

$x \rightarrow y$

$$\begin{aligned}
 \int_0^1 \int_0^1 y e^{xy} dx dy &= \int_0^1 y \left[\frac{e^{xy}}{y} \right]_0^1 dy = \\
 &= \int_0^1 y \left(\frac{e^y}{y} - \frac{1}{y} \right) dy = \int_0^1 (e^y - 1) dy = \\
 &= [e^y - y]_0^1 = e - 1 - 1 + 0 = e - 2
 \end{aligned}$$

 $y \rightarrow x$

$$\int_0^1 \int_0^1 y e^{xy} dy dx$$

$$\begin{aligned}
 \int_0^1 y e^{xy} dy &= \left[y \frac{e^{xy}}{x} \right]_0^1 - \int_0^1 \frac{e^{xy}}{x} dx = \frac{e^x}{x} - 0 - \left[\frac{e^{xy}}{x^2} \right]_0^1 = \\
 &= \frac{e^x}{x} - \left(\frac{e^x}{x^2} - \frac{1}{x^2} \right) = \frac{e^x}{x} - \frac{e^x}{x^2} + \frac{1}{x^2} = \frac{e^x(x-1)+1}{x^2}
 \end{aligned}$$

$$\int_0^1 \int_0^1 y e^{xy} dy dx = \int_0^1 \left(\frac{e^x}{x} - \frac{e^x}{x^2} + \frac{1}{x^2} \right) dx$$

$$\int \frac{e^x}{x} dx = \int \frac{1}{x} e^x dx = \frac{e^x}{x} - \int -\frac{1}{x^2} e^x dx = \frac{e^x}{x} + \int \frac{e^x}{x^2} dx + C$$

$$\int \left(\frac{e^x}{x} - \frac{e^x}{x^2} + \frac{1}{x^2} \right) dx = \int \frac{e^x}{x} dx - \int \frac{e^x}{x^2} dx + \int \frac{1}{x^2} dx =$$

$$= \frac{e^x}{x} + \int \frac{e^x}{x^2} dx - \int \frac{e^x}{x^2} dx - \frac{1}{x} + C$$

$$\int_0^1 \left(\frac{e^x}{x} - \frac{e^x}{x^2} + \frac{1}{x^2} \right) dx = \left[\frac{e^x - 1}{x} \right]_0^1 = e - 1 - \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = e - 2$$