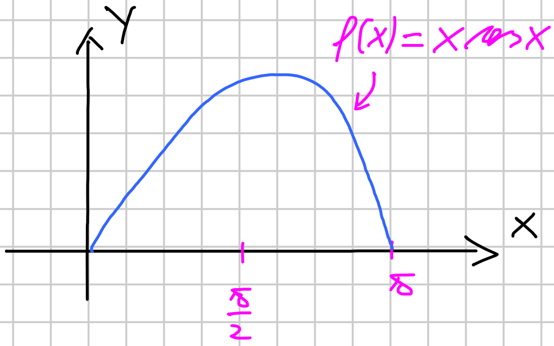
BY GIMUSI 23/06/15

FIGURA: $\{0 \le x \le \pi,\ 0 \le y \le x \operatorname{sen} x\}$



$$\begin{cases} f(x) = x \operatorname{sen} x \\ f'(x) = \operatorname{sen} x + x \cos x = 0 \qquad \operatorname{tg} x = -x \end{cases}$$

ROTAZIONE INTORNO ASSE X

$$V = \int_0^{\pi} \pi f(x)^2 \, dx = \pi \int_0^{\pi} x^2 \operatorname{sen}^2 x \, dx$$

$$\int \overset{f}{x^2} \overset{g'}{\operatorname{sen}^2 x} \, dx = x^2 \left(\frac{x}{2} - \frac{1}{4} \operatorname{sen} 2x \right) - \int 2x \left(\frac{x}{2} - \frac{1}{4} \operatorname{sen} 2x \right) dx$$

$$= \frac{x^3}{2} - \frac{x^2}{4} \operatorname{sen} 2x - \int x^2 \, dx + \int \frac{x}{2} \operatorname{sen} 2x \, dx =$$

$$= \frac{x^3}{2} - \frac{x^2}{4} \operatorname{sen} 2x - \frac{x^3}{3} + \frac{1}{2} \int x \operatorname{sen} 2x \, dx$$

$$\int \overset{f}{x} \overset{g'}{\operatorname{sen} 2x} \, dx = x \left(-\frac{1}{2} \cos 2x \right) + \frac{1}{2} \int \cos 2x \, dx =$$

$$= -\frac{x}{2} \cos 2x + \frac{1}{4} \operatorname{sen} 2x$$

$$\int x^2 \operatorname{sen}^2 x \, dx = \frac{x^3}{6} - \frac{x^2}{4} \operatorname{sen} 2x - \frac{x}{4} \cos 2x + \frac{1}{8} \operatorname{sen} 2x$$

$$V = \pi \int_0^{\pi} x^2 \operatorname{sen}^2 x \, dx = \pi \left[\frac{x^3}{6} - \cancel{\frac{x^2}{4} \operatorname{sen} 2x} - \frac{x}{4} \cos 2x + \cancel{\frac{1}{8} \operatorname{sen} 2x} \right]_0^{\pi} =$$

$$= \pi \left(\frac{\pi^3}{6} - \frac{\pi}{4} \right) = \frac{\pi^4}{6} - \frac{\pi^2}{4}$$

ROTAZIONE INTORNO ASSE Y

$$V = \int_0^{\pi} 2\pi x \cdot f(x)\, dx = \int_0^{\pi} 2\pi x \cdot x \sin x \, dx = 2\pi \int_0^{\pi} x^2 \sin x \, dx$$

$$\int \overset{f}{x^2} \overset{g'}{\sin x} \, dx = x^2(-\cos x) + 2 \int x \cos x \, dx$$

$$\int \overset{f}{x} \overset{g'}{\cos x} \, dx = x \sin x - \int \sin x \, dx = x \sin x + \cos x$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x$$

$$V = 2\pi \int_0^{\pi} x^2 \sin x \, dx = 2\pi \Big[-x^2 \cos x + 2\cancel{x \sin x} + 2 \cos x \Big]_0^{\pi} =$$

$$= 2\pi \left(-\pi^2(-1) - 2 - 2 \right) = 2\pi^3 - 8\pi$$