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$u''+2u'+2u=e^t \sin t$

EQ. IN $\mathbb{C}$

$$\begin{cases} e^{it} = \cos t + i \sin t & \sin t = Im(e^{it}) \\ \leadsto e^t \sin t = Im(e^t \cdot e^{it}) = Im(e^{(1+i)t}) \end{cases}$$

CONSIDERIAMO $z \in \mathbb{C}$ t.c. $u = Im(z)$

$$\leadsto \ddot{z} + 2\dot{z} + 2z = e^{(1+i)t}$$

$$p(s) = s^2 + 2s + 2 \qquad p(1+i) = (1+i)^2 + 2(1+i) + 2 =$$

$$= \cancel{1} - \cancel{1} + 2i + 2 + 2i + 2 = 4 + 4i$$

$$\leadsto z = \frac{e^{(1+i)t}}{p(1+i)} = \frac{e^{(1+i)t}}{4+4i} = \frac{\cancel{4} - \cancel{4}i}{\cancel{32}\ 8} e^{(1+i)t} =$$

$$= \frac{1-i}{8} e^{(1+i)t} \qquad \text{SOL. PART. IN } \mathbb{C}$$

$$u = Im(z) = \frac{e^t}{8} \sin t - \frac{e^t}{8} \cos t$$

SOLUZIONE PART. IN $\mathbb{R}$

VERIFICA

$$\dot{u} = \frac{e^t}{8} \sin t - \cancel{\frac{e^t}{8} \cos t} + \frac{e^t}{8} \sin t + \cancel{\frac{e^t}{8} \cos t}$$

$$\ddot{u} = \frac{e^t}{4} \sin t + \frac{e^t}{4} \cos t$$

$$\leadsto \ddot{u} + 2\dot{u} + 2u = \frac{e^t}{4} \sin t + \cancel{\frac{e^t}{4} \cos t} + \frac{2}{4} e^t \sin t +$$

$$+ \frac{e^t}{4} \sin t - \cancel{\frac{e^t}{4} \cos t} = e^t \sin t$$