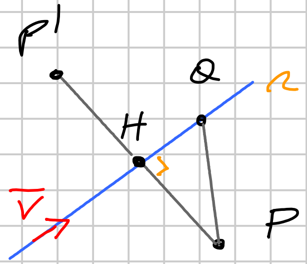
3. Determinare il simmetrico del punto generico (x, y) rispetto ad una retta $ax + by + c$ assegnata.

$P = (x_0, y_0)$ $\qquad r:\ ax + by + c = 0 \qquad \bar{v} = \delta(b, -a)$



$$Q \in r:\quad Q = \left(x,\ -\frac{a}{b}x - \frac{c}{b}\right)$$

$$\overline{PQ} = \left(x - x_0,\ -\frac{a}{b}x - \frac{c}{b} - y_0\right)$$

$$\overline{PQ}\cdot\bar{v} = b(x - x_0) - a\left(-\frac{a}{b}x - \frac{c}{b} - y_0\right) = 0$$

$$bx - bx_0 + \frac{a^2}{b}x + \frac{ac}{b} + ay_0 = 0$$

$$x\left(\frac{a^2 + b^2}{b}\right) = \frac{b^2x_0 - ab\,y_0 - ac}{b}$$

$$x = \frac{b^2x_0 - ab\,y_0 - ac}{a^2 + b^2}$$

$$\overline{PH} = \left(\frac{b^2x_0 - ab\,y_0 - ac}{a^2 + b^2} - x_0,\ -\frac{a}{b}\,\frac{b^2x_0 - ab\,y_0 - ac}{a^2 + b^2} - \frac{c}{b} - y_0\right)$$

$$P' = P + 2\overline{PH} =$$

$$\left(2\,\frac{b^2x_0 - ab\,y_0 - ac}{a^2 + b^2} - x_0,\ -\frac{2a}{b}\,\frac{b^2x_0 - ab\,y_0 - ac}{a^2 + b^2} - \frac{2c}{b} - y_0\right)$$

$$2\,\frac{b^2x_0 - ab\,y_0 - ac}{a^2 + b^2} - x_0 = \frac{2b^2x_0 - 2ab\,y_0 - 2ac - (a^2 + b^2)x_0}{a^2 + b^2} =$$

$$= \frac{(b^2 - a^2)x_0 - 2ab\,y_0 - 2ac}{a^2 + b^2}$$

$$-\frac{2a}{b}\,\frac{b^2x_0-ab\,y_0-ac}{a^2+b^2}-\frac{2c}{b}-y_0=$$

$$=\frac{-2ab^{\cancel{2}}x_0+2a^2\cancel{b}\,y_0+\cancel{2a^2c}-\cancel{2a^2c}-2b^{\cancel{2}}c-y_0\cancel{b}(a^2+b^2)}{\cancel{b}(a^2+b^2)}=$$

$$=\frac{-2ab\,x_0+(a^2-b^2)\,y_0-2bc}{a^2+b^2}$$