BY GIMUSI 09/12/15

1) lim 1/x - [1/x]
x-> 0

$$\lim_{x\to 0} \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor \quad N.E.$$

$$a_n = \frac{1}{n} \to 0 \qquad b_n = \frac{2}{2n+1} \to 0 \qquad n \to \infty$$

$$\begin{cases} \lim_{n\to\infty} f(a_n) = n - \lfloor n \rfloor = 0 & \forall n \in \mathbb{N} \\ \lim_{n\to\infty} f(b_n) = n + \frac{1}{2} - \left\lfloor n + \frac{1}{2} \right\rfloor = \frac{1}{2} & \forall n \in \mathbb{N} \end{cases}$$

2) lim 1/x^2 - [1/x]
x->0

$$\lim_{x\to 0} \frac{1}{x^2} - \left\lfloor \frac{1}{x} \right\rfloor = 0$$

$$\frac{1}{x^2} - \left\lfloor \frac{1}{x} \right\rfloor = \underbrace{\frac{1}{x^2}}_{\to +\infty} \left(1 - \underbrace{x^2 \left\lfloor \frac{1}{x} \right\rfloor}_{\to 0} \right) = +\infty$$

$$\left\lfloor \frac{1}{x} \right\rfloor \le \frac{1}{x}$$

$$\leadsto x^2 \left\lfloor \frac{1}{x} \right\rfloor \le \frac{x^2}{x} \to 0$$

3) lim 1/x^2 [1/x - [1/x]]
x->0

$$\lim_{x\to 0} \frac{1}{x^2} \left\lfloor \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor \right\rfloor = 0$$

$$0 \le \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor < 1 \Rightarrow \left\lfloor \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor \right\rfloor = 0$$