BY GIMUSI 30/01/13

$$\sum_{n=1}^{\infty}\left(\sqrt{1+\frac{(-1)^n}{\sqrt{n}}}-1\right)=$$

$$=\sum_{K=1}^{\infty}\left(\sqrt{1+\frac{(-1)^{2K}}{\sqrt{2K}}}+\sqrt{1+\frac{(-1)^{2K-1}}{\sqrt{2K-1}}}-2\right)=$$

$$=\sum_{K=1}^{\infty}\left(\sqrt{1+\frac{1}{\sqrt{2K}}}+\sqrt{1-\frac{1}{\sqrt{2K-1}}}-2\right)\le$$

$$\overset{(1)}{\le}\sum_{K=1}^{\infty}\left(\sqrt{1+\frac{1}{\sqrt{2K}}}+\sqrt{1-\frac{1}{\sqrt{2K}}}-2\right)$$

(1) $\sqrt{1-\frac{1}{\sqrt{2K}}}\overset{?}{\ge}\sqrt{1-\frac{1}{\sqrt{2K-1}}} \Rightarrow -\frac{1}{\sqrt{2K}}\overset{?}{\ge}-\frac{1}{\sqrt{2K-1}}$

$\Rightarrow \frac{1}{\sqrt{2K}}\overset{?}{\le}\frac{1}{\sqrt{2K-1}} \Rightarrow \frac{\sqrt{2K-1}}{\sqrt{2K}}\le 1$ OK $\forall K\ge 1$

(2) $\sqrt{1+\frac{1}{\sqrt{2K}}}+\sqrt{1-\frac{1}{\sqrt{2K}}}-2\le 0 \qquad 0<A=\frac{1}{\sqrt{2K}}\le 1$

$\Rightarrow \sqrt{1+A}+\sqrt{1-A}\overset{?}{\le}2 \Rightarrow 1+\cancel{A}+1-\cancel{A}+2\sqrt{1-A^2}\overset{?}{\le}4$

$\Rightarrow 2\sqrt{1-A^2}\overset{?}{\le}2 \Rightarrow 1-A^2\overset{?}{\le}1 \Rightarrow -A^2\le 0$ OK $\forall K\ge 1$

$$\sum_{K=1}^{\infty}\left(\sqrt{1+\frac{1}{\sqrt{2K}}}+\sqrt{1-\frac{1}{\sqrt{2K}}}-2\right)$$

È A TERMINI NEGATIVI $a_K\le 0$ $\forall K\ge 1$

AMICO BRUTALE

$$\sqrt{1+\frac{1}{\sqrt{2K}}}+\sqrt{1-\frac{1}{\sqrt{2K}}}-2\sim\cancel{1}+\cancel{\frac{1}{2}\frac{1}{\sqrt{2K}}}+\frac{1}{2}\cdot\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2K}+$$

$$+\cancel{1}-\cancel{\frac{1}{2}\frac{1}{\sqrt{2K}}}+\frac{1}{2}\cdot\frac{1}{2}\left(-\frac{1}{2}\right)\cdot\frac{1}{2K}-\cancel{2}+o\left(\frac{1}{K}\right)=-\frac{1}{8K}+o\left(\frac{1}{K}\right)$$

CONFRONTO ASINTOTICO CON $b_n = \sum_{n=1}^{+\infty} -\frac{1}{K}$

$$\frac{\sqrt{1+\frac{1}{\sqrt{2K}}} + \sqrt{1-\frac{1}{\sqrt{2K}}} - 2}{-1/K} = \frac{-\frac{1}{8K} + o\left(\frac{1}{K}\right)}{-1/K} \to 1/8$$

$$\rightsquigarrow \sum_{K=1}^{\infty}\left(\sqrt{1+\frac{1}{\sqrt{2K}}} + \sqrt{1-\frac{1}{\sqrt{2K}}} - 2\right) \sim \sum_{K=1}^{\infty} -\frac{1}{K} = -\infty$$

$$\rightsquigarrow \sum_{m=1}^{\infty}\left(\sqrt{1+\frac{(-1)^m}{\sqrt{m}}} - 1\right)$$ DIVERGE A $-\infty$