$$f(x,y) = xy^5 - \arctan(xy) \quad \text{su } \mathbb{R}^2$$

## PUNTI STAZIONARI

$$\begin{cases} f_x = y^5 - \dfrac{1}{1+(xy)^2}\cdot y = \dfrac{y^5+x^2y^6-y}{1+(xy)^2} = 0 \\[2ex] f_y = 5xy^3 - \dfrac{1}{1+(xy)^2}\cdot x = \dfrac{5xy^3+5x^3y^5-x}{1+(xy)^2} = 0 \end{cases}$$

$$\begin{cases} y^5+x^2y^6-y = 0 \\ 5xy^3+5x^3y^5-x = 0 \end{cases} \qquad \begin{cases} y(y^3+x^2y^5-1) = 0 \\ x(5y^3+5x^2y^5-1) = 0 \end{cases} \qquad \begin{cases} y = 0 & (1) \\ y^3+x^2y^5-1 = 0 & (2) \end{cases}$$

(1) $y = 0 \rightsquigarrow x(5y^3+5x^2y^5-1) = x(-1) = 0 \rightsquigarrow P_1 = (0,0)$

(2) $y^3+x^2y^5-1 = 0 \rightsquigarrow x(5y^3+5x^2y^5-1) = x(5-1) = 3x = 0 \rightsquigarrow x = 0$

$$\rightsquigarrow y^3 = 1 \quad y = 1 \rightsquigarrow P_2 = (0,1)$$

## STUDIO DEI P.TI STAZIONARI (CONVESSITÀ)

$$f_x = y^5 - \frac{y}{1+(xy)^2} \rightsquigarrow f_{xx} = \frac{y\cdot 2xy^2}{[1+(xy)^2]^2} = \frac{2xy^3}{[1+(xy)^2]^2}$$

$$\rightsquigarrow f_{xy} = 5y^3 - \frac{1+x^2y^2-2x^2y^2}{[1+(xy)^2]^2} = 5y^3 - \frac{1-x^2y^2}{[1+(xy)^2]^2}$$

$$f_y = 5xy^3 - \frac{x}{1+(xy)^2} \rightsquigarrow f_{yy} = 12xy^2 + \frac{2x^3y}{[1+(xy)^2]^2}$$

$$H_f(P_1) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \rightsquigarrow DET = -1 \qquad SEGNATURA: + -$$

$\rightsquigarrow$ $P_1 = (0,0) \equiv$ P.TO DI SELLA

$$H_f(P_2) = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} \rightsquigarrow DET = -9 \qquad SEGNATURA: + -$$

$\rightsquigarrow$ $P_2 = (0,1) \equiv$ P.TO DI SELLA

## SVILUPPI DI TAYLOR

$$(xy^4)_x = y^4 \quad (xy^4)_y = 4xy^3 \quad (xy^4)_{xy} = 4y^3 \quad (xy^4)_{xx} = 0 \quad (xy^4)_{yy} = 12xy^2$$

$$[\operatorname{arctg}(xy)]_x = \frac{y}{1+x^2y^2} \qquad [\operatorname{arctg}(xy)]_y = \frac{x}{1+x^2y^2}$$

$$[\operatorname{arctg}(xy)]_{xy} = \frac{1+x^2y^2-2x^2y^2}{(1+x^2y^2)} = \frac{1-x^2y^2}{(1+x^2y^2)^2}$$

$$[\operatorname{arctg}(xy)]_{xx} = \frac{-2xy^3}{(1+x^2y^2)^2} \qquad [\operatorname{arctg}(xy)]_{yy} = \frac{-2x^3y}{(1+x^2y^2)^2}$$

### CENTRO IN $(0,0)$ $\quad m=2$

$$\begin{cases} xy^4 = o(x^2+y^2) \\ \operatorname{arctg}(xy) = xy + o(x^2+y^2) \end{cases} \rightsquigarrow f(x,y) = -xy + o(x^2+y^2)$$

$\hookrightarrow$ SELLA

### CENTRO IN $(0,1)$ $\quad m=2$

$$\begin{cases} xy^4 = x + 4xy + o(x^2+(y-1)^2) \\ \operatorname{arctg}(xy) = x + xy + o(x^2+(y-1)^2) \end{cases} \rightsquigarrow f(x,y) = 3xy + o(x^2+(y-1)^2)$$

$\hookrightarrow$ SELLA