

$$f(x,y) = x^2 + y^2 - x^2 y^2$$

$$\begin{cases} 2x - 2xy^2 = 0 \\ 2y - 2yx^2 = 0 \end{cases}$$

$$\begin{cases} x(1-y^2) = 0 \\ y(1-x^2) = 0 \end{cases}$$

$$\begin{aligned} x &= 0 \\ \downarrow \\ y &= 0 \end{aligned}$$

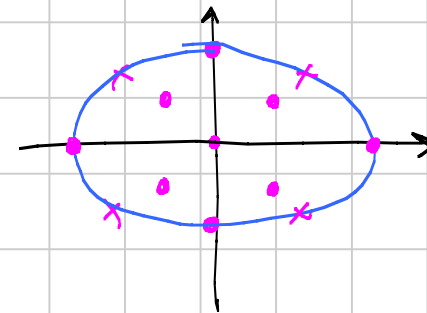
$$\begin{aligned} y &= \pm 1 \\ \downarrow \\ x &= \pm 1 \end{aligned}$$

$$\begin{aligned} (0,0) \\ (\pm 1, \pm 1) \end{aligned}$$

$$\begin{cases} 2x - 2xy^2 = \lambda x \\ 2y - 2yx^2 = \lambda y \\ x^2 + 2y^2 = 4 \end{cases}$$

$$\begin{aligned} x(1-y^2-\lambda) &= 0 \rightarrow x=0 \rightarrow y = \pm\sqrt{2} \\ &\searrow y^2 = 1-\lambda \end{aligned}$$

$$x^2 + 2y^2 \leq 4 \quad \text{INSIEME}$$



$$y(1-x^2-2\lambda)=0 \rightarrow y=0 \rightarrow x=\pm 2$$

$$\searrow x^2=1-2\lambda \rightsquigarrow x^2+2y^2=4$$

$$1-2\lambda+2(1-\lambda)=4$$

$$3-4\lambda=4 \Rightarrow 4\lambda=-1$$

$$\lambda = -\frac{1}{4}$$

se $\lambda \neq 1$

$$\cancel{\sqrt{1-\lambda}} (1-x^2-2\lambda)=0$$

$$x^2=1-2\left(-\frac{1}{4}\right)=1+\frac{1}{2}=\frac{3}{2} \quad y^2=\frac{5}{4}$$

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$$f(x,y) = x + x^2 + y^4 \quad \text{Unico pto stat.} \quad \left(-\frac{1}{2}, 0\right)$$

$$= x^2 + x + \frac{1}{4} - \frac{1}{4} + y^4 = \left(x + \frac{1}{2}\right)^2 + y^4 - \frac{1}{4}$$

Nel punto $\left(-\frac{1}{2}, 0\right)$ vale $-\frac{1}{4}$, altrove vale di $+$ $\Rightarrow \left(-\frac{1}{2}, 0\right)$ p.to min. globale.

$$f(x, y) = 4e^{xy} + x^2 + y^2$$

$$\begin{cases} 4ye^{xy} + 2x = 0 \\ 4xe^{xy} + 2y = 0 \end{cases} \quad \begin{cases} 2xye^{xy} + x^2 = 0 \\ 2yx e^{xy} + y^2 = 0 \end{cases}$$

Sottraggio --,

$$x^2 - y^2 = 0 \rightarrow x = \pm y$$

$$\text{Se } y = x \rightarrow 2xe^{x^2} + x = 0 \rightarrow x(2e^{x^2} + 1) = 0 \rightarrow \boxed{x=0, y=0}$$

$$\text{" } y = -x \rightarrow -2xe^{-x^2} + x = 0 \rightarrow x(-2e^{-x^2} + 1) = 0$$

$$\swarrow$$

$$x=0$$

$$\searrow$$

$$+2e^{-x^2} = +1$$

$$\downarrow$$

$$e^{-x^2} = \frac{1}{2}$$

$$\downarrow$$

$$-x^2 = \log \frac{1}{2} = -\log 2$$

2 POSS,

$$\boxed{\begin{aligned} x &= \pm \sqrt{\log 2} \\ y &= \mp \sqrt{\log 2} \end{aligned}}$$

$$\leftarrow x^2 = \log 2$$

$\leftarrow$