Isometrie del piano 2

Argomenti: isometrie del piano **Difficoltà:** ⋆⋆⋆

Prerequisiti: isometrie nel piano, matrici ortogonali

Nel seguito sono descritte alcune isometrie del piano. Per ciascuna di esse si richiede di

(a) • scrivere l'espressione generale,

(b) • determinare i punti fissi,

(c) • determinare l'immagine della retta $y = 2x - 3$ e della retta $2x + 3y + 5 = 0$,

(d) • determinare la retta che ha come immagine la retta $2x + 3y + 5 = 0$,

(e) • determinare l'immagine della circonferenza di equazione $x^2 + y^2 + 3x - 2y = 10$.

Isometrie da esaminare:

(1) traslazione di vettore $(3, -1)$,

(2) simmetria rispetto all'asse x,

(3) simmetria rispetto all'asse y,

(4) rotazione di $90°$ in senso orario rispetto all'origine,

(5) simmetria rispetto alla retta $x = -8$,

(6) simmetria centrale rispetto al punto $(3, 4)$,

(7) simmetria rispetto alla retta $y = x$,

(8) simmetria rispetto alla retta $y = 4$ seguita da simmetria rispetto alla retta $x = 3$,

(9) simmetria rispetto alla retta $y = 2x$,

(10) simmetria rispetto alla retta $y = 2x$ seguita da simmetria rispetto alla retta $y = 2x - 3$,

(11) rotazione di $30°$ in senso antiorario rispetto al punto $(-2, 3)$,

(12) simmetria rispetto alla retta di equazione $3x + 4y + 7 = 0$,

(13) simmetria rispetto alla retta di equazione $3x + 4y + 7 = 0$ seguita da simmetria rispetto alla retta di equazione $3x - 4y + 11 = 0$,

(14) simmetria rispetto alla retta $y + x = 0$ seguita da traslazione di vettore $(2, -2)$,

(15) simmetria rispetto alla retta di equazione $3x - 4y - 7 = 0$ seguita da rotazione antioraria di $90°$ rispetto al punto $(1, 2)$,

(16) rotazione oraria di $120°$ rispetto al punto $(1, 2)$, seguita da rotazione oraria di $120°$ rispetto al punto $(3, -2)$, seguita da rotazione oraria di $120°$ rispetto al punto $(7, 1)$.

SOLUZIONI ?

BY GIMUSI 31/01/15

(1) traslazione di vettore $(3,-1)$,

(a) $P' = P + (3,-1)$

(b) $P' \neq P \;\; \forall P \rightsquigarrow$ NON ESISTONO P.TI FISSI

(c) $r_2: y = 2x - 3$ $\quad P = (\delta, 2\delta - 3) \in r_2$

$P' = P + (3,-1) = (\delta + 3, 2\delta - 4)$

$r_2': \; y = 2x - 10$ $\quad$ ex $(0,-3) \in r_2 \rightsquigarrow (3,-4) \in r_2'$

$r_2: 2x + 3y + 5 = 0$ $\quad P = (\delta, -\frac{2}{3}\delta - \frac{5}{3}) \in r_2$

$P' = P + (3,-1) = (\delta + 3, -\frac{2}{3}\delta - \frac{8}{3})$

$r_2': \; y = -\frac{2}{3}x + 2 - \frac{8}{3} \quad y = -\frac{2}{3}x - \frac{2}{3} \quad 2x + 3y + 2 = 0$

ex $(-1,-1) \in r_2 \rightsquigarrow (2,-2) \in r_2'$

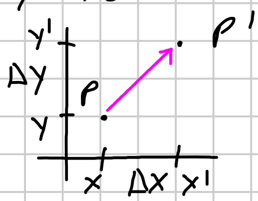
(d) $r': 2x + 3y + 5 = 0$ $\quad P' = (\delta, -\frac{2}{3}\delta - \frac{5}{3}) \in r'$

$P = P' - (3,-1) = (\delta - 3, -\frac{2}{3}\delta - \frac{2}{3})$

$r: \; y = -\frac{2}{3}x - 2 - \frac{2}{3} = -\frac{2}{3}x - \frac{8}{3} \quad 2x + 3y + 8 = 0$

ex $(-1,-1) \in r' \rightsquigarrow (-4,0) \in r$

(e) $c: x^2 + y^2 + 3x - 2y = 10$



$\begin{cases} x' = x + \Delta x \\ y' = y + \Delta y \end{cases} \quad \begin{cases} x = x' - \Delta x \\ y = y' - \Delta y \end{cases}$

$c': \; (x-3)^2 + (y+1)^2 + 3(x-3) - 2(y+1) = 10$

$x^2 - 6x + \cancel{9} + y^2 + \cancel{2y} + 1 + 3x - \cancel{9} - \cancel{2y} - 2 = 10$

$x^2 + y^2 - 3x = 11$

(2) simmetria rispetto all'asse x,

(a) $P' = SP \quad S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(b) $P' = SP = P \rightsquigarrow SP - IP = 0 \quad (S - I)P = \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} P = 0 \quad P = \begin{pmatrix} \delta \\ 0 \end{pmatrix} \equiv$ ASSE X

(c) $r_2: y = 2x - 3 \quad \begin{cases} x = x' \\ y = -y' \end{cases} \rightsquigarrow -y = 2x - 3 \quad y = -2x + 3$ $\quad$ ex $(1,-1) \rightsquigarrow (1,1)$

$r_2: 2x + 3y + 5 = 0 \rightsquigarrow 2x - 3y + 5 = 0$ $\quad$ ex $(-1,-1) \rightsquigarrow (-1,1)$

(d) $r': 2x+3y+5=0$

MATRICE ORTOGONALE

$P = S^{-1}P'$ $\quad S^{-1} = -1\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = S^{T} = S$ $\quad r: 2x-3y+5=0$

(e) $c: x^2+y^2+3x-2y=10 \rightsquigarrow c': x^2+y^2+3x+2y=10$

(3) simmetria rispetto all'asse y,

(a) $P' = SP \quad S = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) $P' = SP = P \rightsquigarrow SP - IP = 0 \quad (S-I)P = \begin{pmatrix} -2 & 0 \\ 0 & 0 \end{pmatrix} P = 0 \quad P = \begin{pmatrix} 0 \\ \delta \end{pmatrix} \equiv$ ASSE Y

(c) $r_2: y = 2x-3 \quad \begin{cases} x = -x' \\ y = y' \end{cases} \rightsquigarrow y = -2x-3$ $\quad$ ex $(1,-1) \rightsquigarrow (-1,-1)$

$r_2: 2x+3y+5=0 \rightsquigarrow -2x+3y+5=0$ $\quad$ ex $(-1,-1) \rightsquigarrow (1,-1)$

(d) $r': 2x+3y+5=0$

$P = S^{-1}P'$ $\quad S^{-1} = S^{T} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = S$ $\quad r: -2x+3y+5=0$

(e) $c: x^2+y^2+3x-2y=10 \rightsquigarrow c': x^2+y^2-3x-2y=10$

(4) rotazione di 90° in senso orario rispetto all'origine,

(a) $P' = SP \quad S = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \underset{\theta = -\frac{\pi}{2}}{=} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(b) $P' = SP = P \rightsquigarrow SP - IP = 0 \quad (S-I)P = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} P = 0 \quad P = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \equiv$ ORIGINE

(c) $r_2: y = 2x-3 \quad \begin{cases} x' = y \\ y' = -x \end{cases} \quad \begin{cases} y = x' \\ x = -y' \end{cases} \quad x = -2y-3 \quad y = -\frac{x}{2} - \frac{3}{2}$ $\quad$ ex $(1,-1) \rightsquigarrow (-1,-1)$

$r_2: 2x+3y+5=0 \rightsquigarrow -2y+3x+5=0$ $\quad$ ex $(-1,-1) \rightsquigarrow (-1,1)$

(d) $r': 2x+3y+5=0$

$P = S^{-1}P'$ $\quad S^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \begin{cases} x' = -y \\ y' = x \end{cases} \quad \begin{cases} y = -x' \\ x = y' \end{cases}$ $\quad r: 2y-3x+5=0$

ex $(-1,-1) \rightsquigarrow (1,-1)$

(e) $c: x^2+y^2+3x-2y=10 \rightsquigarrow c': x^2+y^2-2x-3y=10$

(5) simmetria rispetto alla retta $x = -8$,

(a) $P' - A = S(P-A) \quad A \in r \rightsquigarrow P' = S(P-A) + A \quad S = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad A = (-8, 5)$

(b) $P' = S(P-A) + A = P \rightsquigarrow S(P-A) - I(P-A) = 0 \quad (S-I)(P-A) = \begin{pmatrix} -2 & 0 \\ 0 & 0 \end{pmatrix}(P-A) = 0$

$P - A = \begin{pmatrix} 0 \\ s \end{pmatrix} \rightsquigarrow P = \begin{pmatrix} -8 \\ 5 \end{pmatrix} \in r$

(c) $r_2 : y = 2x - 3 \quad \begin{cases} x' + 8 = -x - 8 \\ y' + 5 = y + 5 \end{cases} \begin{cases} x = -x' - 16 \\ y = y' \end{cases} \rightsquigarrow y = -2x - 32 - 3 \quad y = -2x - 35$

ex $(1, -1) \rightsquigarrow (-17, -1)$

$r_2 : 2x + 3y + 5 = 0 \rightsquigarrow -2x - 32 + 3y + 5 = 0 \quad -2x + 3y - 27 = 0$ ex $(-1, -1) \rightsquigarrow (-15, -1)$

(d) $r' : 2x + 3y + 5 = 0$

$P = S^{-1}(P' - A) + A \quad S^{-1} = S^{5} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = S \quad r : -2x + 3y - 27 = 0$

(e) $c : x^2 + y^2 + 3x - 2y = 10 \rightsquigarrow c' : (-x-16)^2 + y^2 + 3(-x-16) - 2y = 10$

$x^2 + 32x + 256 + y^2 - 3x - 48 - 2y = 10 \quad x^2 + y^2 + 29x - 2y = -198$

(6) simmetria centrale rispetto al punto $(3, 4)$,

(a) $P_0 = (3, 4) \quad \frac{P' + P}{2} = P_0 \rightsquigarrow P' = -P + 2P_0 \quad (S = -I)$

(b) $P' = -P + 2P_0 = P \rightsquigarrow 2P = 2P_0 \quad P = P_0$

(c) $r_2 : y = 2x - 3 \quad \begin{cases} x' = -x + 6 \\ y' = -y + 8 \end{cases} \begin{cases} x = -x' + 6 \\ y = -y' + 8 \end{cases} \rightsquigarrow -y + 8 = 2(-x+6) - 3 = -2x + 9$

$y = 2x - 1$ ex $(3, 3) \rightsquigarrow (3, 5)$

$r_2 : 2x + 3y + 5 = 0 \rightsquigarrow 2(-x+6) + 3(-y+8) + 5 = 0 \quad -2x - 3y + 41 = 0$ ex $(-1, -1) \rightsquigarrow (7, 9)$

(d) $r' : 2x + 3y + 5 = 0 \quad P = -P' + 2P_0$

$\rightsquigarrow r : -2x - 3y + 41 = 0 \quad (S^{-1} = S^{5} = S)$ ex $(7, 9) \rightsquigarrow (-1, -1)$

(e) $c : x^2 + y^2 + 3x - 2y = 10 \rightsquigarrow c' : (-x+6)^2 + (-y+8)^2 + 3(-x+6) - 2(-y+8) = 10$

$x^2 - 12x + 36 + y^2 - 16y + 64 - 3x + 18 + 2y - 16 = 10 \quad x^2 + y^2 - 15x - 14y = -92$

(7) simmetria rispetto alla retta $y = x$,

(a) $P' = SP \quad S = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \overset{\theta = 90}{=} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(b) $P' = SP = P \rightsquigarrow (S-I)P = 0$ $\quad \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} P = 0 \rightsquigarrow P = \begin{pmatrix} \sigma \\ \sigma \end{pmatrix}$ $\quad y = x$

(c) $r_2: y = 2x - 3$ $\quad \begin{cases} x' = x \\ y' = y \end{cases}$ $\quad x = 2y - 3$ $\quad y = \frac{1}{2}x + \frac{3}{2}$ $\quad$ ex $(2,1) \rightsquigarrow (1,2)$

$r_2: 2x + 3y + 5 = 0 \rightsquigarrow 3x + 2y + 5 = 0$ $\quad$ ex $(2,-3) \rightsquigarrow (-3,2)$

(d) $r': 2x + 3y + 5 = 0 \rightsquigarrow 3x + 2y + 5 = 0$ $\quad (S^{-1} = S^{\sigma} = S)$

(e) $c: x^2 + y^2 + 3x - 2y = 10 \rightsquigarrow c': x^2 + y^2 - 2x + 3y = 10$

(8) simmetria rispetto alla retta $y = 4$ seguita da simmetria rispetto alla retta $x = 3$,

(a) SIMM. $y = 5$: $P' = S'(P - A) + A$ $\quad A = (3,5)$ $\quad S' = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \overset{\theta=0}{=} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

SIMM $x = 3$: $P'' = S''(P' - A) + A$ $\quad S'' = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \overset{\theta=180}{=} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

COMPOSIZIONE: $P'' = S''[S'(P-A)] + A = S''S'(P-A) + A = S(P-A) + A$

$$S = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$$

$\rightsquigarrow P'' = -I(P - A) + A$ $\quad P'' = -P + 2A \equiv$ SIMM. CENTR. RISPETTO A

(b) (c) (d) (e) $\rightsquigarrow$ VdC P.TO "6"

(9) simmetria rispetto alla retta $y = 2x$,

(a) $P' = SP$ $\quad S = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ $\quad \theta = 2\alpha$ $\quad \sqrt{5}$, α, 2, 1 $\quad \begin{cases} \cos\alpha = \sqrt{5}/5 \\ \sin\alpha = 2\sqrt{5}/5 \end{cases}$

$$\begin{cases} \cos\theta = \cos 2\alpha = 2\cos^2\alpha - 1 = -3/5 \\ \sin\theta = \sin 2\alpha = 2\sin\alpha\cos\alpha = 4/5 \end{cases} \qquad S = \begin{pmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{pmatrix}$$

(b) $P' = SP = P \rightsquigarrow (S - I)P = 0$ $\quad \begin{pmatrix} -8/5 & 4/5 \\ 4/5 & -2/5 \end{pmatrix} P = 0 \rightsquigarrow P = \begin{pmatrix} \sigma \\ 2\sigma \end{pmatrix}$ $\quad y = 2x$

(c) $r_2: y = 2x - 3$ $\quad \begin{cases} x = -\frac{3}{5}x' + \frac{4}{5}y' \\ y = \frac{4}{5}x' + \frac{3}{5}y' \end{cases}$ $\quad \frac{4}{5}x + \frac{3}{5}y = -\frac{6}{5}x + \frac{8}{5}y - 3$

$5y = 10x + 15$ $\quad y = 2x+3 \quad$ ex $(0,-3) \rightsquigarrow (-12/5, -9/5)$

$r_2: 2x+3y+5=0 \rightsquigarrow 2\left(-\frac{3}{5}x+\frac{4}{5}y\right)+3\left(\frac{4}{5}x+\frac{3}{5}y\right)+5=0$

$-6x+8y+12x+9y+25=0 \quad 6x+17y+25=0 \quad$ ex $(-1,-1) \rightsquigarrow (-1/5, -7/5)$

(d) $r': 2x+3y+5=0 \rightsquigarrow 6x+17y+25=0 \quad (S^{-2}=S^{5}=S)$

(e) $c: x^2+y^2+3x-2y=10$

$\rightsquigarrow c': (-3x+4y)^2+(4x+3y)^2+15(-3x+4y)-10(4x+3y)=250$

$9x^2-\cancel{24xy}+16y^2+16x^2+\cancel{24xy}+9y^2-45x+60y-40x-30y=250$

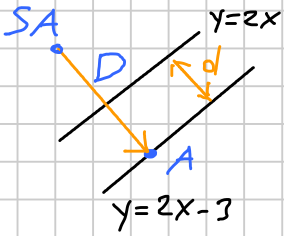
$25x^2+25y^2-95x+30y=250 \quad x^2+y^2-\frac{19}{5}x+\frac{6}{5}y=10$

(10) simmetria rispetto alla retta $y = 2x$ seguita da simmetria rispetto alla retta $y = 2x - 3$,

(a) SIMM. $y=2x$: $\quad P' = SP \quad S = \begin{pmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{pmatrix}$

SIMM. $y=2x-3$: $\quad P'' = S(P'-A)+A \quad A=(\delta, 2\delta-3)$

COMPOSIZIONE: $\quad P'' = S(SP-A)+A = S^2P - SA + A$



$S^2 = S \cdot S = S \cdot S^5 = S \cdot S^{-2} = I$

$P'' = P + A - SA \quad P'' = P + D \quad \equiv$ TRASL. DI VETT. D

$A=(0,-3) \quad SA=(-12/5, -9/5) \quad A-SA = (12/5, -6/5)$

$\|D\|^2 = \|A-SA\|^2 = (144+36)/25 \quad \|D\| = \|A-SA\| = \frac{6}{5}\sqrt{5}$

$d = |3/\sqrt{5}| = \frac{3}{5}\sqrt{5} = \|D\|/2$

(b) $P'' = P + D = P \rightsquigarrow$ NO P.TI FISSI

(c) $r_2: y = 2x-3 \quad \begin{cases} x'' = x + 12/5 \\ y'' = y - 6/5 \end{cases} \quad \begin{cases} x = x'' - 12/5 \\ y = y'' + 6/5 \end{cases} \quad y + \frac{6}{5} = 2\left(x - \frac{12}{5}\right) - 3$

$y = 2x - \frac{24}{5} - \frac{6}{5} - 3 \quad y = 2x - 9 \quad$ ex $(1,-1) \rightsquigarrow (17/5, -11/5)$

$r_2: 2x+3y+5=0 \rightsquigarrow 2(x-12/5)+3(y+6/5)+5=0 \quad 2x+3y-24/5+18/5+5=0$

$2x+3y+19/5=0 \quad$ ex $(-1,-1) \rightsquigarrow (7/5, -11/5)$

(d) $P = P'' - D \quad r': 2x+3y+5=0 \rightsquigarrow 2(x+12/5)+3(y-6/5)+5=0$

$2x+3y+24/5-18/5+5=0 \quad 2x+3y+31/5=0 \quad$ ex $(-1,-1) \rightsquigarrow (-17/5, 1/5)$

(e) $c: x^2+y^2+3x-2y=10$

$\rightsquigarrow c': (x-12/5)^2+(y+6/5)^2+3(x-12/5)-2(y+6/5)=10$

$$x^2-\frac{25}{5}x+\cancel{\frac{155}{25}}+y^2+\frac{12}{5}y+\cancel{\frac{36}{25}}+3x-\cancel{\frac{36}{5}}-2y-\frac{12}{5}=10$$

$$x^2+y^2-\frac{8}{5}x+\frac{2}{5}y=\frac{62}{5}$$

(11) rotazione di 30° in senso antiorario rispetto al punto $(-2, 3)$,

(a) $P'=S(P-P_0)+P_0 \quad P_0=(-2,3) \quad S=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \overset{\theta=\frac{\pi}{6}}{=} \begin{pmatrix}\sqrt{3}/2 & -1/2\\ 1/2 & \sqrt{3}/2\end{pmatrix}$

(b) $P'=S(P-P_0)+P_0=P \rightsquigarrow (S-I)(P-P_0)=0 \quad \begin{pmatrix}\sqrt{3}/2-1 & -1/2\\ 1/2 & \sqrt{3}/2-1\end{pmatrix}(P-P_0)=0 \quad P=P_0$

(c) $r_1: y=2x-3$

MODO 1

$P=(\delta,2\delta-3) \quad P-P_0=(\delta+2,2\delta-6) \quad P'-P_0=\left(\frac{\sqrt{3}}{2}\delta+\sqrt{3}-\delta+3,\ \delta/2+1+\delta\sqrt{3}-3\sqrt{3}\right)$

$P'=(\sqrt{3}+1,-3\sqrt{3}+5)+\delta\left(\frac{\sqrt{3}}{2}-1,\sqrt{3}+\frac{1}{2}\right) \quad y+3\sqrt{3}-5=(x-\sqrt{3}-1)\frac{2\sqrt{3}+1}{\cancel{\sqrt{3}-2}}\frac{(\sqrt{3}+2)}{\cancel{(\sqrt{3}+2)}}$ -1

$y=(x-\sqrt{3}-1)(-6-2-5\sqrt{3})+5-3\sqrt{3}=x(-5\sqrt{3}-8)+15+8\sqrt{3}+5\sqrt{3}+8+5-3\sqrt{3}$

$$y=x(-8-5\sqrt{3})+27+10\sqrt{3}$$

MODO 2

$$\begin{cases} V=(1,2) \rightsquigarrow V'=(\overset{a}{\frac{\sqrt{3}}{2}-1},\ \overset{b}{\frac{1}{2}+\sqrt{3}}) \\ P=(1,-1) \quad P-P_0=(3,-5) \rightsquigarrow P'-P_0=\left(3\frac{\sqrt{3}}{2}+2,\ \frac{3}{2}-2\sqrt{3}\right) \quad P'=\left(\overset{x_0}{\frac{3\sqrt{3}}{2}},\ \overset{y_0}{\frac{9}{2}-2\sqrt{3}}\right)\end{cases}$$

$y-y_0=(x-x_0)\frac{b}{a} \qquad y-y_0=\left(x-\frac{3\sqrt{3}}{2}\right)\frac{1+2\sqrt{3}}{\cancel{\sqrt{3}-2}}\frac{\sqrt{3}+2}{\cancel{\sqrt{3}+2}}=\left(x-\frac{3\sqrt{3}}{2}\right)(-8-5\sqrt{3})$ -1

$y=x(-8-5\sqrt{3})+12\sqrt{3}+\frac{45}{2}+\frac{9}{2}-2\sqrt{3} \qquad y=x(-8-5\sqrt{3})+27+10\sqrt{3}$

$r_2: 2x+3y+5=0$

$$\begin{cases} V=(3,-2) \rightsquigarrow V'=(\overset{a}{3\frac{\sqrt{3}}{2}+1},\ \overset{b}{\frac{3}{2}-\sqrt{3}}) \\ P=(-1,-1) \quad P-P_0=(1,-5) \rightsquigarrow P'-P_0=\left(\frac{\sqrt{3}}{2}+2,\ \frac{1}{2}-2\sqrt{3}\right) \quad P'=\left(\overset{x_0}{\frac{\sqrt{3}}{2}},\ \overset{y_0}{\frac{7}{2}-2\sqrt{3}}\right)\end{cases}$$

$y-y_0=(x-x_0)\frac{3-2\sqrt{3}}{\cancel{3\sqrt{3}+2}}\frac{3\sqrt{3}-2}{\cancel{3\sqrt{3}-2}}\ 23=(x-x_0)\frac{-25+13\sqrt{3}}{23}$

$$y = x\,\frac{-24+13\sqrt3}{23} + \frac{24\sqrt3-39}{46} + \frac{161-92\sqrt3}{46} \qquad y = x\,\frac{-24+13\sqrt3}{23} + \frac{\overset{61}{\cancel{122}}-\overset{34}{\cancel{68}}\sqrt3}{\cancel{46}\;23}$$

$$23y + (24-13\sqrt3)x + (-61+34\sqrt3) = 0$$

(d) $r': 2x+3y+5=0 \rightsquigarrow P = S^{-1}(P'-P_0)+P_0 \qquad S^{-1} = S^{5} = \begin{pmatrix}\sqrt3/2 & 1/2\\ -1/2 & \sqrt3/2\end{pmatrix}$

$$v' = (3,-2) \rightsquigarrow v = (\overset{a}{3\sqrt3/2 - 1}, \overset{b}{-3/2-\sqrt3})$$

$$P' = (-1,-1) \quad P-P_0 = (1,-4) \rightsquigarrow P'-P_0 = (\sqrt3/2-2, -1/2-2\sqrt3) \quad P' = \Big(\overset{x_0}{\tfrac{\sqrt3}{2}-4}, \overset{y_0}{\tfrac52-2\sqrt3}\Big)$$

$$-bx+ay+c=0 \qquad c = bx_0 - ay_0 = -\frac{3\sqrt3}{4}+6-\frac32+4\sqrt3-\frac{13\sqrt3}{4}+8+\frac52-2\sqrt3$$

$$c = 16 + \frac{-3+16-15-8}{4}\sqrt3 = 16-\frac52\sqrt3 \qquad (3+2\sqrt3)x + (3\sqrt3-2)y + (32-5\sqrt3) = 0$$

(e) $c: x^2+y^2+3x-2y=10$

$$(x-x_0)^2+(y-y_0)^2 = R^2 \rightsquigarrow x^2+y^2-2xx_0-2yy_0 = R^2-x_0^2-y_0^2$$

$$x_0 = -\frac32 \quad y_0 = 1 \quad R^2 = 10+\frac94+1 = \frac{53}{4}$$

$$C = \left(-\frac32, 1\right) \quad C-P_0 = \left(\frac12,-2\right) \rightsquigarrow C'-P_0 = \left(\frac{\sqrt3}{4}+1, \frac14-\sqrt3\right) \quad C' = \left(\frac{\sqrt3}{4}-1, \frac{13}{4}-\sqrt3\right)$$

$$x^2+y^2+\left(2-\frac{\sqrt3}{2}\right)x + \left(2\sqrt3-\frac{13}{2}\right) = \frac{53}{4}-\frac{3}{16}-1+\frac{\sqrt3}{2}-\frac{169}{16}-3+\frac{13}{2}\sqrt3$$

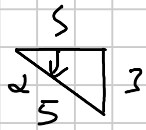
$$x^2+y^2+\left(2-\frac{\sqrt3}{2}\right)x + \left(2\sqrt3-\frac{13}{2}\right)y = \frac{212-3-16-169-48}{16}+7\sqrt3$$

$$c': x^2+y^2+\left(2-\frac{\sqrt3}{2}\right)x + \left(2\sqrt3-\frac{13}{2}\right)y = -\frac32+7\sqrt3$$

(12) simmetria rispetto alla retta di equazione $3x + 4y + 7 = 0$,

(a) $P' = S(P-A)+A \quad A \in r$

$$S = \begin{pmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{pmatrix} \qquad \theta = 2\alpha \qquad \begin{cases}\cos\alpha = 4/5\\ \sin\alpha = -3/5\end{cases}$$



$$\begin{cases}\cos\theta = \cos 2\alpha = 2\cos^2\alpha - 1 = 7/25\\ \sin\theta = \sin 2\alpha = 2\sin\alpha\cos\alpha = -24/25\end{cases} \qquad S = \begin{pmatrix}7/25 & -24/25\\ -24/25 & -7/25\end{pmatrix}$$

(b) $P' = S(P-A)+A = P \rightsquigarrow S(P-A)-(P-A)=0 \quad (S-I)(P-A)=0$

$$\begin{pmatrix} -18/25 & -24/25 \\ -24/25 & -32/25 \end{pmatrix}(P-A)=0 \qquad P-A = \delta(-4,3) \qquad P = A+\delta(-4,3) \in r$$

(c) $r_2: y = 2x-3 \qquad A=(-1,-1) \in r$

$$\begin{cases} v=(1,2) \rightsquigarrow v'=(-41,-38) \\ P=(1,-1) \quad P-A=(2,0) \rightsquigarrow P'-A = \left(\frac{14}{25}, -\frac{48}{25}\right) \quad P' = \left(-\frac{11}{25}, -\frac{73}{25}\right) \end{cases}$$

$$y+\frac{73}{25} = \left(x+\frac{11}{25}\right)\cdot\frac{38}{41} \qquad y = \frac{38}{41}x + \frac{418}{1025} - \frac{2993}{1025} \qquad y = \frac{38}{41}x - \frac{\overset{103}{2575}}{\underset{41}{\cancel{1025}}}$$

$$y = \frac{38}{41}x - \frac{103}{41}$$

$r_2: 2x+3y+5=0$

$v=(3,-2) \rightsquigarrow v'=(69,-58)$

$P=(-1,-1) \quad P-A=0 \rightsquigarrow P'-A=0 \quad P'=A=(-1,-1)$

$$y+1 = (x+1)\,\frac{-58}{69} \qquad 69y+69+58x+58 \qquad 58x+69y+127=0$$

(d) $r': 2x+3y+5=0 \qquad S^{-1}=S^{\delta}=S \rightsquigarrow r: 58x+69y+127=0$

(e) $c: x^2+y^2+3x-2y=10$

$(x-x_0)^2+(y-y_0)^2=R^2 \rightsquigarrow x^2+y^2-2xx_0-2yy_0 = R^2-x_0^2-y_0^2$

$x_0 = -\frac{3}{2} \quad y_0 = 1 \quad R^2 = 10+\frac{9}{4}+1 = \frac{53}{4}$

$C=\left(-\frac{3}{2},1\right) \quad C-A=\left(-\frac{1}{2},2\right) \rightsquigarrow C'-A = \left(-\frac{7}{50}-\frac{48}{25}, \frac{12}{25}-\frac{14}{25}\right) = \left(-\frac{103}{50}, \frac{-2}{25}\right)$

$C' = \left(-\frac{153}{50}, \frac{-27}{25}\right) \qquad x^2+y^2+\frac{153}{25}x+\frac{54}{25}y = \frac{53}{4} - \frac{153^2}{2500} - \frac{27^2}{625} = \frac{6800}{2500}$

$c': x^2+y^2+\frac{153}{25}x+\frac{54}{25}y = \frac{68}{25}$

(13) simmetria rispetto alla retta di equazione $3x + 4y + 7 = 0$ seguita da simmetria rispetto alla retta di equazione $3x - 4y + 11 = 0$,

(a) SIMM. $r': 3x+4y+7=0 \quad P' = S'(P-A') + A' \quad A' \in r' \quad S' = \begin{pmatrix} 7/25 & -24/25 \\ -24/25 & -7/25 \end{pmatrix}$ vd(12)

SIMM. $r'': 3x-4y+11=0 \quad P'' = S''(P'-A'') + A'' \quad A'' \in r'' \quad S'' = \begin{pmatrix} 7/25 & 24/25 \\ 24/25 & -7/25 \end{pmatrix}$

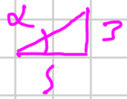
$r' \cap r'': \begin{cases} 3x+4y+7=0 \\ 3x-4y+11=0 \end{cases} \quad \begin{cases} 6x=-18 & x=-3 \\ 4y=2 & y=1/2 \end{cases} \rightsquigarrow A = A' = A'' = (-3, 1/2)$

COMPOSIZIONE: $P'' = S''[S'(P-A)] + A \qquad P'' = S(P-A) + A$

$$S = S''S' = \begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix} = \begin{pmatrix} \cos(-2\theta) & -\sin(-2\theta) \\ \sin(-2\theta) & \cos(-2\theta) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{49-576}{625} & -\frac{336}{625} \\ \frac{336}{625} & \frac{-576+49}{625} \end{pmatrix} = \begin{pmatrix} \frac{-527}{625} & -\frac{336}{625} \\ \frac{336}{625} & -\frac{527}{625} \end{pmatrix}$$

$\equiv$ ROTAZIONE ANTIORARIA DI ANGOLO $-2\theta = 4\alpha$



INTORNO AL P.TO A

(b) $P'' = S(P-A) + A = P \rightsquigarrow (S-I)(P-A) = 0 \quad P-A=0 \qquad P=A$

(c) $r_2: y = 2x-3 \qquad A = (-3, 1/2)$

$$\begin{cases} v = (1,2) \rightsquigarrow v' = (\overset{a}{1199}, \overset{b}{718}) \\ P = (1,-2) \quad P-A = (4, \frac{-3}{2}) \rightsquigarrow P''-A = \left(-\frac{1604}{625}, \frac{4269}{1250}\right) \end{cases} \qquad P'' = \left(\overset{x_0}{-\frac{3479}{625}}, \overset{y_0}{\frac{2447}{625}}\right)$$

$y - y_0 = (x - x_0)\, b/a$

$y - \frac{2447}{625} = \left(x + \frac{3479}{625}\right) \frac{718}{1199} \qquad y = \frac{718}{1199}x + \frac{3479}{625} \cdot \frac{718}{1199} + \frac{2447}{625}$

$y = \frac{718}{1199}x + \frac{8691}{1199}$

$r_2: 2x+3y+5=0$

$v = (3,-2) \rightsquigarrow v' = (-909, 2062)$

$P = (-1,-1) \qquad P-A = (2, -3/2) \rightsquigarrow P''-A = \left(-\frac{22}{25}, \frac{117}{50}\right) \qquad P'' = \left(\frac{-97}{25}, \frac{71}{25}\right)$

$y - \frac{71}{25} = \left(x + \frac{97}{25}\right) \frac{-2062}{909} \qquad 2062x + 909y - \frac{71 \cdot 909}{25} + \frac{2062 \cdot 97}{25} = 0$

$2062x + 909y + 5419 = 0$

(d) $r': 2x+3y+5=0$

$$P = S^{-1}(P'-A)+A \qquad S^{-1}=S^{5}= \begin{pmatrix} -\frac{527}{625} & \frac{336}{625} \\ -\frac{336}{625} & -\frac{527}{625}\end{pmatrix}$$

$$v=(3,-2) \rightsquigarrow v'=(-2253,56)$$

$$P=(-1,-1) \quad P-A=(2,-3/2) \rightsquigarrow P''-A=\left(-\frac{1558}{625},\frac{237}{1250}\right) \quad P''=\left(-\frac{3533}{625},\frac{531}{625}\right)$$

$$y-\frac{531}{625}=\left(x+\frac{3533}{625}\right)\frac{-56}{2253} \qquad 56x+2253y+\frac{-531\cdot 2253+56\cdot 3533}{625}=0$$

$$56x+2253y-1301=0$$

(e) $c: x^2+y^2+3x-2y=10$

$$(x-x_0)^2+(y-y_0)^2=R^2 \rightsquigarrow x^2+y^2-2xx_0-2yy_0=R^2-x_0^2-y_0^2$$

$$x_0=-\frac{3}{2} \quad y_0=1 \quad R^2=10+\frac{9}{4}+1=\frac{53}{4}$$

$$C=\left(-\frac{3}{2},1\right) \quad C-A=\left(\frac{3}{2},\frac{1}{2}\right) \rightsquigarrow C'-A=\left(-\frac{1317}{1250},\frac{581}{1250}\right) \quad C'=\left(-\frac{5667}{1250},\frac{553}{625}\right)$$

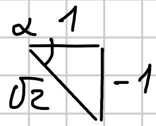
$$x^2+y^2+\frac{5667}{625}x-\frac{1106}{625}y=\frac{53}{4}-\left(\frac{5667}{1250}\right)^2-\left(\frac{553}{625}\right)^2=-\frac{5055}{625}$$

$$c': x^2+y^2+\frac{5667}{625}x-\frac{1106}{625}y=\frac{-5055}{625}$$

(14) simmetria rispetto alla retta $y+x=0$ seguita da traslazione di vettore $(2,-2)$,

(a) $P'=SP+w \quad w=(2,-2)$

$$S=\begin{pmatrix}\cos\theta & \sin\theta \\ \sin\theta & -\cos\theta\end{pmatrix} \quad \theta=2\alpha=-\frac{\pi}{2}$$



$$S=\begin{pmatrix}0 & -1 \\ -1 & 0\end{pmatrix}$$

(b) $P'=SP+w=P \rightsquigarrow (S-I)P=-w \quad \begin{pmatrix}-1 & -1 \\ -1 & -1\end{pmatrix}P=\begin{pmatrix}-2 \\ 2\end{pmatrix}$ NESSUNA SOLUZIONE

(c) $r_2: y=2x-3$

$$\begin{cases} v=(1,2) \rightsquigarrow v'=Sv=(-2,-1)\equiv(\overset{a}{2},\overset{b}{1}) \\ P=(1,-1) \rightsquigarrow P'=(1,-1)+(2,-2)=(\overset{x_0}{3},\overset{y_0}{-3}) \end{cases}$$

$$y+3=(x-3)\frac{1}{2} \quad y=\frac{1}{2}x-\frac{3}{2}-3 \quad y=\frac{1}{2}x-\frac{9}{2}$$

$r_2: 2x+3y+5=0$

$$\begin{cases} v=(3,-2) \rightsquigarrow v' = Sv = \begin{array}{c} \scriptstyle a \quad b \\ (2,-3) \end{array} \\ P=(-1,-1) \rightsquigarrow P' = (1,1)+(2,-2) = \begin{array}{c} \scriptstyle x_0 \quad y_0 \\ (3,-1) \end{array} \end{cases}$$

$y+1 = (x-3)\frac{-3}{2}$ $\quad 2y+2 = -3x+9$ $\quad 3x+2y-7=0$

(d) $r': 2x+3y+5=0$

$P = S^{-1}(P'-W)$ $\quad S^{-1} = S^T = S$

$$\begin{cases} v'=(3,-2) \rightsquigarrow v' = Sv = \begin{array}{c} \scriptstyle a \quad b \\ (2,-3) \end{array} \\ P'=(-1,-1) \rightsquigarrow P'-W = (-3,1) \quad P' = \begin{array}{c} \scriptstyle x_0 \quad y_0 \\ (-1,3) \end{array} \end{cases}$$

$y-3 = (x+1)\frac{-3}{2}$ $\quad 2y-6 = -3x-3$ $\quad 3x+2y-3=0$

(e) $c: x^2+y^2+3x-2y=10$

$(x-x_0)^2+(y-y_0)^2=R^2 \rightsquigarrow x^2+y^2-2xx_0-2yy_0 = R^2-x_0^2-y_0^2$

$x_0 = -\frac{3}{2}$ $\quad y_0 = 1$ $\quad R^2 = 10+\frac{9}{4}+1 = \frac{53}{4}$

$C = \left(-\frac{3}{2}, 1\right) \rightsquigarrow C' = \left(-1, \frac{3}{2}\right)+(2,-2) = (1, -1/2)$

$c': x^2+y^2-2x+y = \frac{53}{4}-1-\frac{1}{4}$ $\quad x^2+y^2-2x+y = 12$

(15) simmetria rispetto alla retta di equazione $3x - 4y - 7 = 0$ seguita da rotazione antioraria di 90° rispetto al punto $(1, 2)$,

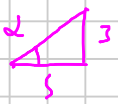
(a) SIMM. $r: 3x-4y-7=0$ $\quad P' = S'(P-A)+A$ $\quad A \in r$ $\quad S' = \begin{pmatrix} 7/25 & 24/25 \\ 24/25 & -7/25 \end{pmatrix}$ vd. 13

ROTAZIONE $\theta = \frac{\pi}{2}$ $B=(1,2)$: $P'' = S''(P'-B)+B$ $\quad S'' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

COMPOSIZ.: $P'' = S''[S'(P-A)+A-B]+B = S''S'(P-A)+S''(A-B)+B$

$$S = S''S' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ \operatorname{sen}\theta & -\cos\theta \end{pmatrix} = \begin{pmatrix} -\operatorname{sen}\theta & \cos\theta \\ \cos\theta & \operatorname{sen}\theta \end{pmatrix} = \begin{pmatrix} \cos(\frac{\pi}{2}+\theta) & \operatorname{sen}(\frac{\pi}{2}+\theta) \\ \operatorname{sen}(\frac{\pi}{2}+\theta) & -\cos(\frac{\pi}{2}+\theta) \end{pmatrix}$$

$S = \begin{pmatrix} -24/25 & 7/25 \\ 7/25 & 24/25 \end{pmatrix}$ $\equiv$ SIMMETRIA RISPETTO A RETTA CHE FORMA UN ANGOLO "$\frac{\pi}{4}+\frac{\theta}{2}$" CON L'ORIGINE



$2\theta = \alpha$

$SP = P \rightsquigarrow (S-I)P = \begin{pmatrix} -49/25 & 7/25 \\ 7/25 & -1/25 \end{pmatrix} P = 0 \rightsquigarrow P = (t, 7t)$

RETTA DI SIMM. PER L'ORIGINE $r_0 : y = 7x$

$$P'' = \underbrace{[SP]}_{\text{SIMM. } r_0} + \underbrace{[-SA + S''(A-B) + B]}_{\text{TRASLAZIONE } W} \qquad P'' = SP + W$$

$$\begin{cases} A = (1,-1) \quad SA = \left(\frac{-31}{25}, \frac{-17}{25}\right) \quad B = (1,2) \quad A-B = (0,-3) \quad S''(A-B) = (3,0) \\ W = \left(\frac{31}{25} + 3 + 1, \ \frac{17}{25} + 0 + 2\right) = \left(\frac{131}{25}, \frac{67}{25}\right) \end{cases}$$

RISPETTO A r_0: $W = W_{//} + W_{\perp}$

$$r_0 \begin{cases} v = (5, 75) \quad \frac{v}{\|v\|} = \left(\frac{1}{5\sqrt2}, \frac{7}{5\sqrt2}\right) \quad \langle W, \frac{v}{\|v\|} \rangle = \frac{131}{125\sqrt2} + \frac{469}{125\sqrt2} = \frac{600}{125\sqrt2} = \frac{24}{5\sqrt2} \\ n = (-75, 5) \quad \frac{n}{\|n\|} = \left(\frac{-7}{5\sqrt2}, \frac{1}{5\sqrt2}\right) \quad \langle W, \frac{n}{\|n\|} \rangle = \frac{-917}{125\sqrt2} + \frac{67}{125\sqrt2} = \frac{-850}{125\sqrt2} = \frac{-34}{5\sqrt2} \end{cases}$$

$$\rightsquigarrow W = \begin{cases} W_{//} = \frac{24}{5\sqrt2} v = \left(\frac{12}{25}, \frac{84}{25}\right) \\ W_{\perp} = \frac{-34}{5\sqrt2} n = \left(\frac{119}{25}, \frac{-17}{25}\right) \end{cases} \qquad P'' = SP + W_{\perp} + W_{//}$$

CONSIDERIAMO IL TERMINE: $SP + W_{\perp}$

$$P_0 \in r_0 \quad P = P_0 + \frac{W_\perp}{2} \rightsquigarrow S\left(P_0 + \frac{W_\perp}{2}\right) + W_\perp = P_0 + \frac{W_\perp}{2}$$

PERTANTO: $P'' = S\left(P - \frac{W_\perp}{2}\right) + \frac{W_\perp}{2} + W_{//}$

= SIMM. RISPETTO A $r // r_0$ (PASSANTE PER $\frac{W_\perp}{2}$)

+ TRASLAZIONE $// r_0$ DI VETTORE $W_{//}$

(b) VERIFICHIAMO CHE NON ESISTONO PUNTI FISSI

$$P'' = SP + W = P \rightsquigarrow (S - I)P = -W \quad \begin{pmatrix} -49/25 & 7/25 \\ 7/25 & -1/25 \end{pmatrix} P = \begin{pmatrix} 131/25 \\ 67/25 \end{pmatrix}$$

$$\begin{pmatrix} -49 & 7 \\ 0 & 0 \end{pmatrix} P = \begin{pmatrix} 131 \\ 569 \end{pmatrix} \rightsquigarrow \text{NESSUNA SOLUZIONE}$$

(c) $r_1 : y = 2x - 3$

$$\begin{cases} v = (1,2) \rightsquigarrow v' = (-10/25, 55/25) \equiv (\overset{a}{-2}, \overset{b}{11}) \\ P = (1,-1) \rightsquigarrow P' = (-31/25, -17/25) + (131/25, 67/25) = (\overset{x_0}{4}, \overset{y_0}{2}) \end{cases}$$

$$y - 2 = (x - 4)\frac{-11}{2} \qquad y = -\frac{11}{2}x + 22 + 2 \qquad y = -\frac{11}{2}x + 24$$

$r_2: 2x+3y+5=0$

$$\begin{cases} v=(3,-2) \rightsquigarrow v'=(-86/25,\,-27/25) \equiv (\overset{a}{86},\overset{b}{27}) \\ P=(-1,-1) \rightsquigarrow P'=(17/25,\,-31/25)+(131/25,\,67/25)=(\overset{x_0}{158/25},\,\overset{y_0}{36/25}) \end{cases}$$

$$y-\frac{36}{25}=\left(x-\frac{158}{25}\right)\frac{27}{86} \qquad 27x-86y+\frac{86\cdot 36-158\cdot 27}{25}=0 \qquad 27x-86y-36=0$$

(d) $r': 2x+3y+5=0$

$$P=S^{-1}(P'-w) \qquad S^{-1}=S^T=S=\begin{pmatrix} -24/25 & 7/25 \\ 7/25 & 24/25 \end{pmatrix}$$

$$v'=(3,-2) \rightsquigarrow v'=(-86/25,\,-27/25) \equiv (\overset{a}{86},\overset{b}{27})$$

$$P'=(-1,-1) \rightsquigarrow P'-w=\left(-1-\frac{131}{25},\,-1-\frac{67}{25}\right)=(-156/25,\,-92/25)$$

$$P=\left(\frac{156\cdot 24-92\cdot 7}{625},\,\frac{-156\cdot 7-92\cdot 24}{625}\right)=\left(\overset{x_0}{\frac{124}{25}},\,\overset{y_0}{-\frac{132}{25}}\right)$$

$$y+\frac{132}{25}=\left(x-\frac{124}{25}\right)\frac{27}{86} \qquad y=\frac{27}{86}x-\frac{132\cdot 86+124\cdot 27}{25\cdot 86} \qquad y=\frac{27}{86}x-\frac{294}{43}$$

(e) $c: x^2+y^2+3x-2y=10$

$$(x-x_0)^2+(y-y_0)^2=R^2 \rightsquigarrow x^2+y^2-2xx_0-2yy_0=R^2-x_0^2-y_0^2$$

$$x_0=-\frac{3}{2} \qquad y_0=1 \qquad R^2=10+\frac{9}{4}+1=\frac{53}{4}$$

$$C=\left(-\frac{3}{2},1\right) \rightsquigarrow C'=\left(36/25+7/25,\,-21/50+24/25\right)+(131/25,\,67/25)=$$

$$=\left(43/25+131/25,\,27/50+134/50\right)=\left(\frac{174}{25},\,\frac{161}{50}\right)$$

$$c': x^2+y^2-\frac{348}{25}x-\frac{161}{25}y=\frac{53}{4}-\left(\frac{174}{25}\right)^2-\left(\frac{161}{50}\right)^2$$

$$x^2+y^2-\frac{348}{25}x-\frac{161}{25}y=\frac{-1138}{25}$$

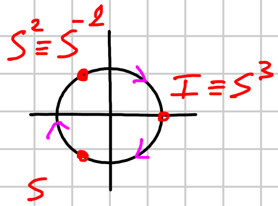
(16) rotazione oraria di 120° rispetto al punto (1, 2), seguita da rotazione oraria di 120° rispetto al punto (3, −2), seguita da rotazione oraria di 120° rispetto al punto (7, 1).

(a) ROTAZIONE $P_1=(1,2)$ $\quad P' = S(P-P_1)+P_1 \quad S=\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix} = \begin{pmatrix}-1/2 & \sqrt3/2 \\ -\sqrt3/2 & -1/2\end{pmatrix}$ $\quad \theta=-120°$

ROTAZIONE $P_2=(3,-2)$ $\quad P''=S(P'-P_2)+P_2$

ROTAZIONE $P_3=(7,1)$ $\quad P'''=S(P''-P_3)+P_3$

COMPOSIZIONE: $P'''=S\{S[S(P-P_1)+P_1-P_2]+P_2-P_3\}+P_3 =$



$$= S^2[S(P-P_1)+P_1-P_2]+S(P_2-P_3)+P_3 =$$

$$= S^3(P-P_1)+S^2(P_1-P_2)+S(P_2-P_3)+P_3$$

$S^3=I \rightsquigarrow S^2=S^{-1}=S^{\sigma}$

$$= P+S^{\sigma}(P_1-P_2)+S(P_2-P_3)+(P_3-P_1)$$

$\rightsquigarrow P'''=P+w \equiv$ TRASLAZIONE DI VETTORE w

$$\begin{cases} P_1-P_2=(-2,4) \rightsquigarrow S^{\sigma}(P_1-P_2)=\begin{pmatrix}-1/2 & -\sqrt3/2 \\ \sqrt3/2 & -1/2\end{pmatrix}\begin{pmatrix}-2 \\ 4\end{pmatrix}=(1-2\sqrt3,-\sqrt3-2) \\ P_2-P_3=(-4,-3) \rightsquigarrow S(P_2-P_3)=\begin{pmatrix}-1/2 & \sqrt3/2 \\ -\sqrt3/2 & -1/2\end{pmatrix}\begin{pmatrix}-4 \\ -3\end{pmatrix}=(2-3\sqrt3/2,2\sqrt3+3/2) \\ P_3-P_1=(6,-1) \rightsquigarrow w=(9-7\sqrt3/2,-3/2+\sqrt3) \end{cases}$$

(b) NON ESISTONO P.TI FISSI : $P'''=P+w=P \quad (w\neq 0)$

(c) $r_1: y=2x-3 \qquad \begin{cases} x'=x+9-7\sqrt3/2 \\ y'=y-\frac32+\sqrt3 \end{cases} \qquad \begin{cases} x=x'-9+7\sqrt3/2 \\ y=y'+3/2-\sqrt3 \end{cases}$

$\rightsquigarrow y+\frac32-\sqrt3=2x-18+7\sqrt3-3 \qquad y=2x-45/2+8\sqrt3$

$r_2: 2x+3y+5=0 \rightsquigarrow 2x-18+7\sqrt3+3y+9/2-3\sqrt3+5=0$

$$2x+3y-17/2+4\sqrt3=0$$

(d) $r': 2x+3y+5=0 \qquad P=P'''-w$

$\rightsquigarrow 2x+18-7\sqrt3+3y-9/2+3\sqrt3+5=0 \qquad 2x+3y+37/2-4\sqrt3=0$

(e) $c: x^2+y^2+3x-2y=10$

$$(x-x_0)^2+(y-y_0)^2=R^2 \rightsquigarrow x^2+y^2-2xx_0-2yy_0=R^2-x_0^2-y_0^2$$

$$x_0=-\frac{3}{2} \quad y_0=1 \quad R^2=10+\frac{9}{4}+1=\frac{53}{4}$$

$$C=\left(-\frac{3}{2},1\right) \rightsquigarrow C'=\left(-\frac{3}{2}+8-7\sqrt{3}/2,\ 1-3/2+\sqrt{3}\right)=\left(13/2-7\sqrt{3}/2,\ -1/2+\sqrt{3}\right)$$

$$c': x^2+y^2+(-15+7\sqrt{3})x+(1-2\sqrt{3})y=\frac{53}{4}-\frac{225}{4}-\frac{147}{4}+\frac{210\sqrt{3}}{4}-\frac{1}{4}-3+\sqrt{3}$$

$$x^2+y^2+(-15+7\sqrt{3})x+(1-2\sqrt{3})y=-83+\frac{107\sqrt{3}}{2}$$