Titolo nota 19/09/2007

DISEQUAZIONI

$x > a$ $\quad]a, +\infty[\quad (a, +\infty[$

$x \geq a$ $\quad [a, +\infty[$

$x < a$ $\quad]-\infty, a[\quad]-\infty, a)$

$x \leq a$ $\quad]-\infty, a]$

$a < x < b$ $\quad]a, b[\quad (a, b)$

$a \leq x < b$ $\quad [a, b[\quad [a, b)$

$a < x \leq b$ $\quad]a, b] \quad (a, b]$

$a \leq x \leq b$ $\quad [a, b]$

DISEQUAZIONI DI PRIMO GRADO

$2x > 3 \qquad x > 3/2 \qquad [3/2, +\infty[$

$2x + 1 \geq x + 3$

$2x - x \geq 3 - 1$

$x \geq 2$ $\qquad [2, +\infty[$

$-x > 2$

$x < -2$ $\qquad]-\infty, -2[$

$-2x \leq -6$

$2x \geq 6$ $\qquad x \geq 6/2 = 3$ $\qquad [3, +\infty[$

DISEQUAZIONI DI SECONDO GRADO

$ax^2 + bx + c > 0$ $\qquad$ $x^2 + 3x - 1 \geq 0$

$a > 0$

$ax^2 + bx + c = 0$ ha 2 RADICI DISTINTE E REALI

x_1, x_2

$x_1 > x_2$

$ax^2 + bx + c > 0$ se $x > x_1$

oppure $x < x_2$

quando? in $]-\infty, x_2[\cup]x_1, +\infty[$

$ax^2 + bx + c = 0$ per $x = x_1$ oppure

$x = x_2$

$ax^2 + bx + c < 0$ se $x_2 < x < x_1$

in $]x_2, x_1[$

1 sola radice reale (doppia) x_1

$ax^2 + bx + c > 0$ se $x \neq x_1$ cioè

in $]-\infty, x_1[\cup]x_1, +\infty[$

$ax^2 + bx + c = 0$ se $x = x_1$

$ax^2 + bx + c < 0$ MAI!! $\emptyset$ INSIEME VUOTO

NON CI SONO RADICI REALI

$ax^2+bx+c>0$ SEMPRE

$]-\infty,+\infty[$ oppure $\mathbb{R}$

$ax^2+bx+c=0$ MAI $\cancel{\emptyset}$

$ax^2+bx+c<0$ MAI $\cancel{\emptyset}$

$x^2-2x+1\geq 0$ $(x-1)^2$

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + 2bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - ac}}{a}$$

$$x^2 - 2x + 1$$

$$x_{1,2} = 1 \pm \sqrt{1 - 1}$$

$$= 1$$

$x^2 - 2x + 1 \geq 0$ sempre $\mathbb{R}$ $]-\infty, +\infty[$

$x^2 - 2x + 1 > 0$ $x \neq 1$ $]-\infty, 1[\cup]1, +\infty[$

$x^2 + x + 1 < 0$ $\Delta < 0$ non ci sono radici reali

↑

Soluzione : $\emptyset$ NON CI SONO SOLUZIONI

$x^2 - 2x - 3 \geq 0$ $x_1 = 3$ $x_2 = -1$

$$x_{1,2} = 1 \pm \sqrt{1+3} = 1 \pm 2 \begin{cases} 3 \\ -1 \end{cases}$$

$x \leq -1$ oppure $x \geq 3$ $\quad]-\infty, -1] \cup [3, +\infty[$

$ax^2 + bx + c$ ha 2 radici x_1, x_2
distinte

$$ax^2+bx+c = a(x-x_1)(x-x_2)$$

$ax^2 + bx + c > 0 \qquad a(x-x_1)(x-x_2) > 0$

Prodotto di 2 fattori è >0 se sono tutti e 2 >0 oppure tutti e 2 minori di 0

Tutti e 2 >0 $\quad x - x_1 > 0$ e $x - x_2 > 0$

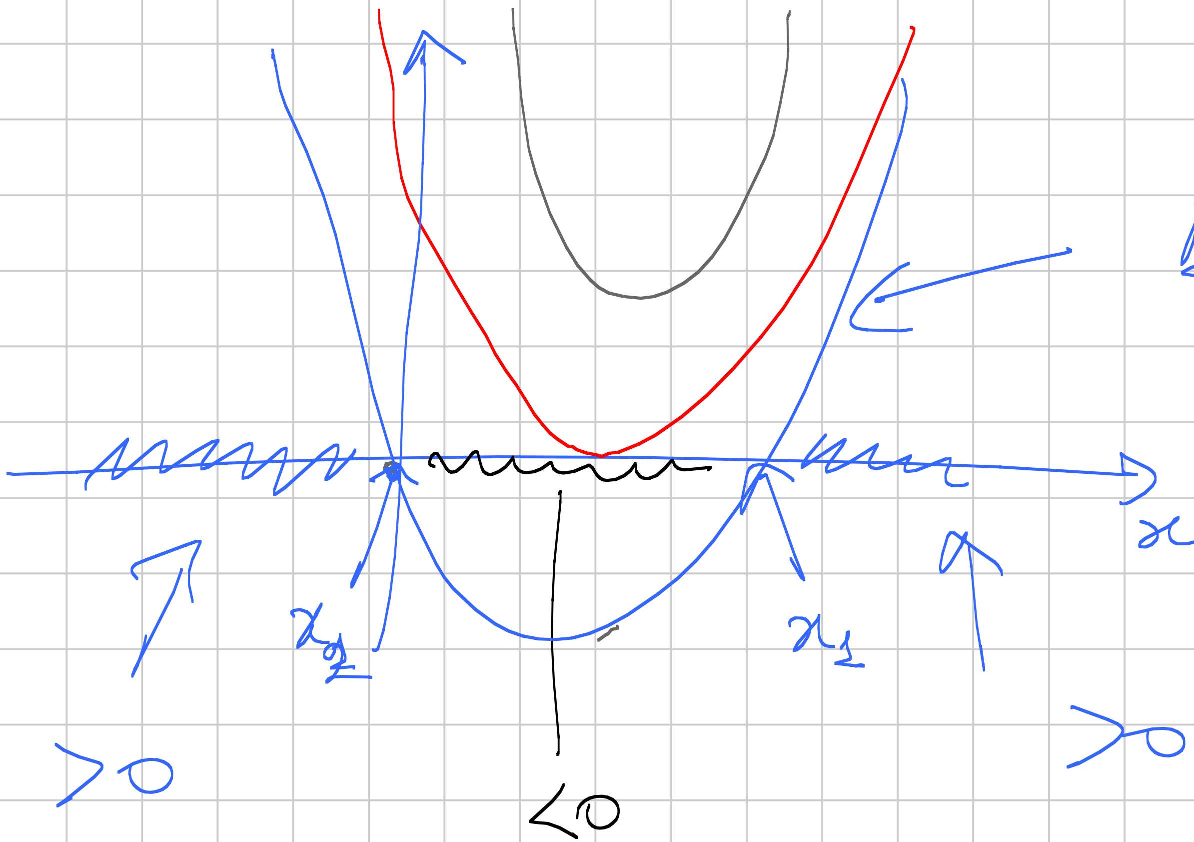
vuol dire $\quad x > x_1$ e $x > x_2$

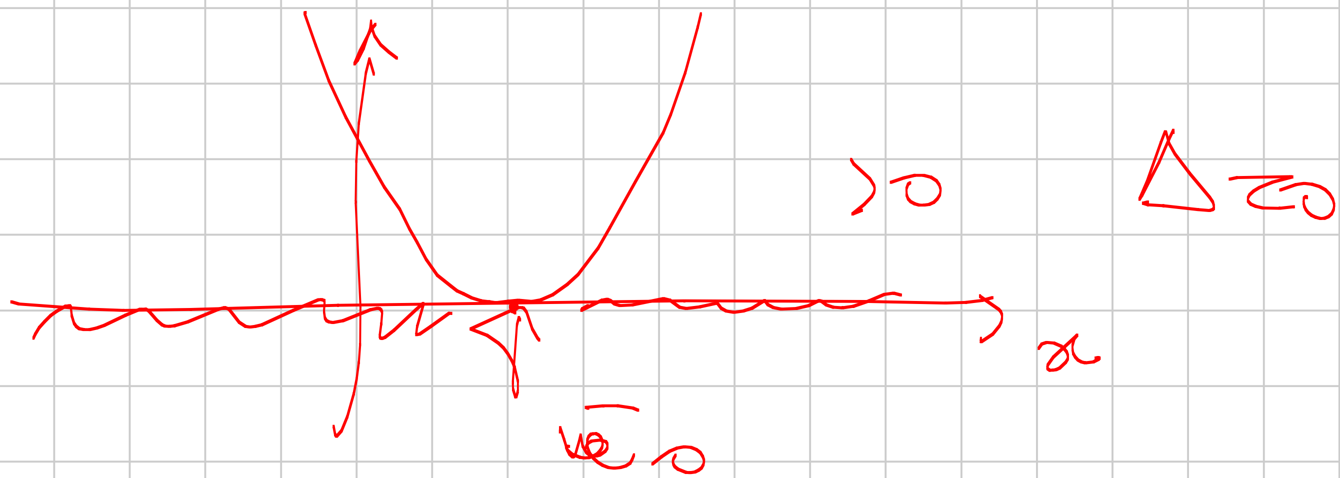
Tutti e 2 <0 $\quad x - x_1 < 0$ e $x - x_2 < 0$

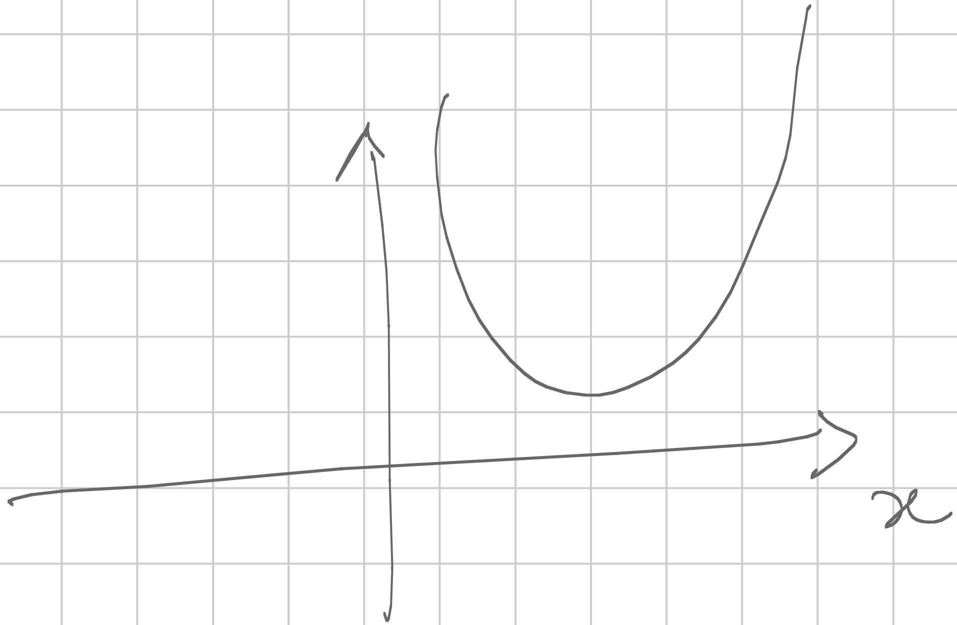
x più piccola della più piccola radice

ax^2+bx+c

$a>0$







$\Delta < 0$

$a < 0$

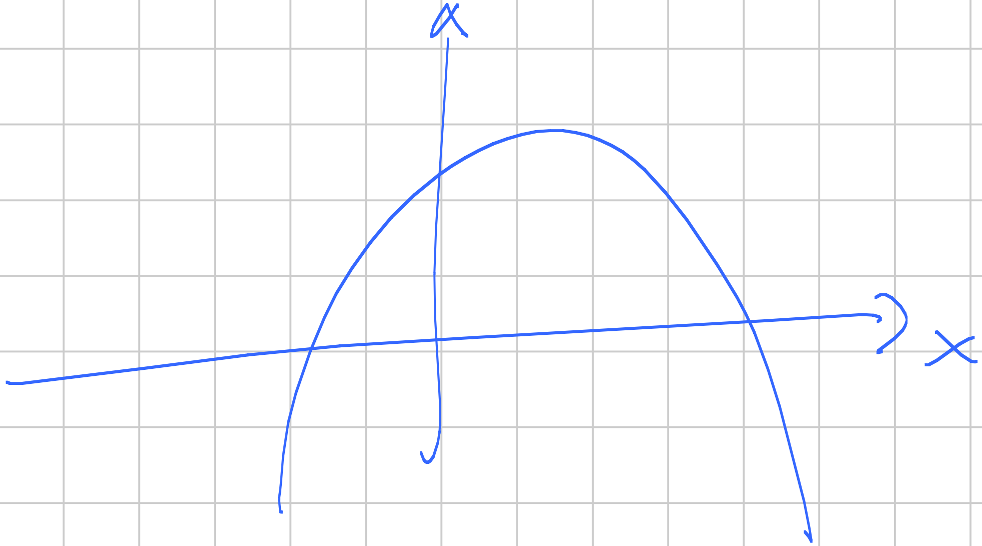
$ax^2 + bx + c > 0$

$-x^2 + x + 1 > 0$

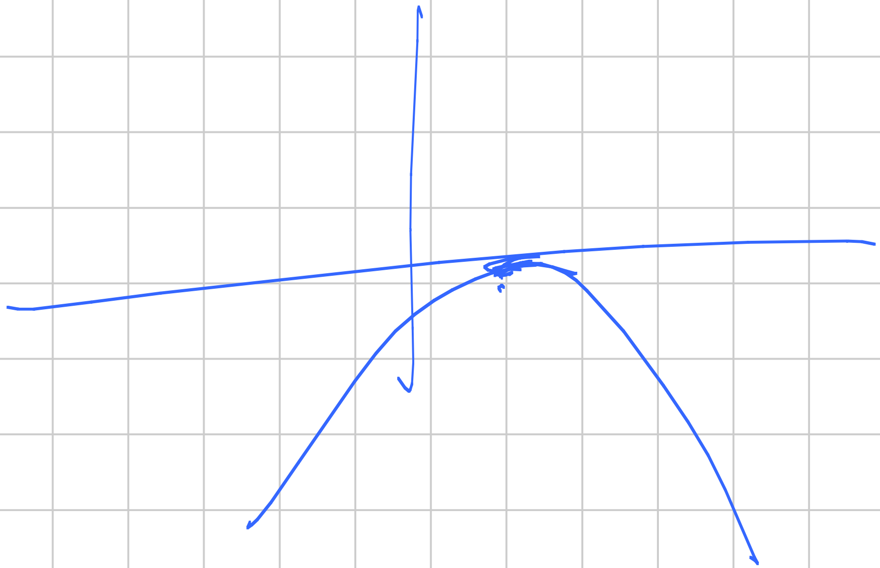
$\downarrow$

$x^2 - x - 1 < 0$

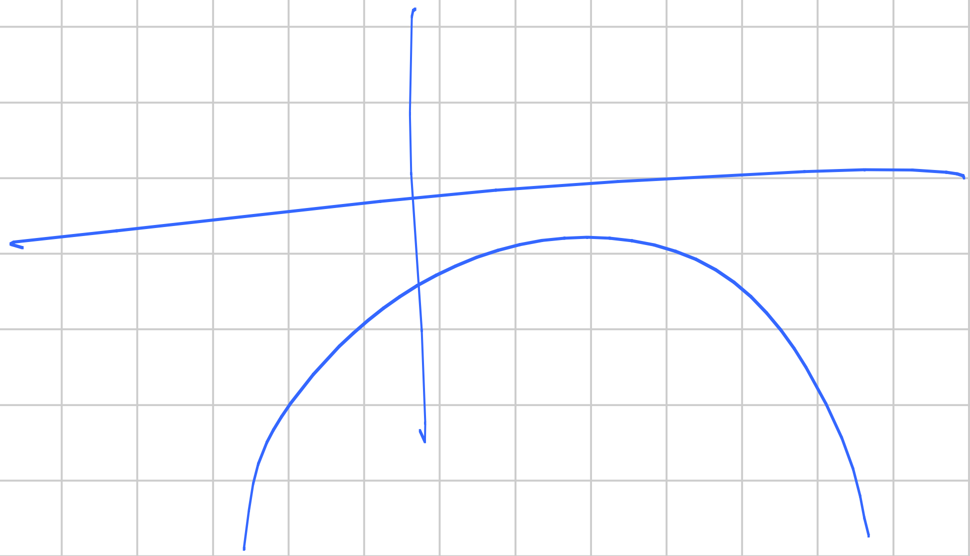
$a < 0$



$\Delta > 0$



$\Delta = 0$



$\Delta < 0$

Titolo nota 19/09/2007

$x^2 - 4x + 3 \leq 0$

$x^2 - 4x + 3 = 0$

$x_1 = 3 \qquad x_2 = 1$

$1 \leq x \leq 3$

$[1, 3]$

$x^2 - 4x + 3 > 0$

$x < 1 \ \cup \ x > 3$

$]-\infty, 1[\ \cup \ ]3, +\infty[$

$-x^2 + 2x + 3 \geq 0$ diventa $x^2 - 2x - 3 \leq 0$

$x_1 = 3$ $x_2 = -1$

$-1 \leq x \leq 3$ $[-1, 3]$

$-x^2 + 2x + 3 > 0$ diventa $x^2 - 2x - 3 < 0$

$-1 < x < 3$ $]-1, 3[$

$x^2 + 4x + 4 \geq 0$

$x^2 + 4x + 4 = 0$

solo per $x = -2$

$(x+2)^2 = x^2 + 4x + 4$

Sempre: è > 0 per $x \neq -2$ e $= 0$ per $x = -2$

$\mathbb{R} =]-\infty, +\infty[$

$x^2 + 4x + 4 > 0$ $\quad x \neq -2 \quad]-\infty, -2[\cup]-2, +\infty[$

$x^2 + 4x + 4 < 0$ MAI! $\emptyset$

$x^2 + 4x + 4 \leq 0$ può essere < 0 oppure

≥ 0 NON SUCCEDE

SOLUZIONE

$x = -2$ $\{-2\}$ $x = -2$

$x^2 - 4x + 5 \geq 0$ $\Delta = 16 - 20 < 0$

$x^2 - 4x + 5$ è sempre > 0 $\Rightarrow$

$x^2 - 4x + 5 \geq 0$ SEMPRE $\mathbb{R}$ $]-\infty, +\infty[$

$x^2 - 4x + 5 \leq 0$ $\emptyset$ MAI XCHÉ É SEMPRE > 0

$x^2 \geq 4$ ~~$x \geq \pm 2$~~ NO

$x \leq -2 \cup x \geq 2$

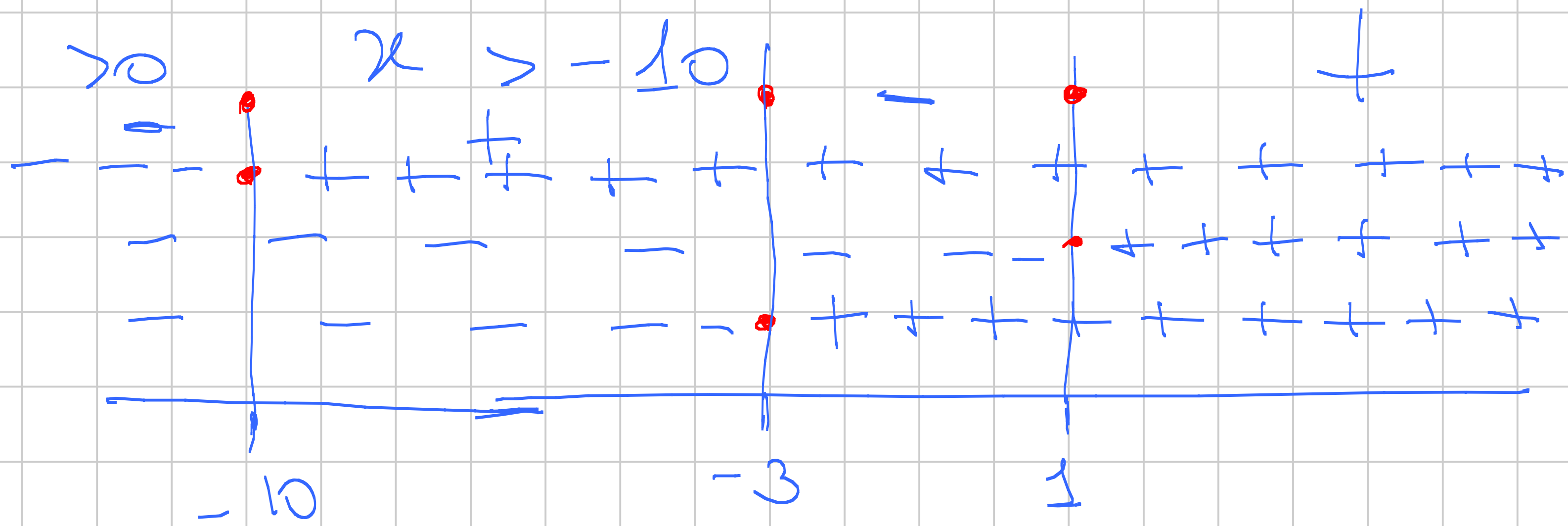
$]-\infty, -2] \cup [2, +\infty[$

$$(x+3)(x-1)(x+10) > 0$$

$x + 3 > 0 \qquad x > -3$

$x - 1 > 0 \qquad x > 1$

$x + 10 > 0 \qquad x > -10$

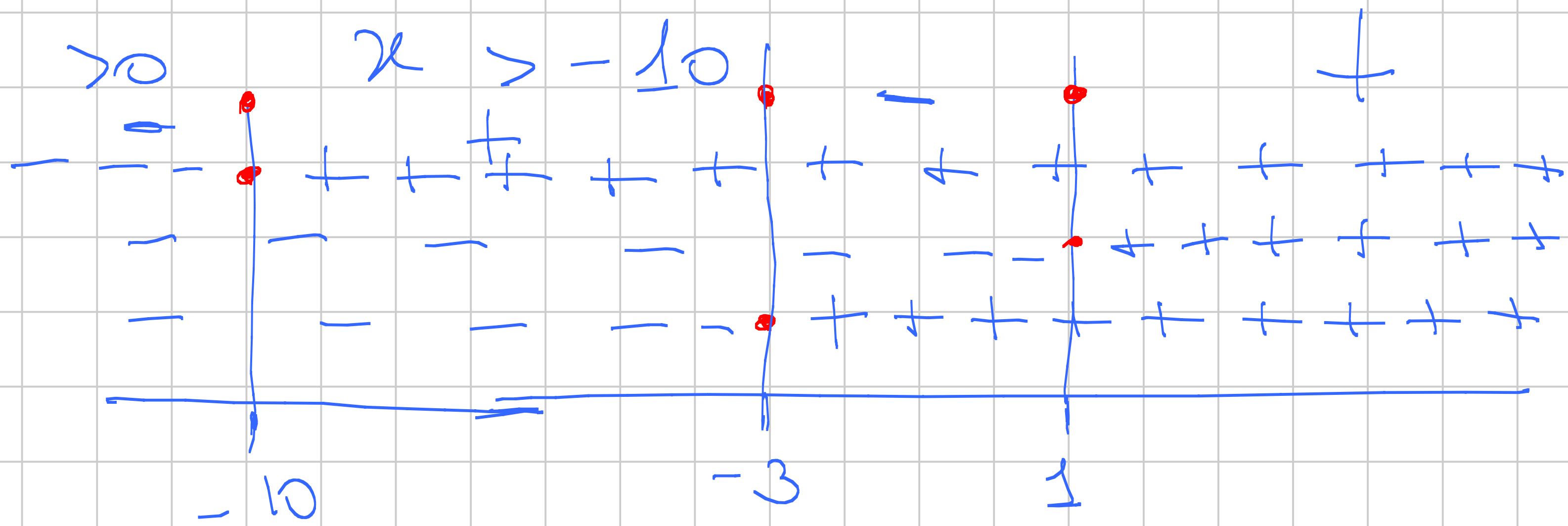


$$(x+3)(x-1)(x+10) > 0$$

$x + 3 > 0 \qquad x > -3$

$x - 1 > 0 \qquad x > 1$

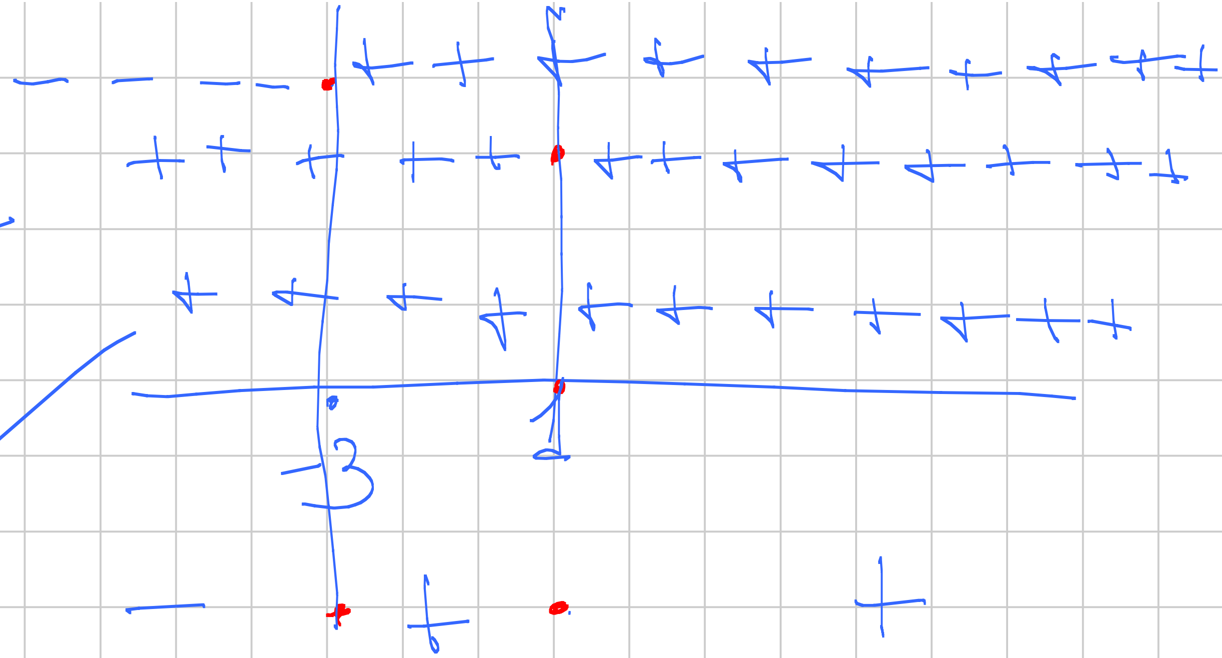
$x + 10 > 0 \qquad x > -10$



$x+3>0$

$(x-1)^2$

x^2+1



SOLUZIONE È $x<-3$

$$(x^2+1)(x-1)^2(x+3)\leq 0$$

SOLUZIONE: $x\leq -3 \vee x=1$

$$]-\infty, -3] \cup \{1\}$$

$$(x^2 - x)(x+1) \geq 0$$

$$x^2 - x \geq 0$$

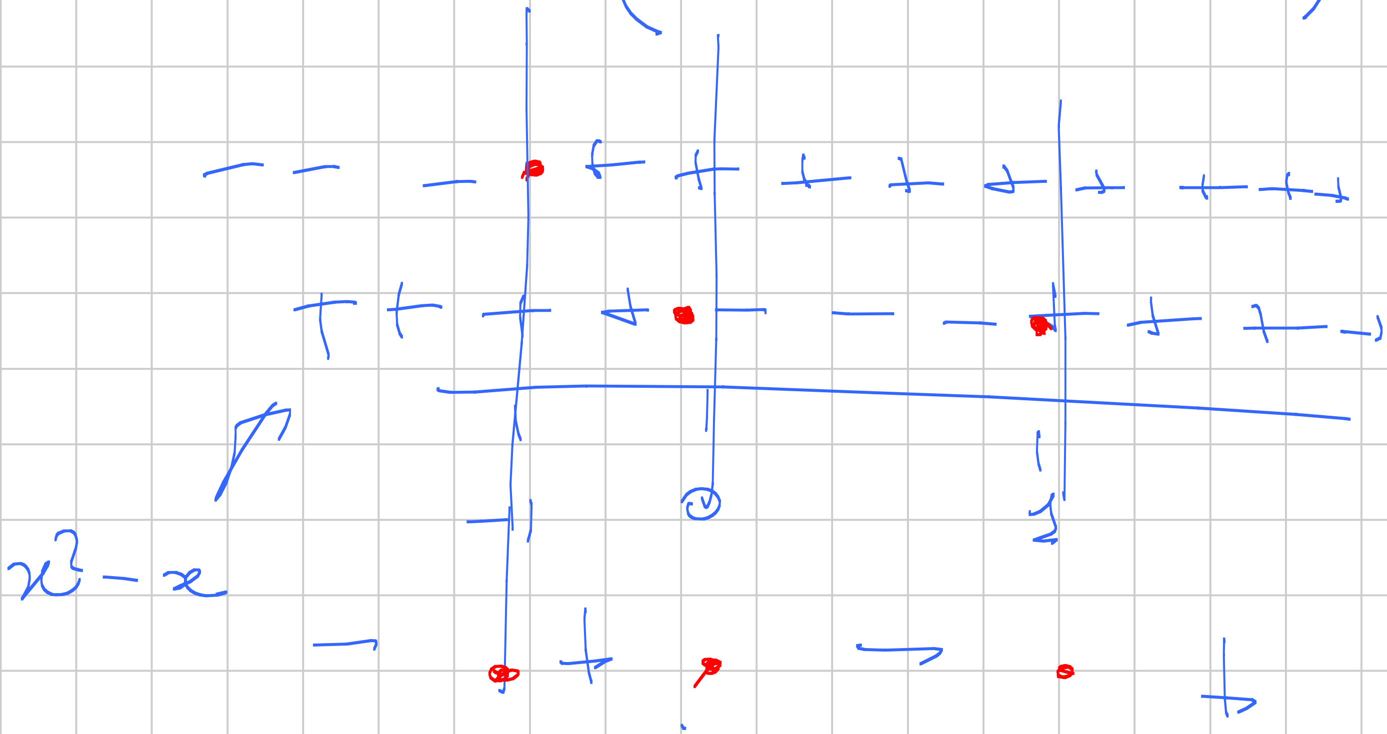
$$\parallel$$

$$x(x-1)$$ $x(x-1) = 0$ per $x = 0$

oppure $x = 1$

$x^2 - x > 0$ se $x < 0 \lor x > 1$ $\left[= 0 \text{ se } x = 0 \lor x = 1 \right]$

$x + 1 \geq 0 \qquad x \geq -1 \qquad (= 0 \text{ se } x = -1)$

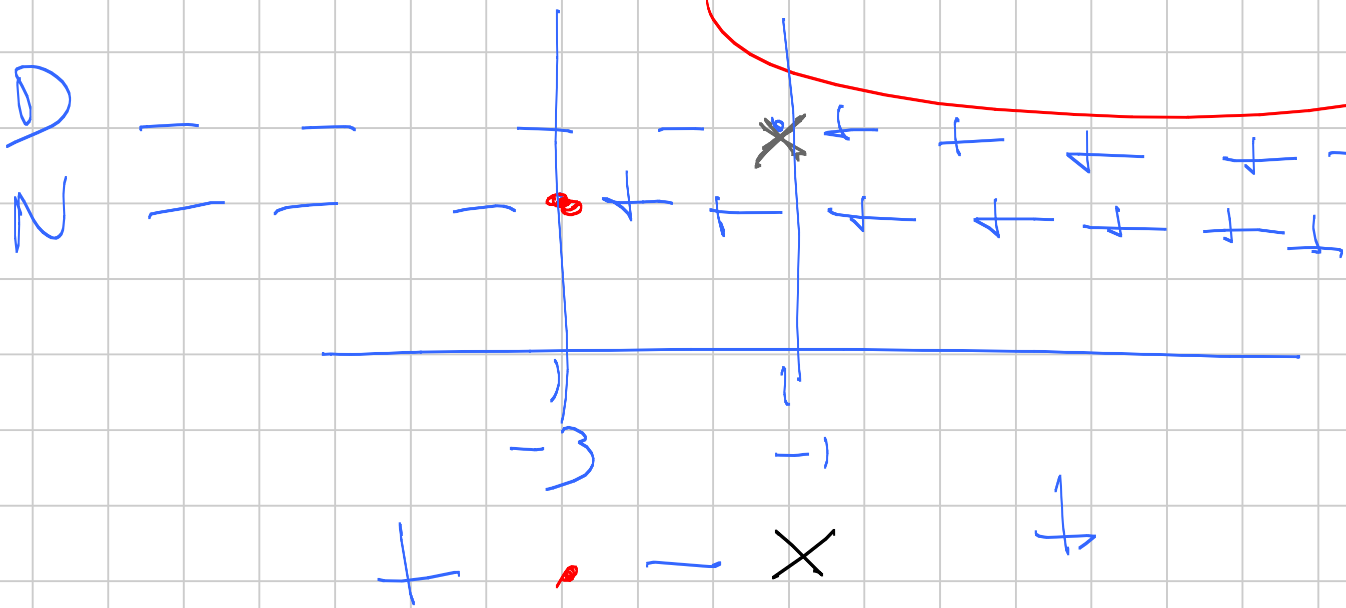


Soluz. $-1 \leq x \leq 0 \cup x \geq 1$

$$\frac{x+3}{x+1} \geq 0$$

N $\quad x+3 > 0 \qquad x > -3 \quad (=0 \text{ se } x=-3)$

D $\quad x+1 > 0 \qquad x > -1 \quad (=0 \text{ se } x=-1)$



SOLUZIONE $x \leq -3 \cup x > -1$

$$\frac{1}{x} > 1$$

$\cancel{1 > x}$ NO

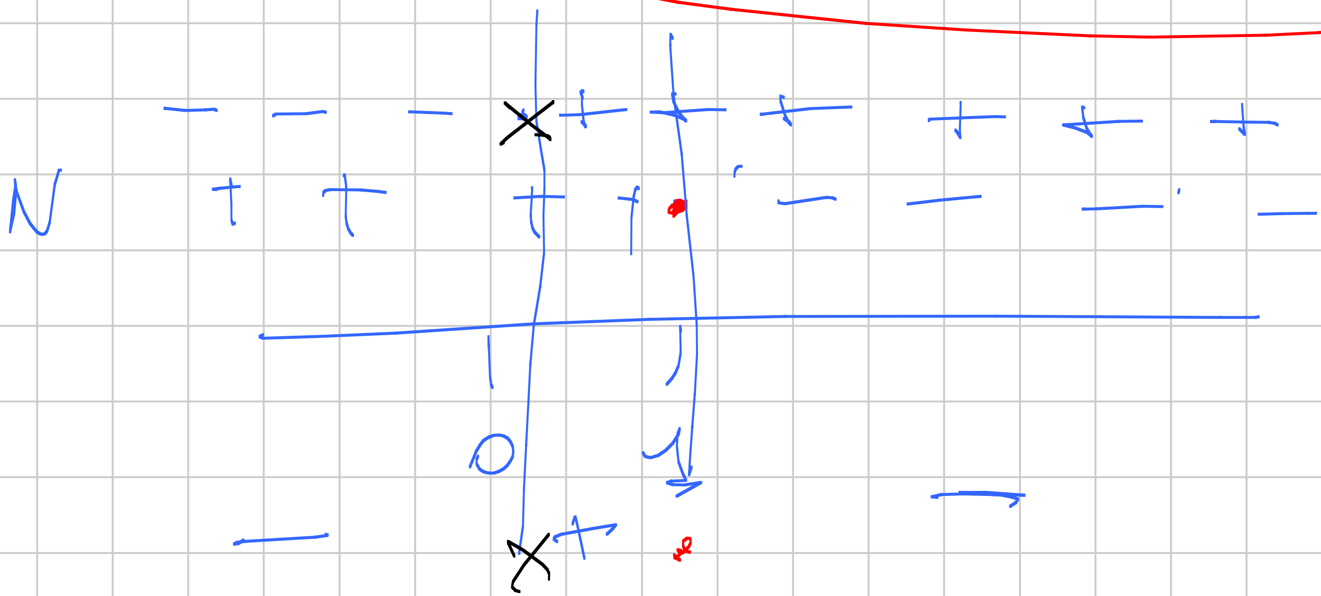
x può essere < 0

$$\frac{1}{x} - 1 > 0$$

$$\frac{1-x}{x} > 0$$

N $\quad 1 - x > 0 \qquad x < 1 \qquad (=0 \quad x \quad x = 1)$

D $\quad x > 0 \qquad (=0 \quad x \quad x = 0)$



SOLUZIONE

$$0 < x < 1$$

Titolo nota 19/09/2007

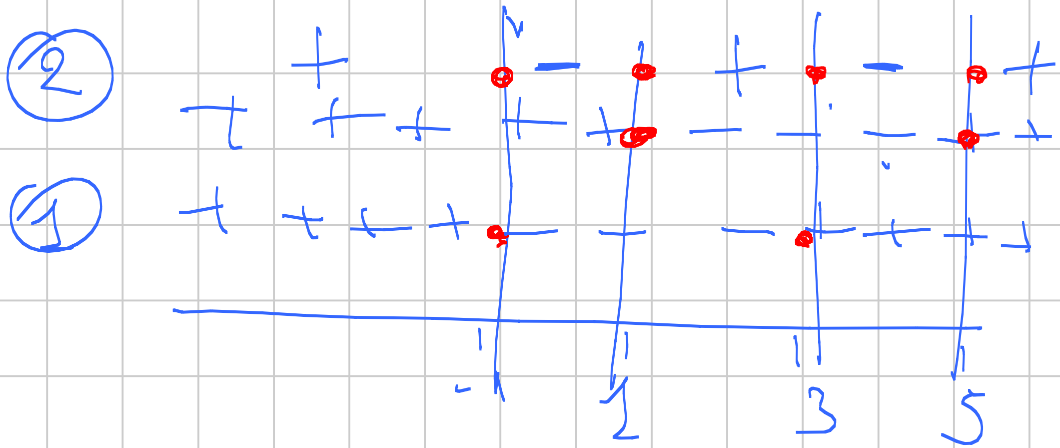
$$(x^2-2x-3)(x^2-6x+5)\geq 0$$

(1) $x^2-2x-3>0$ se $x<-1 \vee x>3$ ($=0$ se $x=-1$ oppure $x=3$)

Radici di x^2-2x-3 $x_1=3$ $x_2=-1$

(2) $x^2-6x+5>0$ se $x<1 \vee x>5$ ($=0$ se $x=1$ oppure $x=5$)

RADICI $x_1=5$ $x_2=1$



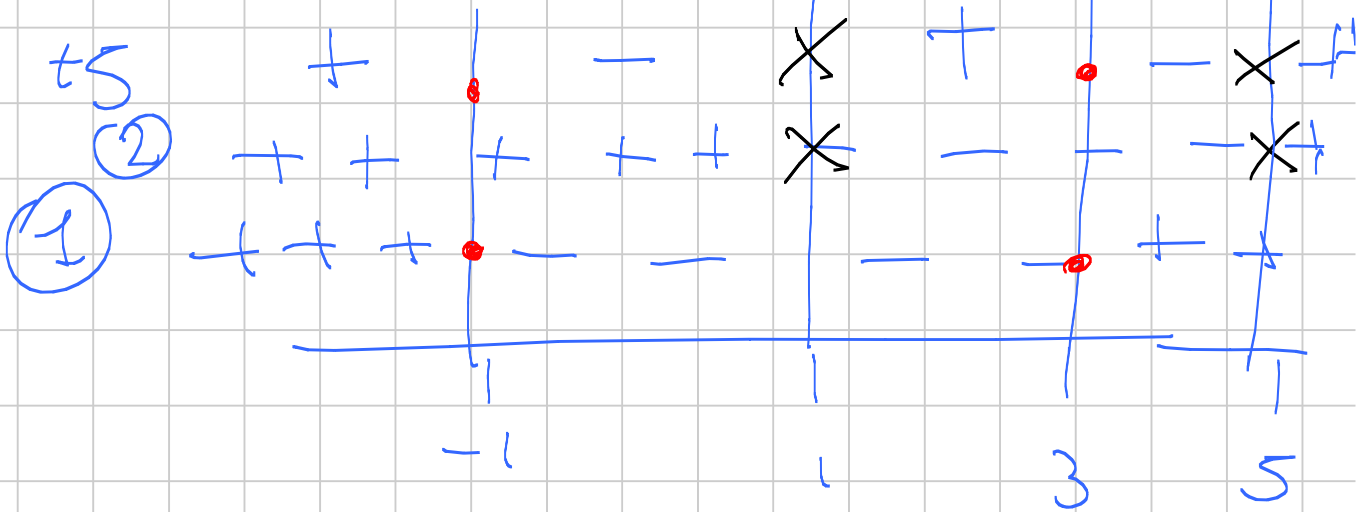
SOLUZIONE

$x \le -1 \cup 1 \le x \le 3 \cup x \ge 5$

$(x^2-2x-3)(x^2-6x+5)>0$

SOLUZIONE $x<-1 \vee 1<x<3 \vee x>5$

$$\frac{x^2-2x-3}{x^2-6x+5} \geq 0$$



SOLUZIONE

$x \leq -1 \quad \vee \quad 1 < x \leq 3 \quad \vee \quad x > 5$

$$(x^2-2x-3)^{2000}(x^2-6x+5)^{1999} \leq 0$$

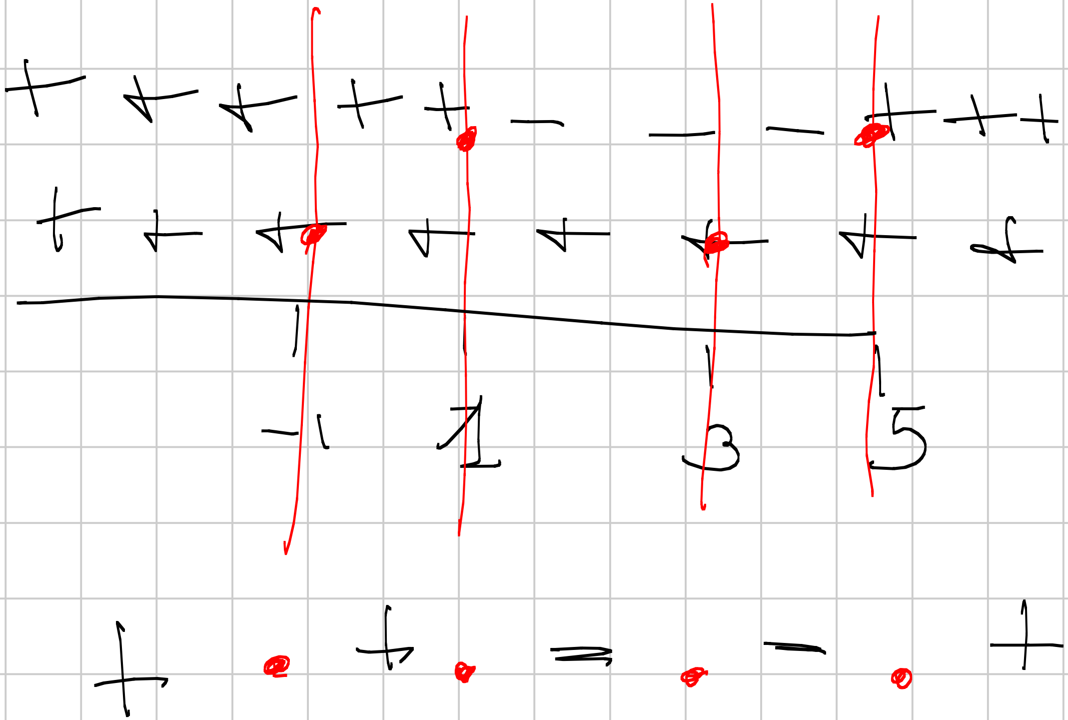
① $(x^2-2x-3)^{2000} \geq 0$ sempre $=0$ se

$x=3$ oppure $x=-1$

② $(x^2-6x+5)^{1999} \geq 0$ se e solo se ($\Longleftrightarrow$) $x^2-6x+5 \geq 0$

$x^2-6x+5>0$ se $x<1$ oppure $x>5$

(ed è $=0$ se $x=1$ oppure $x=5$)



SOLUZIONE:

$$\{-1\} \cup 1 \le x \le 5$$