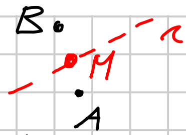
7. Consideriamo nel piano cartesiano i punti $A=(3,1)$ e $B=(4,-4)$.

- (a) Determinare il luogo dei punti P tali che $PA=PB$.
- (b) Determinare il luogo dei punti P tali che $PA=3PB$.
- (c) Più in generale, determinare al variare del parametro $\lambda>0$, il luogo dei punti P tali che $PA=\lambda PB$.

$$P=(x,y) \Rightarrow PA=\|(x-3,y-1)\| \quad PB=\|(x-4,y+4)\|$$



(a) $$PA^2=PB^2 \Rightarrow \cancel{x^2}+9-6x+\cancel{y^2}+1-2y=\cancel{x^2}+16-8x+\cancel{y^2}+16+8y$$

$$\Rightarrow 2x-10y-22=0 \Rightarrow x-5y-11=0$$

$\left(M=\left(\frac{7}{2},-\frac{3}{2}\right)\in r\right)$

(b) $$PA^2=9PB^2 \Rightarrow x^2+9-6x+y^2+1-2y=9x^2+144-72x+9y^2+144+72y$$

$$\Rightarrow 8x^2+8y^2-66x+74y+278=0$$

$$8\left(x-\frac{33}{8}\right)^2+8\left(y+\frac{37}{8}\right)^2-\frac{1089}{8}-\frac{1369}{8}+278=0$$

$$8\left(x-\frac{33}{8}\right)^2+8\left(y+\frac{37}{8}\right)^2-\frac{\overset{234}{1089+1369-2224}}{8}=0$$

$$\left(x-\frac{33}{8}\right)^2+\left(y+\frac{37}{8}\right)^2-\frac{117}{32}=0$$

(c) $$x^2+9-6x+y^2+1-2y=\lambda^2x^2+16\lambda^2-8\lambda^2x+\lambda^2y^2+16\lambda^2+8\lambda^2y$$

$$\Rightarrow (\lambda^2-1)x^2+(6-8\lambda^2)x+(\lambda^2-1)y^2+(2+8\lambda^2)y+(32\lambda^2-10)=0$$

$\lambda=1 \Rightarrow$ RETTA

$$\lambda\neq 1 \Rightarrow x^2+\frac{6-8\lambda^2}{\lambda^2-1}x+y^2+\frac{2+8\lambda^2}{\lambda^2-1}y+\frac{32\lambda^2-10}{\lambda^2-1}=0$$

$$\left[x^2+\frac{6-8\lambda^2}{\lambda^2-1}x+\frac{(6-8\lambda^2)^2}{4(\lambda^2-1)^2}\right]+\left[y^2+\frac{2+8\lambda^2}{\lambda^2-1}y+\frac{(2+8\lambda^2)^2}{4(\lambda^2-1)^2}\right]$$

$$+\left(\frac{32\lambda^2-10}{\lambda^2-1}-\frac{(\cancel{6}^{3}-\cancel{8}^{4}\lambda^2)^2}{\cancel{4}(\lambda^2-1)^2}-\frac{(\cancel{2}^{1}+\cancel{8}^{4}\lambda^2)^2}{\cancel{4}(\lambda^2-1)^2}\right)$$

$$\Rightarrow\left[x+\frac{6-8\lambda^2}{2(\lambda^2-1)}\right]^2+\left[y+\frac{2+8\lambda^2}{2(\lambda^2-1)}\right]^2+(\lambda^2-1)^{-2}\left[\cancel{32\lambda^4}-10\lambda^2-32\lambda^2+\cancel{10}-\cancel{9}-\cancel{16\lambda^4}+24\lambda^2-\cancel{1}-\cancel{16\lambda^4}-8\lambda^2\right]$$

$$R^2=-(\lambda^2-1)^{-2}\left[-26\lambda^2\right]=26\lambda^2/(\lambda^2-1)^2$$