BY GIMUSI 06/06/15

$$f(x,y)=\frac{xy}{1+x^4+y^4}\qquad (x,y)\in\mathbb{R}^2$$

$$f(0,0)=0\qquad \lim_{(x,y)\to\infty} f(x,y)=\lim_{\rho\to+\infty}\frac{\rho^2\frac{1}{2}\sin 2\theta}{1+\rho^4(\cos^4\theta+\sin^4\theta)}=0$$

$\rightsquigarrow$ x W. GENERAL. $\exists$ MAX E MIN

PUNTI STAZIONARI

$$\begin{cases} f_x=\dfrac{y(1+x^4+y^4)-4x^3\cdot xy}{(1+x^4+y^4)^2}=\dfrac{y+x^4y+y^5-4x^4y}{(1+x^4+y^4)^2}=\dfrac{y+y^5-3x^4y}{(1+x^4+y^4)^2}=0 \\ f_y=\dfrac{x+x^5-3xy^4}{(1+x^4+y^4)^2}=0 \end{cases}$$

SIST. 1

$$\begin{cases} y(1+y^4-3x^4)=0 \\ x(1+x^4-3y^4)=0 \end{cases}\rightsquigarrow\begin{cases} x=0 \\ y=0 \end{cases}\rightsquigarrow P_0=(0,0)$$

SIST. 2

$$\begin{cases} 1+y^4-3x^4=0 \\ 1+x^4-3y^4=0 \end{cases}\quad\begin{cases} y^4=3x^4-1 \\ 1+x^4-9x^4+3 \end{cases}\quad\begin{cases} y^4=1/2 & y=\pm\sqrt[4]{1/2} \\ x^4=1/2 & x=\pm\sqrt[4]{1/2} \end{cases}$$

$$\rightsquigarrow P_{1,2}=\left(\pm\sqrt[4]{1/2},\ \pm\sqrt[4]{1/2}\right)\qquad P_{3,4}=\left(\pm\sqrt[4]{1/2},\ \mp\sqrt[4]{1/2}\right)$$

STUDIO PUNTI STAZIONARI (CONVESSITÀ)

$$f_{xx}=\frac{-12x^3y(1+x^4+y^4)^{\cancel{2}}-2\cdot 4x^3\cancel{(1+x^4+y^4)}(y+y^5-3x^4y)}{(1+x^4+y^4)^{\cancel{4}\ 3}}$$

$$=\frac{4x^3y(-3-3x^4-3y^4-2-2y^4+6x^4)}{(1+x^4+y^4)^3}=\frac{4x^3y(3x^4-5y^4-5)}{(1+x^4+y^4)^3}$$

$$f_{yy}=\frac{4xy^3(3y^4-5x^4-5)}{(1+x^4+y^4)^3}$$

$$f_{xy} = \frac{(1+5y^4-3x^4)(1+x^4+y^4)^{\cancel{2}} - 2\cdot 4y^3\cancel{(1+x^4+y^4)}(y+y^5-3x^4y)}{(1+x^4+y^4)^{\cancel{4}\,3}}$$

$$= \frac{1+x^4+y^4+5y^4+5x^4y^4+5y^8-3x^4-3x^8-3x^4y^4-8y^4-8y^8+24x^4y^4}{(1+x^4+y^4)^3} =$$

$$= \frac{1-2x^4-2y^4+26x^4y^4-3x^8-3y^8}{(1+x^4+y^4)^3}$$

$$\begin{cases} f_{xx}(P_0)=0 \quad f_{yy}(P_0)=0 \quad f_{xy}(P_0)=f_{yx}(P_0)=1 \\ Hf(P_0)=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \det Hf(P_0) = -1 \end{cases} \rightsquigarrow P_0 \text{ P.TO DI SELLA}$$

$$\begin{cases} f_{xx}(P_1) = \dfrac{\cancel{2}(3/2-5/2-5)}{2^{\cancel{3}\,2}} = -\dfrac{3}{2} \quad f_{yy}(P_1) = -\dfrac{3}{2} \\ f_{xy}(P_1) = f_{yx}(P_1) = \dfrac{\cancel{1}-\cancel{1}-1+26/4-3/4-3/4}{2^3} = \dfrac{1}{8}\,\dfrac{-4+26-6}{4} \\ \qquad = \dfrac{1}{\cancel{8}}\,\dfrac{\cancel{16}^{\,\cancel{8}\,1}}{\cancel{4}\,2} = \dfrac{1}{2} \\ Hf(P_1)=\begin{pmatrix} -3/2 & 1/2 \\ 1/2 & -3/2 \end{pmatrix} \quad \det Hf(P_1) = 2 \end{cases} \rightsquigarrow P_1 \text{ P.TO DI MAX}$$

$$\begin{cases} f_{xx}(P_2) = -\dfrac{3}{2} \quad f_{yy}(P_2) = \dfrac{-3}{2} \quad f_{xy}(P_2)=f_{yx}(P_2)=\dfrac{1}{2} \\ Hf(P_2)=\begin{pmatrix} -3/2 & 1/2 \\ 1/2 & -3/2 \end{pmatrix} \quad \det Hf(P_2) = 2 \end{cases} \rightsquigarrow P_2 \text{ P.TO DI MAX}$$

$$\begin{cases} f_{xx}(P_3) = \dfrac{-\cancel{2}(3/2 - 5/2 - 5)}{2^{\cancel{3}\,2}} = \dfrac{3}{2} \qquad f_{yy}(P_3) = \dfrac{3}{2} \\[2ex] f_{xy}(P_3) = f_{yx}(P_3) = \dfrac{1-1-1 \;+26/5 - 3/5 - 3/5}{2^3} = \dfrac{1}{2} \\[2ex] Hf(P_3) = \begin{pmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{pmatrix} \quad \det Hf(P_3) = 2 \rightsquigarrow P_3 \text{ P.TO DI MIN} \end{cases}$$

$$\begin{cases} f_{xx}(P_5) = 3/2 \quad f_{yy}(P_5) = 3/2 \quad f_{xy}(P_3) = f_{yx}(P_3) = 1/2 \\[2ex] Hf(P_3) = \begin{pmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{pmatrix} \quad \det Hf(P_5) = 2 \rightsquigarrow P_5 \text{ P.TO DI MIN} \end{cases}$$