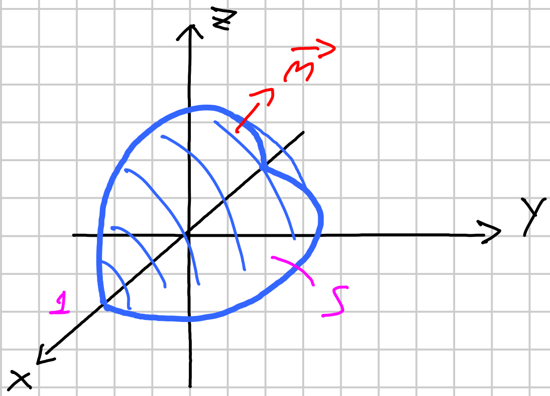
BY GIMUSI 15/07/2015

$$\begin{cases} S: x^2+y^2+z^2=1 \quad y,z \geq 0 \\ \vec{F}=(xz,\ y^2,\ y\sin z) \end{cases}$$

MODO 1 – TEOREMA DI STOKES

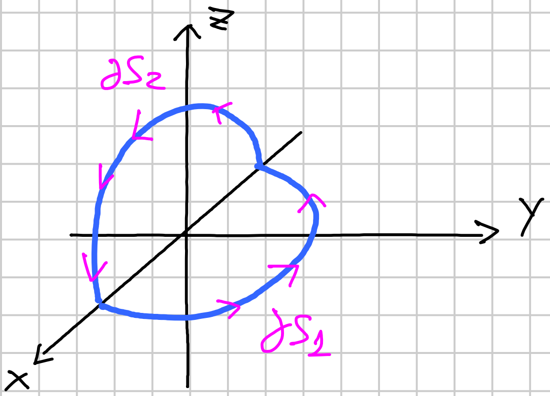


$$\int_S \operatorname{rot}\vec{F}\cdot\vec{n}\,d\sigma = \int_{\partial S}\vec{F}\cdot\vec{\tau}\,ds$$

$$\partial S = \partial S_1 \cup \partial S_2$$

$$\partial S_1 = \{(\cos\vartheta, \sin\vartheta, 0),\ \vartheta\in[0,\pi]\}$$

$$\partial S_2 = \{(-\cos\vartheta, 0, \sin\vartheta),\ \vartheta\in[0,\pi]\}$$



$$\int_{\partial S}\vec{F}\cdot\vec{\tau}\,ds = \int_{\partial S_1}\vec{F}\cdot\vec{\tau}\,ds + \int_{\partial S_2}\vec{F}\cdot\vec{\tau}\,ds$$

$$\int_{\partial S_1}\vec{F}\cdot\vec{\tau}\,ds = \int_0^\pi y^2\,dy = \int_0^\pi \sin^2\vartheta\ d(\sin\vartheta) =$$

$$= \frac{1}{3}\left[\sin^3\vartheta\right]_0^\pi = 0$$

$$\int_{\partial S_2}\vec{F}\cdot\vec{\tau}\,ds = \int_0^\pi xz\,dx = \int_0^\pi -\cos\vartheta\sin\vartheta\ d(-\cos\vartheta) =$$

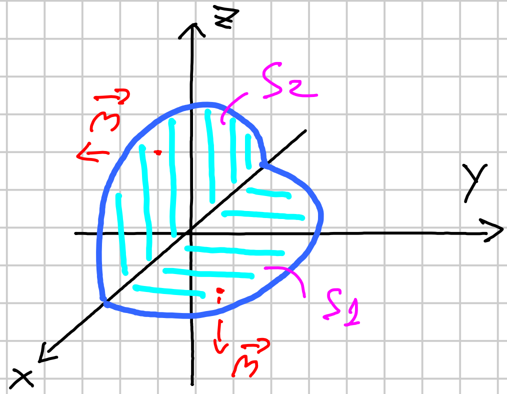
$$= \int_0^\pi -\cos\vartheta\sin^2\vartheta\,d\vartheta = -\int_0^\pi \sin^2\vartheta\,d(\sin\vartheta) = 0$$

$$\leadsto \int_S \operatorname{rot}\vec{F}\cdot\vec{n}\,d\sigma = \int_{\partial S}\vec{F}\cdot\vec{\tau}\,ds = 0$$

MODO 2 – TEOR. DI GG + CALCOLO FLUSSO

$$\mathrm{rot}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial x & \partial y & \partial z \\ xz & y^2 & y\sin z \end{vmatrix} = \sin z\,\hat{i} + x\,\hat{j} + 0\cdot\hat{k}$$

$$\mathrm{div}(\mathrm{rot}\,\vec{F}) = 0$$



$$\int_{S\cup S_1\cup S_2} \mathrm{rot}\,\vec{F}\cdot\vec{n}\,d\sigma = \int_{\Omega} \mathrm{div}(\mathrm{rot}\,\vec{F})\,dx\,dy\,dz = 0$$

$$\int_{S} \mathrm{rot}\,\vec{F}\cdot\vec{n}\,d\sigma = -\int_{S_1} \mathrm{rot}\,\vec{F}\cdot\vec{n}\,d\sigma - \int_{S_2} \mathrm{rot}\,\vec{F}\cdot\vec{n}\,d\sigma$$

$$\int_{S_1} \mathrm{rot}\,\vec{F}\cdot\vec{n}\,d\sigma = \int_{S_2} 0\,d\sigma = 0$$

$$\int_{S_2} \mathrm{rot}\,\vec{F}\cdot\vec{n}\,d\sigma = \int_{S_2} -x\,d\sigma = 0 \quad (\text{x SIMMETRIA})$$

$$\rightsquigarrow \int_{S} \mathrm{rot}\,\vec{F}\cdot\vec{n}\,d\sigma = 0$$