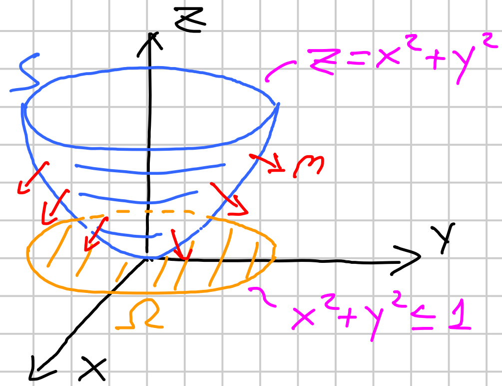
BY GIMUSI 26/06/15

$\vec{E} = (y^3, z-x, x^2)$ $\qquad$ $S:\ z = x^2+y^2,\ x^2+y^2 \leq 1$

MODO 3



PARAMETRIZZAZIONE DI S

$$\begin{cases} \phi(\rho,\theta) = (\rho\cos\theta, \rho\sin\theta, \rho^2) \\ \Omega = \{(\rho\cos\theta, \rho\sin\theta) : 0\leq\rho\leq 1,\ 0\leq\theta\leq 2\pi\} \end{cases}$$

VERSORE USCENTE $\quad n = \dfrac{\vec{N}}{|\vec{N}|}$

$$\begin{pmatrix} \phi_\rho \\ \phi_\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 2\rho \\ -\rho\sin\theta & \rho\cos\theta & 0 \end{pmatrix} \rightsquigarrow \vec{N}' = (M_1, M_2, M_3) = (-2\rho^2\cos\theta, -2\rho^2\sin\theta, \rho)$$

$$(\rho,\theta) = (1,0) \rightsquigarrow \vec{N}' = (-2,0,1) \rightsquigarrow \vec{N} = -\vec{N}' = (2\rho^2\cos\theta, 2\rho^2\sin\theta, -\rho)$$

CALCOLO DEL FLUSSO

$$\int_S \vec{E}\cdot\vec{n}\, d\sigma = \int_\Omega \left(\rho^3\sin^3\theta\cdot 2\rho^2\cos\theta + (\rho^2-\rho\cos\theta)2\rho^2\sin\theta - \rho^3\cos^2\theta\right) d\rho\, d\theta =$$

$$= \iint_\Omega \left(\underbrace{\cancel{2\rho^5\sin^3\theta\cos\theta}}_{=0} + \underbrace{\cancel{2\rho^5\sin\theta}}_{=0} - \underbrace{\cancel{2\rho^3\cos\theta\sin\theta}}_{=0} - \rho^3\cos^2\theta\right) d\rho\, d\theta =$$

$$\int_\Omega 2\rho^5\sin^3\theta\cos\theta\, d\rho\, d\theta = \int_0^{2\pi}\int_0^1 2\rho^5\sin^3\theta\cos\theta\, d\rho\, d\theta = \int_0^1 2\rho^5 \int_0^{2\pi} \sin^3\theta\, d(\sin\theta) = 0$$

$$= -\int_0^{2\pi}\int_0^1 \rho^3\cos^2\theta\, d\theta = -\int_0^{2\pi} \frac{1}{5}\cos^2\theta\, d\theta = -\frac{\pi}{5}$$