7. Consideriamo l'equazione

BY GIMUSI 26/11/16

$$x^5 - x\log(1+x^2) + \sin(x^4) = n.$$

(c) (Bonus question) Studiare, al variare ~~del~~ dei ~~parametro~~ parametri ~~reale~~ reali ~~positivo~~ positivi a, b la convergenza della serie

$$\sum_{n=n_0}^{\infty} (x_n - n^a)^{\cancel{2}\, b}.$$

(c) $\lim\limits_{x\to+\infty} f(x) = +\infty \Rightarrow n \to +\infty \quad x_n \to +\infty$

$$n^a = \left(x_n^5 - x_n\log(1+x_n^2) + \sin(x_n^4)\right)^a =$$

$$= x_n^{5a}\left(1 - \frac{1}{x_n^4}\left(\log x_n^2 + \log\left(1+\frac{1}{x_n^2}\right)\right) + \frac{\sin x_n^4}{x_n^5}\right)^a =$$

$$= x_n^{5a}\left(1 - \frac{1}{x_n^4}\left(2\log x_n + \frac{1}{x_n^2} + o\left(\frac{1}{x_n^2}\right)\right) + \frac{\sin x_n^4}{x_n^5}\right)^a =$$

$$= x_n^{5a}\left(1 - \frac{2\log x_n}{x_n^4} + \frac{\sin x_n^4}{x_n^5} + \frac{1}{x_n^6} + o\left(\frac{1}{x_n^6}\right)\right)^a =$$

$$\frac{\sin x_n^4}{x_n^5} = \frac{\log x_n}{x_n^4} \cdot \frac{\sin x_n^4}{x_n \log x_n} \to 0 \qquad \frac{\log x_n}{x_n^4} \to 0$$

$$= x_n^{5a}\left(1 - \frac{2\log x_n}{x_n^4} + o\left(\frac{\log x_n}{x_n^4}\right)\right)^a =$$

$$= x_n^{5a}\left(1 - \frac{2a\log x_n}{x_n^4} + o\left(\frac{\log x_n}{x_n^4}\right)\right) =$$

$$= x_n^{5a} - \frac{2a\log x_n}{x_n^{4-5a}} + o\left(\frac{\log x_n}{x_n^{4-5a}}\right)$$

$$x_n - n^a = x_n - x_n^{5a} + \frac{2a\log x_n}{x_n^{4-5a}} + o\left(\frac{\log x_n}{x_n^{4-5a}}\right) \qquad n \to +\infty$$

SIA $y_n = x_n - x_n^{5a} + \frac{2a\log x_n}{x_n^{4-5a}}$

PER $a \leq 1/5$ $\quad x_n - n^a > 0 \quad \wedge \quad y_n > 0 \quad (\to +\infty)$

$$\Rightarrow \frac{(x_n - n^a)^b}{y_n^b} \to 1 \qquad (x_n - n^a)^b \geq 0 \qquad y_n^b > 0$$

PER CONFRONTO ASINTOTICO $\displaystyle\sum_{n=n_0}^{+\infty} (x_n - n^a)^2$ E $\displaystyle\sum_{n=n_0}^{+\infty} y_n^2$

HANNO LO STESSO COMPORTAMENTO

$a = 1/5$

$$\sum_{n=n_0}^{+\infty} y_n^b = \sum_{n=n_0}^{+\infty} \left(\frac{2a \log x_n}{x_n^3}\right)^b = \sum_{n=n_0}^{+\infty} \frac{(2a)^b \log^b x_n}{x_n^{3b}}$$

CONVERGE PER $3b > 1$ PER C.A. CON $\displaystyle\sum_{n=n_0}^{+\infty} \frac{1}{x_n^\beta}$ $\quad \beta = \frac{1+3b}{2}$

$a < 1/5$

$$\lim_{n\to+\infty} y_n^b = \lim_{n\to+\infty} \left(x_n - x_n^{5a} + \frac{2a \log x_n}{x_n^{5-5a}}\right)^b = +\infty$$

$$\lim_{n\to+\infty} \left(x_n - x_n^{5a}\right) = +\infty \qquad \lim_{n\to+\infty} \frac{2a \log x_n}{x_n^{5-5a}} = 0$$

PER $a > 1/5$ $\quad x_n - n^a < 0 \quad \wedge \quad y_n < 0 \quad (\to -\infty)$

SE $(x_n - n^a)^b \wedge y_n^b$ SONO DEFINITE, HANNO LO STESSO SEGNO E SI APPLICA ANCORA IL CRITERIO DEL C.A.

$$\lim_{n\to+\infty} y_n^b = \lim_{n\to+\infty} x_n^{ab} \left(x_n^{1-5a} - 1 + \frac{2a \log x_n}{x_n^5}\right)^b = +\infty \cdot (-1)^b$$

$$\rightsquigarrow \sum_{n=n_0}^{+\infty} (x_n - n^a)^b \begin{cases} = +\infty & \text{PER } a < 1/5 \quad \forall b > 0 \\ < +\infty & \text{PER } a = 1/5 \quad b > 1/3 \\ = +\infty \cdot (-1)^b & \text{PER } a > 1/5 \quad \forall b \text{ S.C. } (-1)^b \text{ È DEFINITO} \end{cases}$$