BY GIMUSI 23/06/15

$$f(x,y) = x^2+y^2+5\,\mathrm{arctg}(xy)$$

STUDIO CON WEIERSTRASS

1) $f(x,y)$ È CONTINUA IN $\mathbb{R}^2$

2) $$\lim_{x^2+y^2\to+\infty} f(x,y) = \lim_{\rho\to+\infty} \rho^2+5\,\mathrm{arctg}(\rho^2\cos\theta\sin\theta) = +\infty$$

INFATTI: $$\underbrace{\rho^2+5\,\mathrm{arctg}(\rho^2\cos\theta\sin\theta)}_{\to+\infty} \geq \underbrace{\rho^2-1000}_{\to+\infty}$$

3) $f(0,0)=0$

$\rightsquigarrow$ $f(x,y)$ AMMETTE MASSIMO E MINIMO

PUNTI STAZIONARI

$$\begin{cases} f_x = 2x + \dfrac{5y}{1+(xy)^2} = 0 \\ f_y = 2y + \dfrac{5x}{1+(xy)^2} = 0 \end{cases} \qquad \begin{cases} 2x+2x^3y^2+5y=0 \\ 2y+2x^2y^3+5x=0 \end{cases}$$

$x=0 \quad y=0 \rightsquigarrow P_0=(0,0)$

$$x\neq0 \quad y\neq0 \rightsquigarrow \begin{cases} 2xy+2x^3y^3+5y^2=0 \\ 2yx+2x^3y^3+5x^2=0 \end{cases} \rightsquigarrow \begin{cases} 5y^2-5x^2=0 \\ y^2=x^2 \quad y=\pm x \end{cases}$$

$$\rightsquigarrow \begin{cases} y=x \rightsquigarrow 2x+2x^5+5x=0 \quad x(x^5+3)=0 \\ y=-x \rightsquigarrow 2x+2x^5-5x=0 \quad x(x^5-1)=0 \rightsquigarrow x=\pm1 \end{cases}$$

$\rightsquigarrow P_1=(1,-1) \quad P_2=(-1,1) \quad f(P_1)=f(P_2)=2-5\pi<0$

$$\rightsquigarrow \begin{cases} \mathrm{SUP}\, f = +\infty \\ \mathrm{INF}\, f = \mathrm{MIN}\, f = 2-5\pi \text{ IN } P_1 \text{ E } P_2 \end{cases}$$