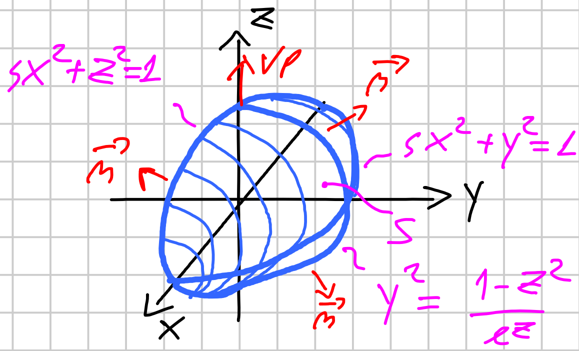
BY GIMUSI 08/06/15

$$\vec{F} = (\overset{A}{\sin y}, \overset{B}{x e^z}, \overset{C}{x z^2})$$

$$S:\ 5x^2 + e^z y^2 + z^2 = 1,\quad y \geq 0$$

NORMALE IN $P=(0,0,1)$: $V_P = (0,0,1)$



$$\mathrm{rot}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial x & \partial y & \partial z \\ A & B & C \end{vmatrix} =$$

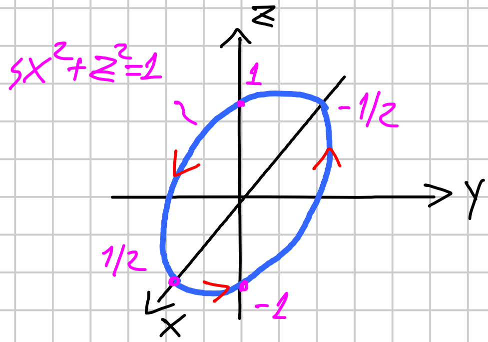
$$= (C_y - B_z)\hat{i} + (A_z - C_x)\hat{j} + (B_x - A_y)\hat{k} =$$

$$= (0 - x e^z)\hat{i} + (0 - z^2)\hat{j} + (e^z - \cos y)\hat{k} = (-x e^z, -z^2, e^z - \cos y)$$

FORMULA DI STOKES: $\displaystyle\int_S \mathrm{rot}\,\vec{F}\cdot\vec{n}\,d\sigma = \int_{\partial S} \vec{F}\cdot\vec{\tau}\,ds = \int_{\partial S} A\,dx + B\,dy + C\,dz$

$\partial S: \{(x,0,z) : 5x^2 + z^2 = 1\}$

SENSO DI PERCORRENZA: ANTIORARIO RISP. A y^+



$$\int_{\partial S} \overset{=0}{A\,dx} + \overset{=0}{B\,dy} + C\,dz = \int_{\partial S} x z^2\,dz$$

PARAMETRIZZAZIONE DI ∂S: $\left(\frac{\cos\theta}{2}, 0, -\sin\theta\right)$ $\quad 0 \leq \theta \leq 2\pi$

$$\int_{\partial S} x z^2\,dz = \int_0^{2\pi} \frac{\cos\theta}{2}\sin^2\theta\,(-\cos\theta)\,d\theta = \int_0^{2\pi} -\frac{1}{2}\cos^2\theta\,\sin^2\theta\,d\theta =$$

$$= -\frac{1}{8}\int_0^{2\pi} \sin^2 2\theta\,d\theta = -\frac{1}{16}\int_0^{2\pi} \sin^2 2\theta\,d(2\theta) = -\frac{1}{16}\,2\pi = -\frac{\pi}{8}$$