\alpha\right) &= -\sin\alpha\\ \sin\left(\frac{\pi}{2}+\alpha\right) &= +\cos\alpha\\ \tan\left(\frac{\pi}{2}+\alpha\right) &= -\frac{1}{\tan\alpha}\end{aligned}\right\}$$

PUNTO P''



FORMULE DI ADDIZIONE

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \quad (1)$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \quad (2)$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \quad (3)$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \quad (4)$$

CASO $\alpha=\beta \rightarrow$ DUPLICAZIONE

USO (3)

$$\sin(2\alpha) = \sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$$

$$\boxed{\sin(2\alpha) = 2\sin\alpha\cos\alpha}$$

$$\cos(2\alpha) = \cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$$
$$= \cos^2\alpha - \sin^2\alpha$$
$$= \cos^2\alpha - (1-\cos^2\alpha) = 2\cos^2\alpha - 1$$
$$= (1-\sin^2\alpha) - \sin^2\alpha = 1 - 2\sin^2\alpha$$

$$\boxed{\begin{aligned} \cos(2\alpha) &= \cos^2\alpha - \sin^2\alpha \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha \end{aligned}}$$

— o — o — o —

FORMULE PRODOTTO → SOMMA

$\boxed{5}$ $$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right] \qquad (1) + (2)$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] \qquad (2) - (1)$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right] \qquad 3 + 4$$

— o — o —

FORMULE SOMMA → PRODOTTO

$$\boxed{\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$$

Dalla $\boxed{5}$ abbiamo che

$$2\cos\underset{\frac{x+y}{2}}{\alpha}\cos\underset{\frac{x-y}{2}}{\beta} = \cos\underbrace{(\alpha+\beta)}_{x}+\cos\underbrace{(\alpha-\beta)}_{y}$$

$\alpha+\beta = x$ Sommo: $2\alpha = x+y$ $\alpha = \frac{x+y}{2}$

$\alpha-\beta = y$

Sottraggo: $2\beta = x-y$ $\beta = \frac{x-y}{2}$

$$\boxed{\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)}$$

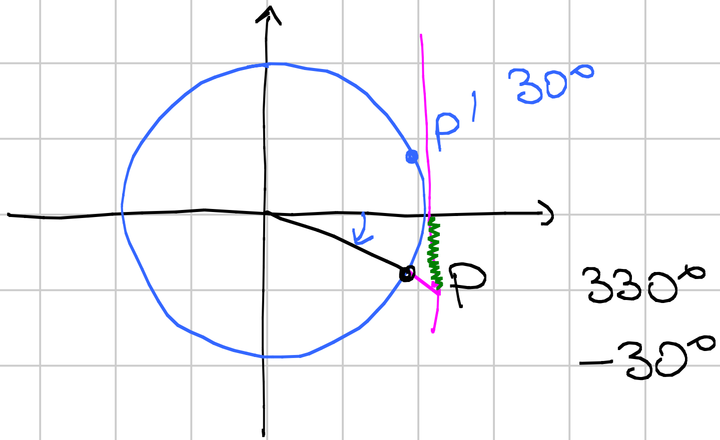
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)$$

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cdot\cos\left(\frac{x+y}{2}\right)$$

$330°$



$$\sin(330°) = \sin(-30°)$$
$$= -\sin 30° = -\frac{1}{2}$$

$$\cos(330°) = \cos(-30°)$$
$$= \cos(30°) = \frac{\sqrt{3}}{2}.$$

$$\tan(330°) = -\frac{1}{\sqrt{3}}$$

$$15° = 45° - 30°$$

$$\cos(15°) = \cos(45° - 30°) =$$ Formula addizione

$$= \cos 45° \cos 30° + \sin 45° \sin 30°$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin(15^\circ) = \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos(75^\circ) = \cos(90^\circ - 15^\circ) = \sin(15^\circ)$$

Archi associati $\cos\left(\frac{\pi}{2} - \alpha\right)$

$$\sin(75^\circ) = \sin(90^\circ - 15^\circ) = \cos(15^\circ)$$

$$\cos(-75^\circ) = \cos(75^\circ)$$

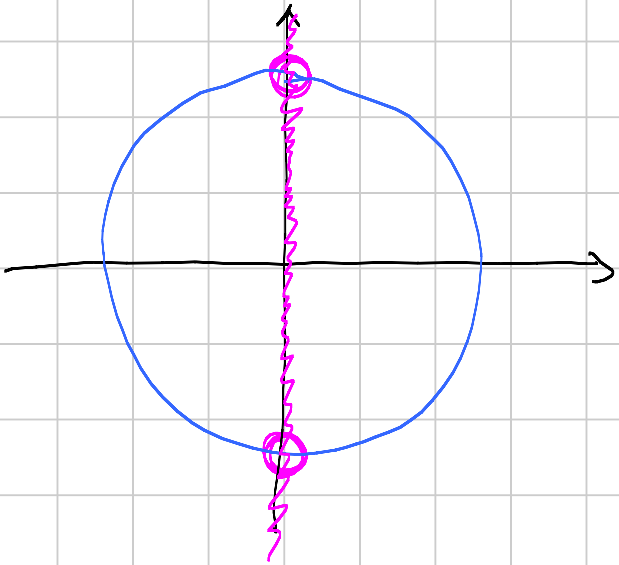
$$\sin(-75^\circ) = -\sin(75^\circ)$$

EQUAZIONI

① Trovare le soluzioni di $\cos x = 0$ nell'intervallo $[0, 3\pi]$ → 1.5 giri

Guardare il cerchio !!!

Trovare i p.ti della circ. trigo. con coord. $x = 0$

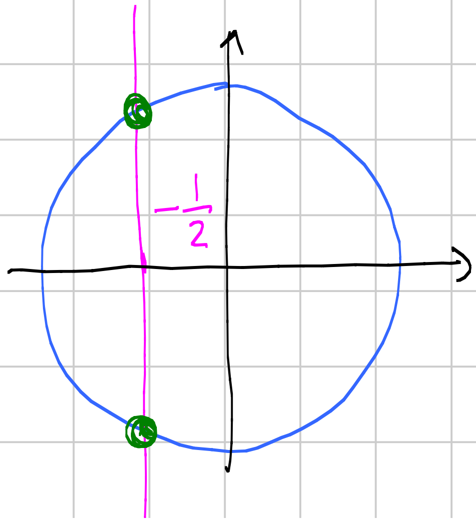


3 soluzioni

$\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2} = 2\pi + \frac{\pi}{2}$

② $\cos x = -\frac{1}{2}$

in $[0, 2\pi]$

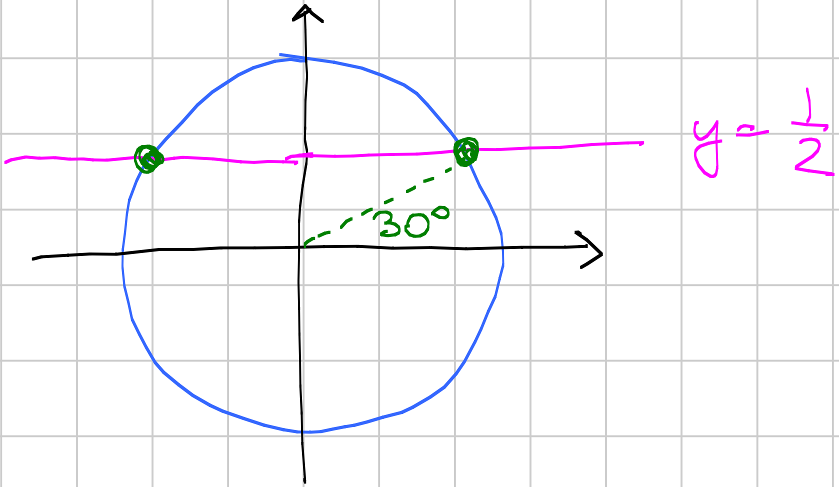


2 soluzioni:

$x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$

(3) $\sin x = \frac{1}{2}$ in $[0, 2\pi]$

coordinata y del punto $= \frac{1}{2}$

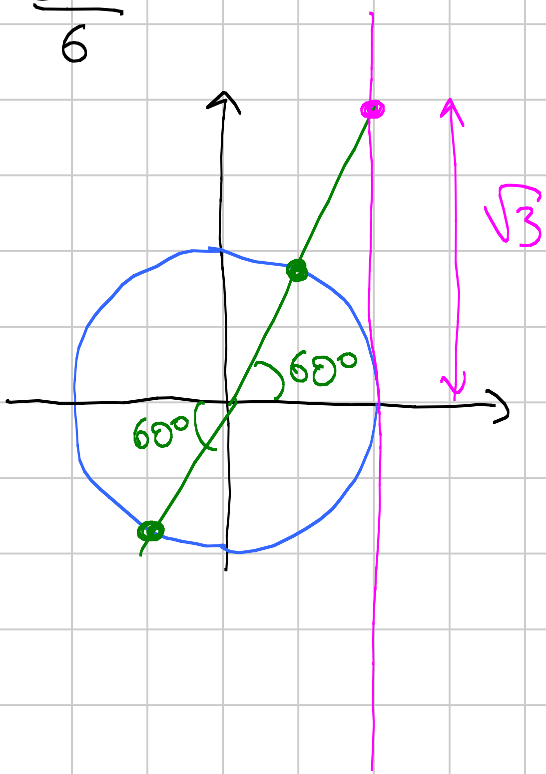


2 soluzioni: $x = \frac{\pi}{6}$; $x = \frac{5\pi}{6}$

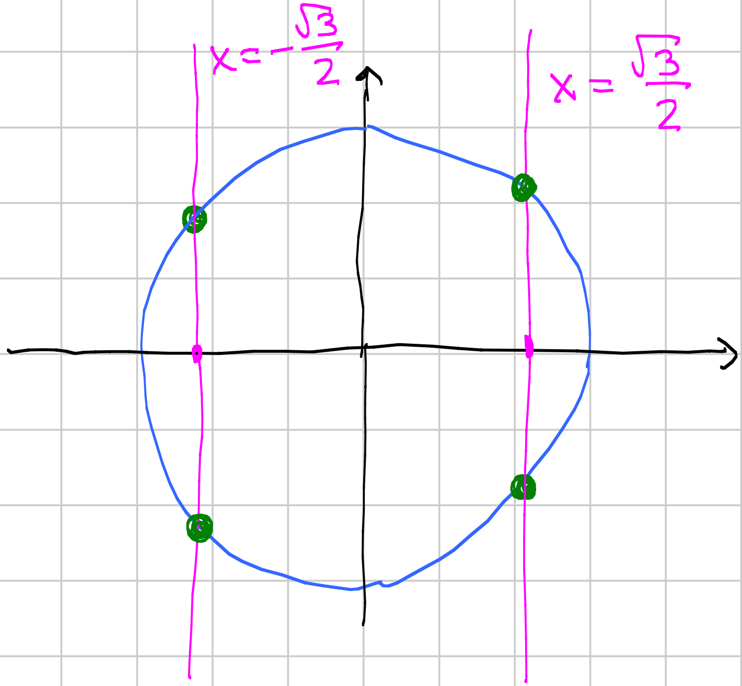
(4) $\tan x = \sqrt{3}$ in $[0, 2\pi]$

2 soluzioni:

$x = \frac{\pi}{3}$, $x = \frac{4\pi}{3}$



(5) $4\cos^2 x = 3$; $\cos^2 x = \frac{3}{4}$; $\cos x = \pm\frac{\sqrt{3}}{2}$



4 soluzioni:

$\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, $\frac{11\pi}{6}$

$\frac{7\pi}{6} \to \pi + \frac{\pi}{6}$

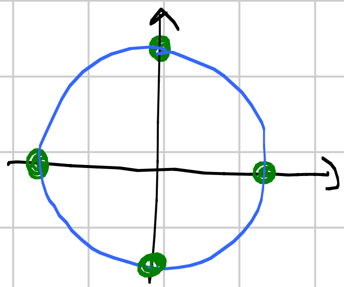
$\frac{11\pi}{6} \to 2\pi - \frac{\pi}{6}$

(6) $\cos^3 x = \cos x$ $\boxed{\cos^2 x = 1 \quad \text{NO!!!!}}$ $\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1) = 0$ $\to \cos x = 0$

$\to \cos^2 x - 1 = 0 \to \cos^2 x = 1 \to \cos x = \pm 1$

$\cos x = \begin{cases} 0 \\ 1 \\ -1 \end{cases}$



In $[0, 2\pi]$ ci sono 5 soluzioni:

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

DISEQUAZIONI

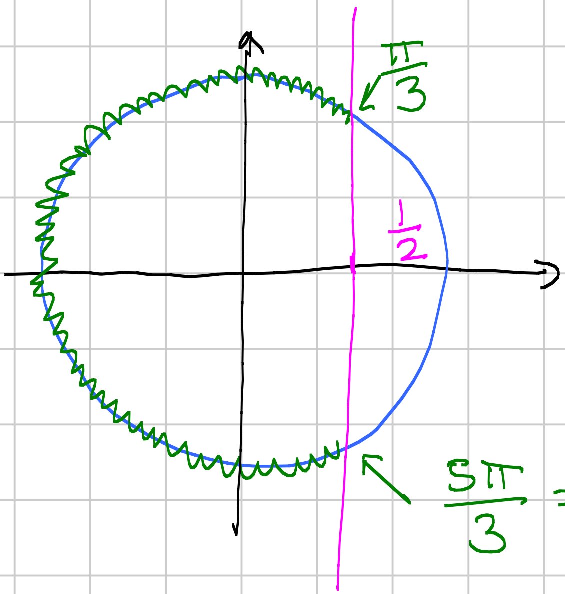
Guardare il cerchio !!!!

① $2\cos x < 1$ in $[0, 2\pi]$

$\cos x < \frac{1}{2}$

↓

Cerco i p.ti della circ. trig. con coord. x minore di $\frac{1}{2}$



$\frac{5\pi}{3} = 2\pi - \frac{\pi}{3}$

Soluzione della diseq. in $[0, 2\pi]$:

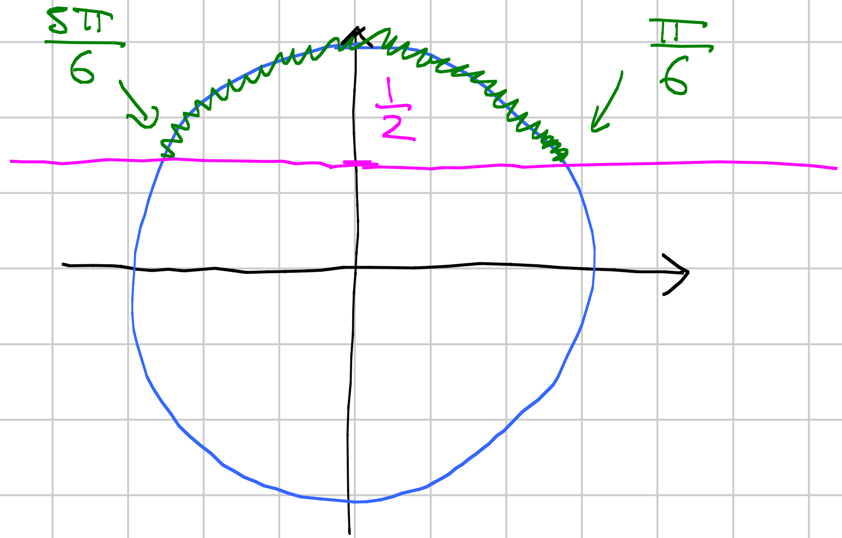
$$\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$$

② $2\sin x > 1$ in $[0, 2\pi]$

$\sin x > \frac{1}{2}$

coordinata y maggiore di $\frac{1}{2}$

$$\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$$

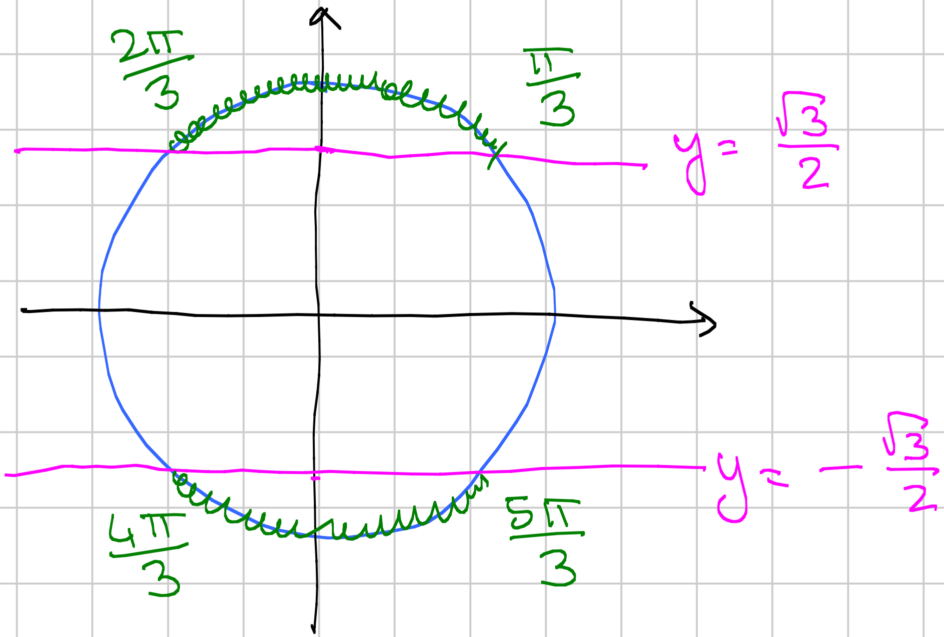


③ $4\sin^2 x > 3$ in $[0, 2\pi]$ $\sin^2 x > \frac{3}{4}$

$t = \sin x$ $t^2 > \frac{3}{4}$, $t^2 - \frac{3}{4} > 0$ $\longrightarrow$ VALORI ESTERNI

Valori esterni: $\sin x > \frac{\sqrt{3}}{2}$ oppure

$\sin x < -\frac{\sqrt{3}}{2}$



$$\left(\frac{\pi}{3}, \frac{2\pi}{3}\right) \cup \left(\frac{4\pi}{3}, \frac{5\pi}{3}\right)$$