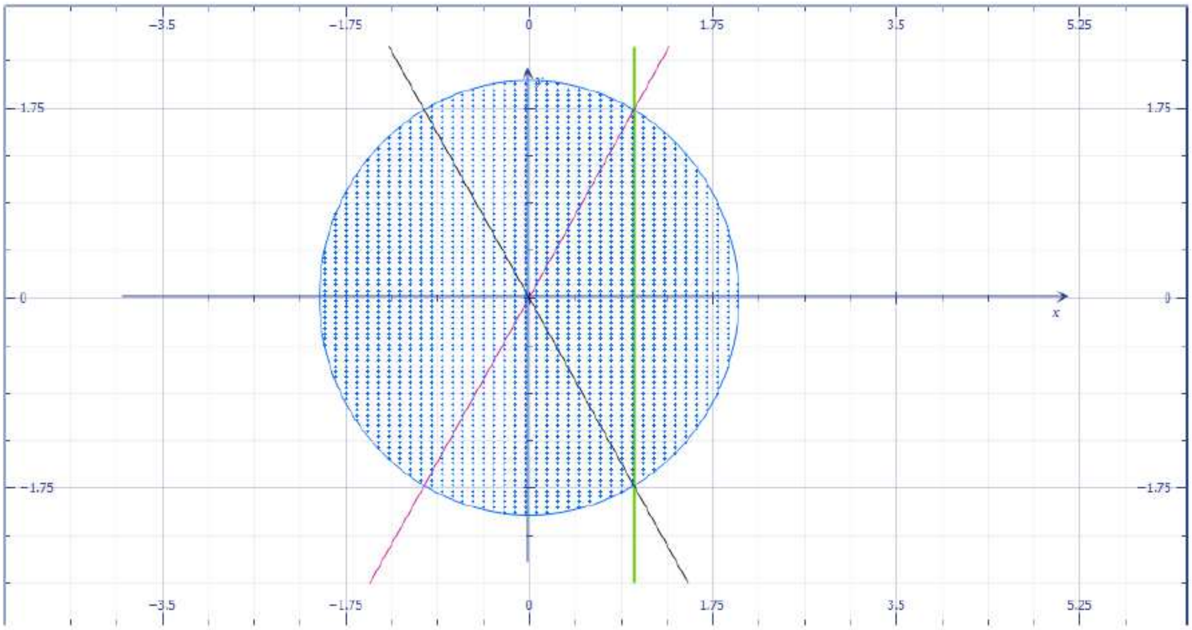
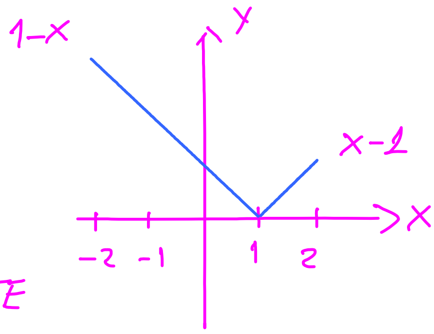
$x^2 + y^2 <= 4$ f(x,y)=|x-1| $= \begin{cases} x-1 & x \geq 1 \\ 1-x & x \leq 1 \end{cases}$

Integrale su A+



Integrale sui triangolini

$$\int_0^1 \int_{(-\sqrt{3})x}^{\sqrt{3}x} \underbrace{x - 1}_{\leq 0} dy\, dx \rightarrow \text{È NEGATIVO}$$



Integrale su il settore circolare

$$\int_0^2 \int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} (\rho\cos(\theta) - 1)\rho d\theta\, d\rho \rightarrow$$ "CONTIENE" ANCHE L'INTEGRALE PRECEDENTE

E poi faccio

$$-\int_0^1 \int_{(-\sqrt{3})x}^{\sqrt{3}x} x - 1dy\, dx + \int_0^2 \int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} (\rho\cos(\theta) - 1)\rho d\theta\, d\rho$$

Integrale A-

Integrale su il restante insieme

$$\int_0^2 \int_{\frac{\pi}{3}}^{\frac{5}{3}\pi} (1 - \rho\cos(\theta))\rho d\theta\, d\rho$$

CON SEGNO −

Dopo si che faccio

$$\int_0^2 \int_{\frac{\pi}{3}}^{\frac{5}{3}\pi} (1 - \rho\cos(\theta))\rho d\theta\, d\rho \,-\!\cancel{+} \int_0^1 \int_{(-\sqrt{3})x}^{\sqrt{3}x} x - 1dy\, dx$$

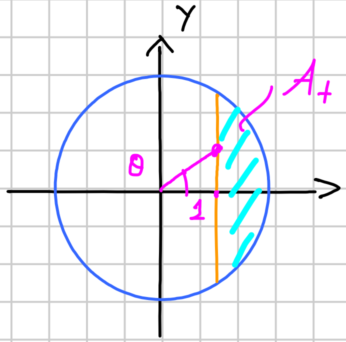
A questo punto faccio la somma di A+ e A-

$$\int_A |x-1|\, dA = \int_A (1-x)\, dA + 2\int_{A_+} (x-1)\, dA$$

$$\int_A (1-x)\, dA = \int_0^{2\pi}\int_0^2 (1-\rho\cos\theta)\rho\, d\rho\, d\theta = \int_0^{2\pi}\int_0^2 (\rho-\rho^2\cos\theta)\, d\rho\, d\theta =$$

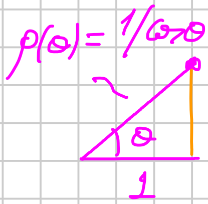
$$= \int_0^{2\pi} \left[\frac{\rho^2}{2} - \frac{\rho^3}{3}\cos\theta\right]_0^2 d\theta = \int_0^{2\pi}\left(2-\frac{8}{3}\cos\theta\right) d\theta =$$

$$= \left[2\theta - \frac{8}{3}\sin\theta\right]_0^{2\pi} = 4\pi$$



$$\int_{A_+} (x-1)\, dA = \int_{-\frac{\pi}{3}}^{\pi/3}\int_{1/\cos\theta}^{2} (\rho\cos\theta-1)\,\rho\, d\rho\, d\theta =$$

$$= 2\int_0^{\pi/3}\int_{1/\cos\theta}^{2} (\rho^2\cos\theta-\rho)\, d\rho\, d\theta =$$



$$= 2\int_0^{\pi/3} \left[\frac{\rho^3}{3}\cos\theta - \frac{\rho^2}{2}\right]_{1/\cos\theta}^{2} d\theta =$$

$$= 2\int_0^{\pi/3}\left(\frac{8}{3}\cos\theta - 2 - \frac{1}{3}\frac{1}{\cos^2\theta} + \frac{1}{2}\frac{1}{\cos^2\theta}\right) d\theta = 2\int_0^{\pi/3}\left(\frac{8}{3}\cos\theta - 2 + \frac{1}{6\cos^2\theta}\right) d\theta$$

$$= 2\left[\frac{8}{3}\sin\theta - 2\theta + \frac{1}{6}\operatorname{tg}\theta\right]_0^{\pi/3} = 2\left(\frac{8}{3}\frac{\sqrt{3}}{2} - \frac{2\pi}{3} + \frac{1}{6}\sqrt{3}\right) =$$

$$= \left(\frac{8}{3}+\frac{1}{3}\right)\sqrt{3} - \frac{4\pi}{3} = 3\sqrt{3} - \frac{4\pi}{3}$$

$$\rightsquigarrow \int_A |x-1|\, dA = 4\pi + 2\left(3\sqrt{3} - \frac{4}{3}\pi\right) = 4\pi - \frac{8}{3}\pi + 6\sqrt{3} = \frac{4}{3}\pi + 6\sqrt{3}$$