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Limiti 5

Argomenti: limiti di funzioni **Difficoltà**: ⋆⋆

Prerequisiti: limiti notevoli, confronto di ordini di infinito e di infinitesimo

In ogni riga è assegnata una funzione, di cui si chiede di calcolare il limite per x tendente a ciascuno dei valori indicati (se la richiesta non ha senso, accorgersene e segnalarlo).

	Funzione	$x \to$	a) Limite	$x \to$	b) Limite	$x \to$	c) Limite	$x \to$	d) Limite
1)	$\frac{x^3+x}{x^3-x}$	$-\infty$	1	0	-1	1^+	$+\infty$	$+\infty$	1
2)	$\frac{x^3+2\sqrt{x}}{3x^2-\sqrt{x}}$	$-\infty$	\	0^+	-2	1	3/2	$+\infty$	$+\infty$
3)	x^3-3^x	$-\infty$	$-\infty$	0^+	-1	1	-2	$+\infty$	$-\infty$
5)	$x+\log(x^{10})$	$-\infty$	$-\infty$	0^-	$-\infty$	1	1	$+\infty$	$+\infty$
5)	$\frac{x^2+2^x}{3x^2+3^x}$	$-\infty$	1/3	0^-	1	0^+	1	$+\infty$	0
6)	$\frac{x^2+1}{x-1}$	$-\infty$	$-\infty$	0	-1	1^-	$-\infty$	$+\infty$	$+\infty$
7)	$\frac{\log x}{x}$	0^-	\	0^+	$-\infty$	1	0	$+\infty$	0
8)	$\frac{x^2-2\log x}{3x^2+\log x}$	$-\infty$	\	0^+	-2	1	1/3	$+\infty$	1/3
9)	$\frac{x^2+\sin x}{\arctan x}$	$-\infty$	$-\infty$	0^-	1	0^+	1	$+\infty$	$+\infty$
10)	$\frac{x+2\sin x}{x+3\arctan x}$	$-\infty$	1	0^-	3/5	0^+	3/5	$+\infty$	1
11)	$\frac{2^x+3^{x^2}}{x^3}$	$-\infty$	$-\infty$	0^-	$-\infty$	0^+	$+\infty$	$+\infty$	$+\infty$
12)	$\frac{x^2+\cos x}{x-\pi}$	$-\infty$	$-\infty$	0	$-1/\pi$	π^-	$-\infty$	$+\infty$	$+\infty$
13)	$\frac{2\sqrt[3]{x}+\sin x}{\sqrt[3]{x}-\arctan x}$	$-\infty$	2	0^-	2	0^+	2	$+\infty$	2
14)	$\frac{x+\sin x}{\log x}$	$-\infty$	\	0^+	0	1	\	$+\infty$	$+\infty$

 Test di allenamento n. 28

1.a) $\lim_{x\to-\infty} \frac{x^3+x}{x^3-x} = 1$ $\quad \frac{x^3+x}{x^3-x} = \frac{x}{x}\,\frac{x^2+1}{x^2-1} \to 1$ ($\frac{x}{x}\to 1$, $\frac{x^2+1}{x^2-1}\to 1$)

1.b) $\lim_{x\to 0} \frac{x^3+x}{x^3-x} = -1$ $\quad \frac{x^3+x}{x^3-x} = \frac{x}{x}\,\frac{x^2+1}{x^2-1} \to -1$ ($\frac{x}{x}\to 1$, $\frac{x^2+1}{x^2-1}\to -1$)

1.c) $\lim_{x\to 1^+} \frac{x^3+x}{x^3-x} = +\infty$ $\quad \frac{x^3+x}{x^3-x} \to \frac{2}{0^+} \to +\infty$

1.d) $\lim_{x\to+\infty} \frac{x^3+x}{x^3-x} = 1$ $\quad \frac{x^3+x}{x^3-x} = \frac{x}{x}\,\frac{x^2+1}{x^2-1} \to 1$ ($\frac{x}{x}\to 1$, $\frac{x^2+1}{x^2-1}\to 1$)

2.a) $\lim_{x\to-\infty} \frac{x^3+2\sqrt{x}}{3x^2-\sqrt{x}} =$ \

2.b) $\lim_{x\to 0^+} \frac{x^3+2\sqrt{x}}{3x^2-\sqrt{x}} = -2$ $\quad \frac{x^3+2\sqrt{x}}{3x^2-\sqrt{x}} = \frac{\sqrt{x}}{\sqrt{x}}\,\frac{x^{5/2}+2}{3x^{3/2}-1} \to -2$

2.c) $\lim_{x\to 1} \frac{x^3+2\sqrt{x}}{3x^2-\sqrt{x}} = \frac{3}{2}$

2.d) $\lim_{x\to+\infty} \frac{x^3+2\sqrt{x}}{3x^2-\sqrt{x}} = +\infty$ $\quad \frac{x^3+2\sqrt{x}}{3x^2-\sqrt{x}} = \frac{x^2}{x^2}\,\frac{x+\frac{2\sqrt{x}}{x^2}}{3-\frac{\sqrt{x}}{x^2}} \to +\infty$ ($\frac{x^2}{x^2}\to 1$, $x+\frac{2\sqrt{x}}{x^2}\to+\infty$, $3-\frac{\sqrt{x}}{x^2}\to 3$)

3.a) $\lim_{x\to-\infty} x^3-3^x = -\infty$ $\quad x^3-3^x \to -\infty - 0$

3.b) $\lim_{x\to 0^+} x^3-3^x = -1$ $\quad x^3-3^x \to 0-1$

3.c) $\lim_{x\to 1} x^3-3^x = -2$ $\quad x^3-3^x \to 1-3$

3.d) $\lim_{x\to+\infty} x^3-3^x = -\infty$ $\quad x^3-3^x = 3^x\left(\frac{x^3}{3^x}-1\right) \to -\infty$ ($3^x\to+\infty$, $\frac{x^3}{3^x}-1\to -1$)

4.a) $\lim_{x\to-\infty} x+\log(x^{10}) = -\infty$ $\quad x+\log(x^{10}) = x\left(1+\frac{10\log|x|}{x}\right) \to -\infty$ ($x\to-\infty$, $\frac{10\log|x|}{x}\to 1$)

4.b) $\lim_{x\to 0^-} x+\log(x^{10}) = -\infty$ $\quad x+\log(x^{10}) \to 0+(-\infty)$

4.c) $\lim_{x\to 1} x+\log(x^{10}) = 1$ $\quad x+\log(x^{10}) \to 1+0$

4.d) $\lim_{x\to+\infty} x+\log(x^{10}) = +\infty$ $\quad x+\log(x^{10}) = x\left(1+\frac{10\log|x|}{x}\right) \to +\infty$ ($x\to+\infty$, $\frac{10\log|x|}{x}\to 1$)

5.a) $\lim_{x\to-\infty} \frac{x^2+2^x}{3x^2+3^x} = 1/3$ $\quad \frac{x^2+2^x}{3x^2+3^x} = \frac{x^2}{x^2}\,\frac{1+\frac{2^x}{x^2}}{3+\frac{3^x}{x^2}} \to 1/3$ ($\frac{x^2}{x^2}\to 1$, $\frac{2^x}{x^2}\to 0$, $\frac{3^x}{x^2}\to+\infty$)

5.b) $\lim_{x\to 0^-} \frac{x^2+2^x}{3x^2+3^x} = 1$

5.c) $\lim_{x\to 0^+} \frac{x^2+2^x}{3x^2+3^x} = 1$

5.d) $\lim_{x\to+\infty} \frac{x^2+2^x}{3x^2+3^x} = 0$ $\quad \frac{x^2+2^x}{3x^2+3^x} = \frac{2^x}{3^x}\,\frac{\frac{x^2}{2^x}+1}{\frac{3x^2}{3^x}+1} \to 0$ ($\frac{2^x}{3^x}\to 0$, $\frac{x^2}{2^x}\to 1$, $\frac{3x^2}{3^x}\to 1$)

6.a) $\lim_{x\to-\infty} \frac{x^2+1}{x-1} = -\infty$ $\quad \frac{x^2+1}{x-1} = \frac{x^2}{x}\,\frac{1+\frac{1}{x^2}}{1-\frac{1}{x}} \to -\infty$ ($\frac{x^2}{x}\to-\infty$, $\frac{1+\frac{1}{x^2}}{1-\frac{1}{x}}\to 1$)

6.b) $\lim_{x\to 0} \frac{x^2+1}{x-1} = -1$

6.c) $\lim_{x\to 1^-} \frac{x^2+1}{x-1} = -\infty$ $\quad \frac{x^2+1}{x-1} \to \frac{2}{0^-} \to -\infty$

6.d) $\lim_{x\to+\infty} \frac{x^2+1}{x-1} = +\infty$ $\quad \frac{x^2+1}{x-1} = \frac{x^2}{x}\,\frac{1+\frac{1}{x^2}}{1-\frac{1}{x}} \to +\infty$ ($\frac{x^2}{x}\to+\infty$, $\frac{1+\frac{1}{x^2}}{1-\frac{1}{x}}\to 1$)

7.a) $\lim_{x\to 0^-} \frac{\log x}{x} = \setminus$

7.b) $\lim_{x\to 0^+} \frac{\log x}{x} = -\infty$ $\qquad \frac{\log x}{x} \to \frac{-\infty}{0^+} \to -\infty$

7.c) $\lim_{x\to 1} \frac{\log x}{x} = 0$

7.d) $\lim_{x\to +\infty} \frac{\log x}{x} = 0$ $\qquad \begin{cases} \log x = y \\ x = e^y \end{cases}$ $\lim_{x\to+\infty} \frac{\log x}{x} = \lim_{y\to+\infty} \frac{y}{e^y} = 0$

8.a) $\lim_{x\to -\infty} \frac{x^2 - 2\log x}{3x^2 + \log x} = \setminus$

8.b) $\lim_{x\to 0^+} \frac{x^2 - 2\log x}{3x^2 + \log x} = -2$ $\qquad \frac{x^2 - 2\log x}{3x^2 + \log x} = \frac{\log x}{\log x} \frac{\frac{x^2}{\log x} - 2}{\frac{3x^2}{\log x} + 1} \to -2$

8.c) $\lim_{x\to 1} \frac{x^2 - 2\log x}{3x^2 + \log x} = \frac{1}{3}$

8.d) $\lim_{x\to +\infty} \frac{x^2 - 2\log x}{3x^2 + \log x} = \frac{1}{3}$ $\qquad \frac{x^2 - 2\log x}{3x^2 + \log x} = \frac{x^2}{x^2} \frac{1 - \frac{2\log x}{x^2}}{3 + \frac{\log x}{x^2}} \to \frac{1}{3}$

9.a) $\lim_{x\to -\infty} \frac{x^2 + \sin x}{\arctan x} = -\infty$ $\qquad \frac{x^2 + \sin x}{\arctan x} = x^2 \frac{1 + \frac{\sin x}{x^2}}{\arctan x} \to -\infty$ ($1 + \frac{\sin x}{x^2} \to -2/\pi$)

9.b) $\lim_{x\to 0^-} \frac{x^2 + \sin x}{\arctan x} = 1$ $\qquad \frac{x^2 + \sin x}{\arctan x} = \frac{x}{x} \frac{x + \frac{\sin x}{x}}{\frac{\arctan x}{x}} \to 1$ ($\frac{x}{x} \to 1$, $x + \frac{\sin x}{x} \to 1$)

9.c) $\lim_{x\to 0^+} \frac{x^2 + \sin x}{\arctan x} = 1$ $\qquad \frac{x^2 + \sin x}{\arctan x} = \frac{x}{x} \frac{x + \frac{\sin x}{x}}{\frac{\arctan x}{x}} \to 1$ ($\frac{x}{x} \to 1$, $x + \frac{\sin x}{x} \to 1$)

9.d) $\lim_{x\to +\infty} \frac{x^2 + \sin x}{\arctan x} = +\infty$ $\qquad \frac{x^2 + \sin x}{\arctan x} = x^2 \frac{1 + \frac{\sin x}{x^2}}{\arctan x} \to +\infty$ ($1 + \frac{\sin x}{x^2} \to 2/\pi$)

10.a) $\lim_{x\to-\infty} \frac{x+2\sin x}{x+3\arctan x} = 1$

$\frac{x+2\sin x}{x+3\arctan x} = \frac{x}{x}\cdot\frac{1+\frac{2\sin x}{x}}{1+3\frac{\arctan x}{x}} \to 1$ ($\frac{x}{x}\to 1$, $\frac{2\sin x}{x}\to 1$)

10.b) $\lim_{x\to 0^-} \frac{x+2\sin x}{x+3\arctan x} = \frac{3}{5}$

$\frac{x+2\sin x}{x+3\arctan x} = \frac{x}{x}\cdot\frac{1+\frac{2\sin x}{x}}{1+3\frac{\arctan x}{x}} \to 3/5$ ($\frac{x}{x}\to 1$, $\frac{2\sin x}{x}\to 1$)

10.c) $\lim_{x\to 0^+} \frac{x+2\sin x}{x+3\arctan x} = \frac{3}{5}$

$\frac{x+2\sin x}{x+3\arctan x} = \frac{x}{x}\cdot\frac{1+\frac{2\sin x}{x}}{1+3\frac{\arctan x}{x}} \to 3/5$ ($\frac{x}{x}\to 1$, $\frac{2\sin x}{x}\to 1$)

10.d) $\lim_{x\to+\infty} \frac{x+2\sin x}{x+3\arctan x} = 1$

$\frac{x+2\sin x}{x+3\arctan x} = \frac{x}{x}\cdot\frac{1+\frac{2\sin x}{x}}{1+3\frac{\arctan x}{x}} \to 1$ ($\frac{x}{x}\to 1$, $\frac{2\sin x}{x}\to 1$)

11.a) $\lim_{x\to-\infty} \frac{2^x+3^{x^2}}{x^3} = -\infty$

$\frac{2^x+3^{x^2}}{x^3} = \frac{2^x}{x^3} + \frac{3^{x^2}}{x^3} \to -\infty$ ($2^x\to 0$, $3^{x^2}\to-\infty$)

11.b) $\lim_{x\to 0^-} \frac{2^x+3^{x^2}}{x^3} = -\infty$

$\frac{2^x+3^{x^2}}{x^3} \to -\infty$ ($2^x\to 2$, $x^3\to 0^-$)

11.c) $\lim_{x\to 0^+} \frac{2^x+3^{x^2}}{x^3} = +\infty$

$\frac{2^x+3^{x^2}}{x^3} \to +\infty$ ($2^x\to 2$, $x^3\to 0^+$)

11.d) $\lim_{x\to+\infty} \frac{2^x+3^{x^2}}{x^3} = +\infty$

$\frac{2^x+3^{x^2}}{x^3} = \frac{3^{x^2}}{x^3}\cdot\frac{\frac{2^x}{3^{x^2}}+1}{1} \to +\infty$ ($\frac{3^{x^2}}{x^3}\to+\infty$, $\frac{2^x}{3^{x^2}}\to 1$)

12.a) $\lim_{x\to-\infty} \frac{x^2+\cos x}{x-\pi} = -\infty$

$\frac{x^2+\cos x}{x-\pi} = \frac{x^2}{x}\cdot\frac{1+\frac{\cos x}{x^2}}{1-\pi/x} \to -\infty$ ($x^2\to-\infty$, $\frac{x}{x}\to 1$)

12.b) $\lim_{x\to 0} \frac{x^2+\cos x}{x-\pi} = -\frac{1}{\pi}$

12.c) $\lim_{x\to\pi^-} \frac{x^2+\cos x}{x-\pi} = -\infty$

$\frac{x^2+\cos x}{x-\pi} \to -\infty$ ($x^2+\cos x\to\pi^2-1>0$, $x-\pi\to 0^-$)

12.d) $\lim_{x\to+\infty} \frac{x^2+\cos x}{x-\pi} = +\infty$

$\frac{x^2+\cos x}{x-\pi} = \frac{x^2}{x}\cdot\frac{1+\frac{\cos x}{x^2}}{1-\pi/x} \to +\infty$ ($x^2\to+\infty$, $\frac{x}{x}\to 1$)

13.a) $\lim_{x\to-\infty} \frac{2\sqrt[3]{x}+\sin x}{\sqrt[3]{x}-\arctan x} = 2$

$\frac{2\sqrt[3]{x}+\sin x}{\sqrt[3]{x}-\arctan x} = \frac{\sqrt[3]{x}}{\sqrt[3]{x}} \cdot \frac{2+\frac{\sin x}{\sqrt[3]{x}}}{1-\frac{\arctan x}{\sqrt[3]{x}}} \to 2$ ($\sqrt[3]{x}\to 1$, $\frac{\sin x}{\sqrt[3]{x}}\to 2$)

13.b) $\lim_{x\to 0^-} \frac{2\sqrt[3]{x}+\sin x}{\sqrt[3]{x}-\arctan x} = 2$

$\frac{2\sqrt[3]{x}+\sin x}{\sqrt[3]{x}-\arctan x} = \frac{\sqrt[3]{x}}{\sqrt[3]{x}} \cdot \frac{2+\frac{\sin x}{\sqrt[3]{x}}}{1-\frac{\arctan x}{\sqrt[3]{x}}} \to 2$ ($\sqrt[3]{x}\to 1$, $\frac{\sin x}{\sqrt[3]{x}}\to 2$)

13.c) $\lim_{x\to 0^+} \frac{2\sqrt[3]{x}+\sin x}{\sqrt[3]{x}-\arctan x} = 2$

$\frac{2\sqrt[3]{x}+\sin x}{\sqrt[3]{x}-\arctan x} = \frac{\sqrt[3]{x}}{\sqrt[3]{x}} \cdot \frac{2+\frac{\sin x}{\sqrt[3]{x}}}{1-\frac{\arctan x}{\sqrt[3]{x}}} \to 2$ ($\sqrt[3]{x}\to 1$, $\frac{\sin x}{\sqrt[3]{x}}\to 2$)

13.d) $\lim_{x\to+\infty} \frac{2\sqrt[3]{x}+\sin x}{\sqrt[3]{x}-\arctan x} = 2$

$\frac{2\sqrt[3]{x}+\sin x}{\sqrt[3]{x}-\arctan x} = \frac{\sqrt[3]{x}}{\sqrt[3]{x}} \cdot \frac{2+\frac{\sin x}{\sqrt[3]{x}}}{1-\frac{\arctan x}{\sqrt[3]{x}}} \to 2$ ($\sqrt[3]{x}\to 1$, $\frac{\sin x}{\sqrt[3]{x}}\to 2$)

14.a) $\lim_{x\to-\infty} \frac{x+\sin x}{\log x} = \setminus$

14.b) $\lim_{x\to 0^+} \frac{x+\sin x}{\log x} = 0$

$\frac{x+\sin x}{\log x} \to 0$ ($x+\sin x \to 0$, $\log x \to -\infty$)

14.c) $\lim_{x\to 1} \frac{x+\sin x}{\log x} = \setminus$

$\frac{x+\sin x}{\log x} \to \begin{cases} 0^+ \\ 0^- \end{cases}$ ($x+\sin x \to 1+\sin 1$)

14.d) $\lim_{x\to+\infty} \frac{x+\sin x}{\log x} = +\infty$

$\frac{x+\sin x}{\log x} = \frac{\log x}{\log x} \cdot \frac{\frac{x}{\log x}+\frac{\sin x}{\log x}}{1} \to +\infty$ ($\log x \to 1$, $\frac{x}{\log x}\to+\infty$)