Forme quadratiche 2

Argomenti: segnatura di forme quadratiche **Difficoltà:** ⋆⋆⋆

Prerequisiti: criteri per la segnatura (completamento quadrati, Sylvester, Cartesio)

1. Consideriamo le seguenti forme quadratiche $q(x, y)$ in $\mathbb{R}^2$:
$$x^2 + 2y^2 + axy, \qquad x^2 + ay^2 + 5xy, \qquad ax^2 - 2axy - 3y^2, \qquad -y^2 + axy.$$
Determinare, per ciascuna di esse, per quali valori del parametro reale a risulta
   (a) definita positiva,
   (b) indefinita,
   (c) nulla sul sottospazio $V = \{(x, y) \in \mathbb{R}^2 : x + 3y = 0\}$,
   (d) definita positiva sul sottospazio V di cui al punto precedente.

2. Consideriamo le seguenti forme quadratiche $q(x, y, z)$ in $\mathbb{R}^3$:
$$x^2 + 2y^2 + 3z^2 + axz, \qquad x^2 + ay^2 + 3z^2 + 6xy - yz, \qquad y^2 + axz - 4ayz.$$
Determinare, per ciascuna di esse, per quali valori del parametro reale a risulta
   (a) definita positiva,
   (b) indefinita,
   (c) indefinita, ma definita negativa su almeno un sottospazio di dimensione 2,
   (d) nulla su almeno un sottospazio di dimensione 2,
   (e) nulla sul sottospazio generato da $(1, 2, 3)$,
   (f) definita positiva su almeno un sottospazio di dimensione 2,
   (g) definita negativa su almeno un sottospazio di dimensione 1,
   (h) definita positiva sul sottospazio generato da $(1, 1, 3)$ e $(0, 2, 1)$.

3. Determinare, al variare del parametro reale a, la segnatura delle seguenti forme quadratiche $q(x, y, z, w)$ in $\mathbb{R}^4$:
$$\begin{array}{cc} x^2 - y^2 + az^2 - w^2 & x^2 + ay^2 - w^2 \\ ax^2 - y^2 - (a+3)z^2 - w^2 & -x^2 - ay^2 + (a+4)z^2 - (2a+1)w^2 \\ x^2 - y^2 + 2z^2 - w^2 - 2xz + ayw & az^2 + 2y^2 + 3z^2 - x^2 + 2axy + 2yw \\ 2x^2 + 3y^2 + 4w^2 + axz & x^2 + ay^2 + w^2 + 2yz \\ ax^2 + w^2 + 2ayz + 2xz & x^2 + ay^2 + 3z^2 + 4w^2 + 2xz + 4zw + 2ayz \end{array}$$

4. Consideriamo la seguente forma quadratica in $\mathbb{R}^3$:
$$q(x, y, z) = ax^2 + 2y^2 + 4z^2 + 2xy + byz.$$
Determinare per quali valori dei parametri reali a e b la forma risulta semidefinita positiva e nulla sul sottospazio generato da $(-2, 2, 1)$.

SOLUZIONI?

BY GIMUSI 18/01/15

1. Consideriamo le seguenti forme quadratiche $q(x,y)$ in $\mathbb{R}^2$:

1) x^2+2y^2+axy, 2) x^2+ay^2+5xy, 3) $ax^2-2axy-3y^2$, 4) $-y^2+axy$.

Determinare, per ciascuna di esse, per quali valori del parametro reale a risulta

(a) definita positiva,

(b) indefinita,

(c) nulla sul sottospazio $V=\{(x,y)\in\mathbb{R}^2 : x+3y=0\}$,

(d) definita positiva sul sottospazio V di cui al punto precedente.

1) $x^2+2y^2+axy \rightsquigarrow \begin{pmatrix} 1 & a/2 \\ a/2 & 2 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & a/2 \\ a/2 & 2-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)(2-\lambda)-a^2/4 = \\ \lambda^2-3\lambda+2-a^2/4=0 \end{cases}$

$$\lambda_{1,2} = \frac{3\pm\sqrt{9-8+a^2}}{2} = \frac{3\pm\sqrt{1+a^2}}{2}$$

(a) $\rightsquigarrow 3-\sqrt{1+a^2}>0$ $\sqrt{1+a^2}<3$ $1+a^2<9$ $a^2<8$ $-2\sqrt{2}<a<2\sqrt{2}$

(b) $\rightsquigarrow 3-\sqrt{1+a^2}<0$ $a\in\{(-\infty,-2\sqrt{2})\cup(2\sqrt{2},+\infty)\}$

(c) $v=(-3\delta,\delta)\in V$ $q(\delta)=9\delta^2+2\delta^2-3a\delta^2=(11-3a)\delta^2=0$ $\begin{cases}\delta=0\\ a=11/3\end{cases}$

(d) $q(\delta)=(11-3a)\delta^2>0 \rightsquigarrow 11-3a>0$ $a<11/3$

2) $x^2+ay^2+5xy \rightsquigarrow \begin{pmatrix} 1 & 5/2 \\ 5/2 & a \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 5/2 \\ 5/2 & a-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)(a-\lambda)-25/4 = \\ =\lambda^2-(a+1)\lambda+a-25/4=0 \end{cases}$

$$\lambda_{1,2} = \frac{(a+1)\pm\sqrt{(a+1)^2-4a+25}}{2}$$

(a) $\left.\begin{array}{l} \lambda_1\cdot\lambda_2 = DET(A) = a-25/4>0 \rightsquigarrow a>25/4 \\ \lambda_1+\lambda_2 = TR(A) = a+1>0 \rightsquigarrow a>-1 \end{array}\right\} \rightsquigarrow a>25/4$

(b) $DET(A) = a-25/4<0 \rightsquigarrow a<25/4$

(c) $v=(-3\delta,\delta)\in V$ $q(\delta)=9\delta^2+a\delta^2-15\delta^2=(a-6)\delta^2=0$ $\begin{cases}\delta=0\\ a=6\end{cases}$

(d) $q(\delta)=(a-6)\delta^2>0$ $a>6$

3) $ax^2-2axy-3y^2 \rightsquigarrow \begin{pmatrix} a & -a \\ -a & -3 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} a-\lambda & -a \\ -a & -3-\lambda \end{vmatrix} = \lambda^2+(3-a)\lambda-3a-a^2$

$$\lambda_{1,2} = \frac{(a-3)\pm\sqrt{(a-3)^2+12a+4a^2}}{2}$$

(a) $\lambda_1 \cdot \lambda_2 = DET(A) = -3a - a^2 > 0 \rightsquigarrow -3 < a < 0$

$\lambda_1 + \lambda_2 = TR(A) = a - 3 > 0 \rightsquigarrow a > 3$ } IMPOSSIBILE

(b) $DET(A) = -3a - a^2 < 0 \rightsquigarrow a \in \{(-\infty, -3) \cup (0, +\infty)\}$

(c) $v = (-3b, b) \in V \quad q(b) = 9ab^2 + 6ab^2 - 3b^2 = (15a - 3) b^2 = 0 \quad a = 1/5$

(d) $q(b) = (15a - 3) b^2 > 0 \quad a > 1/5$

5) $-y^2 + axy \rightsquigarrow \begin{pmatrix} 0 & a/2 \\ a/2 & -1 \end{pmatrix} \quad |A - \lambda I| = \begin{vmatrix} -\lambda & a/2 \\ a/2 & -1-\lambda \end{vmatrix} = \lambda^2 + \lambda - a^2/4 = 0$

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1 + a^2}}{2}$$

(a) $\lambda_1 = \dfrac{-1 - \sqrt{1 + a^2}}{2} < 0$ IMPOSSIBILE

$(\lambda_1 \cdot \lambda_2 = DET(A) = -a^2/4 \,, \ \lambda_1 + \lambda_2 = TR(A) = -1)$

(b) $DET(A) = -a^2/4 < 0 \quad a \neq 0 \quad \left[q(v) = -\left(y - \frac{ax}{2}\right)^2 + \frac{a^2 x}{4} \right]$

(c) $v = (-3b, b) \quad q(b) = -b^2 - 3ab^2 = -(1 + 3a) b^2 \quad a = -1/3$

(d) $q(b) = -(1 + 3a) b^2 > 0 \quad 1 + 3a < 0 \quad a < -1/3$

2. Consideriamo le seguenti forme quadratiche $q(x, y, z)$ in $\mathbb{R}^3$:

1) $x^2 + 2y^2 + 3z^2 + axz$, 2) $x^2 + ay^2 + 3z^2 + 6xy - yz$, 3) $y^2 + axz - 4ayz$.

Determinare, per ciascuna di esse, per quali valori del parametro reale a risulta

(a) definita positiva,
(b) indefinita,
(c) indefinita, ma definita negativa su almeno un sottospazio di dimensione 2,
(d) nulla su almeno un sottospazio di dimensione 2,
(e) nulla sul sottospazio generato da $(1, 2, 3)$,
(f) definita positiva su almeno un sottospazio di dimensione 2,
(g) definita negativa su almeno un sottospazio di dimensione 1,
(h) definita positiva sul sottospazio generato da $(1, 1, 3)$ e $(0, 2, 1)$.

1) $x^2+2y^2+3z^2+axz \rightsquigarrow \begin{pmatrix}1&0&a/2\\0&2&0\\a/2&0&3\end{pmatrix}$ $\quad |A-\lambda I| = \begin{vmatrix}1-\lambda&0&a/2\\0&2-\lambda&0\\a/2&0&3-\lambda\end{vmatrix} =$

$=(1-\lambda)(2-\lambda)(3-\lambda)-\frac{a^2}{4}(2-\lambda)=(2-\lambda)\left(\lambda^2-5\lambda+6-a^2/4\right)=0$

$\lambda_1=2 \quad \lambda_{2,3}=\frac{5\pm\sqrt{25-24+a^2}}{2}=\frac{5\pm\sqrt{1+a^2}}{2}$

(a) $\lambda_3>0 \quad 5-\sqrt{1+a^2}>0 \quad 1+a^2<25 \quad a^2<24 \quad -2\sqrt{6}<a<2\sqrt{6}$

IN ALTERNATIVA $\quad DET(A)>0 \rightsquigarrow 6-a^2/4>0 \quad a^2<24$

(b) $\lambda_3<0 \quad a\in\{(-\infty,-2\sqrt{6})\cup(2\sqrt{6},+\infty)\}$

(c) IMPOSSIBILE (AL PIU SOLO $\lambda_3<0 \rightsquigarrow$ MAX DIM $=1$)

(d) IMPOSSIBILE (AL PIU SOLO $\lambda_3=0 \rightsquigarrow$ MAX DIM $=1$)

(e) $(x,y,z)=\delta(1,2,3) \rightsquigarrow q(\delta)=\delta^2+8\delta^2+27\delta^2+3a\delta^2=0$
$\rightsquigarrow (36+3a)\delta^2=0 \qquad a=-12$

(f) $\lambda_1>0 \quad \lambda_2>0 \quad \forall a\in\mathbb{R}$

(g) $\lambda_3<0 \quad a\in\{(-\infty,-2\sqrt{6})\cup(2\sqrt{6},+\infty)\}$

(h) $(x,y,z)=\delta(1,1,3)+s(0,2,1) = (\delta,\delta+2s,3\delta+s)$

$\rightsquigarrow q(\delta,s)=\delta^2+2(\delta+2s)^2+3(3\delta+s)^2+a\delta(3\delta+s)=$

$=\delta^2+2(\delta^2+4s^2+4s\delta)+3(9\delta^2+s^2+6s\delta)+3a\delta^2+as\delta=$

$=\delta^2+2\delta^2+8s^2+8s\delta+27\delta^2+3s^2+18s\delta+3a\delta^2+as\delta=$

$=(30+3a)\delta^2+11s^2+(26+a)s\delta$

$\begin{vmatrix}30+3a & 13+a/2\\ 13+a/2 & 11\end{vmatrix}>0 \qquad 330+33a-169-\frac{a^2}{4}-13a=$
$=-\frac{a^2}{4}+20a+161=-a^2+80a+644$

$a=\frac{-80\pm\sqrt{6400+2576}}{-2}=40\pm2\sqrt{561}$

$DET(A)>0 \rightsquigarrow \quad 40-2\sqrt{561}<a<40+2\sqrt{561}$

2) $x^2 + a y^2 + 3z^2 + 6xy - yz \rightsquigarrow \begin{pmatrix} 1 & 3 & 0 \\ 3 & a & -1/2 \\ 0 & -1/2 & 3 \end{pmatrix} \quad |A - \lambda I| = \begin{vmatrix} 1-\lambda & 3 & 0 \\ 3 & a-\lambda & -1/2 \\ 0 & -1/2 & 3-\lambda \end{vmatrix} =$

$= (1-\lambda)(a-\lambda)(3-\lambda) - \frac{1}{4}(1-\lambda) - 9(3-\lambda)$

COMPL. QUADRATI: $(x+3y)^2 - 9y^2 + ay^2 + 3\left(z - \frac{y}{6}\right)^2 - \frac{y^2}{12} =$

$= (x+3y)^2 + 3\left(z - \frac{y}{6}\right)^2 + \left(a - 9 - \frac{1}{12}\right) y^2 =$

$= (x+3y)^2 + 3\left(z - \frac{y}{6}\right)^2 + \left(\frac{12a - 109}{12}\right) y^2$

(a) $12a - 109 > 0 \rightsquigarrow a > 109/12$

(b) $a < 109/12$

(c) IMPOSSIBILE

(d) IMPOSSIBILE

(e) $(x,y,z) = \sigma(1,2,3) \rightsquigarrow q(\sigma) = \sigma^2 + 4a\sigma^2 + 27\sigma^2 + 12\sigma^2 - 6\sigma^2 =$

$= \sigma^2(34 + 4a) = 0 \rightsquigarrow a = -17/2$

(f) $\forall a \in \mathbb{R}$

(g) $a < 109/12 \quad V = \sigma(0,1,0)$

(h) $(x,y,z) = \sigma(1,1,3) + s(0,2,1) = (\sigma, \sigma + 2s, 3\sigma + s)$

$\rightsquigarrow q(\sigma, s) = \sigma^2 + a(\sigma+2s)^2 + 3(3\sigma+s)^2 + 6\sigma(\sigma+2s) - (\sigma+2s)(3\sigma+s) =$

$= \sigma^2 + a\sigma^2 + 4as^2 + 4as\sigma + 27\sigma^2 + 3s^2 + 18s\sigma + 6\sigma^2 + 12s\sigma - 3\sigma^2 - s\sigma +$

$- 6s\sigma - 2s^2 = (31+a)\sigma^2 + (4a+1)s^2 + (4a+23)s\sigma$

$\begin{vmatrix} 31+a & 2a+23/2 \\ 2a+23/2 & 4a+1 \end{vmatrix} > 0 \qquad (31+a)(4a+1) - (2a + 23/2)^2 =$

$= \cancel{4a^2} + 125a + 31 - \cancel{4a^2} - 46a - \frac{529}{4} =$

$= 79a + \frac{124 - 529}{4} = 79a - \frac{405}{4} > 0 \rightsquigarrow a > \frac{405}{316}$

oss. $31 + a > 0 \rightsquigarrow a > -31$

3) $y^2+axz-4ayz \rightsquigarrow \begin{pmatrix}0&0&a/2\\0&1&-2a\\a/2&-2a&0\end{pmatrix}$ $\quad |A-\lambda I| = \begin{vmatrix}-\lambda&0&a/2\\0&1-\lambda&-2a\\a/2&-2a&-\lambda\end{vmatrix} =$

$= \lambda^2(1-\lambda) - \frac{a^2}{4}(1-\lambda) + 4a^2\lambda = -\lambda^3+\lambda^2+\left(4a^2+\frac{a^2}{4}\right)\lambda - \frac{a^2}{4} = 0$

COMPL. QUADRATI: $(y-2az)^2-4a^2z^2+axz = (y-2az)^2-\left(2az-\frac{x}{4}\right)^2+\frac{x^2}{16}$

VER: $y^2+\cancel{4a^2z^2}-4ayz-\cancel{4a^2z^2}-\cancel{\frac{x^2}{16}}+azx+\cancel{\frac{x^2}{16}}$

(a) $a=0 \rightsquigarrow q(v)=q(y)=y^2$

(b) $a\neq 0 \rightsquigarrow$ CARTESIO VPV $n_0=0$ $n_+=2$ $n_-=1$

(c) IMPOSSIBILE

(d) $a=0 \quad q(v)=0 \quad \forall\ v=\delta(1,0,0)+s(0,0,1)$

(e) $v=\delta(1,2,3) \rightsquigarrow q(v)=4\delta^2+3a\delta^2-24a\delta^2=(4-21a)\delta^2=0 \quad a=4/21$

(f) $a\neq 0 \quad v=\delta(1,0,0)+s(0,1,0) \quad q(v)>0 \quad s,\delta\neq 0$

(g) $a\neq 0 \quad v=\delta(0,2a,1) \quad q(v)<0$

(h) $(x,y,z)=\delta(1,1,3)+s(0,2,1) = (\delta,\ \delta+2s,\ 3\delta+s)$

$\rightsquigarrow q(\delta,s) = (\delta+2s)^2+a\,\delta(3\delta+s)-4a\,(\delta+2s)(3\delta+s) =$

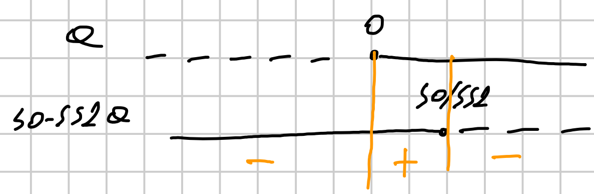
$= \delta^2+4s^2+4s\delta+3a\delta^2+as\delta-12a\delta^2-8as^2-28as\delta =$

$= (1-9a)\delta^2+(4-8a)s^2+(4-27a)s\delta$

$\begin{vmatrix}1-9a & 2-27a/2\\ 2-27a/2 & 4-8a\end{vmatrix} = (1-9a)(4-8a)-\left(2-\frac{27}{2}a\right)^2 =$

$= 72a^2-44a+\cancel{4}-\cancel{4}+54a-\frac{729}{4}a^2 = 10a-441a^2 =$

$= a(10-441a)$



$\begin{cases}1-9a>0\\4-8a>0\end{cases} \rightsquigarrow \begin{cases}a<1/9\\a<1/2\end{cases}$ $\quad\rightsquigarrow 0<a<10/441$

3. Determinare, al variare del parametro reale a, la segnatura delle seguenti forme quadratiche $q(x,y,z,w)$ in $\mathbb{R}^4$:

(a) $x^2 - y^2 + az^2 - w^2$

(b) $x^2 + ay^2 - w^2$

(c) $ax^2 - y^2 - (a+3)z^2 - w^2$

(d) $-x^2 - ay^2 + (a+4)z^2 - (2a+1)w^2$

(e) $x^2 - y^2 + 2z^2 - w^2 - 2xz + ayw$

(f) $az^2 + 2y^2 + 3z^2 - x^2 + 2axy + 2yw$

(g) $2x^2 + 3y^2 + 4w^2 + axz$

(h) $x^2 + ay^2 + w^2 + 2yz$

(i) $ax^2 + w^2 + 2ayz + 2xz$

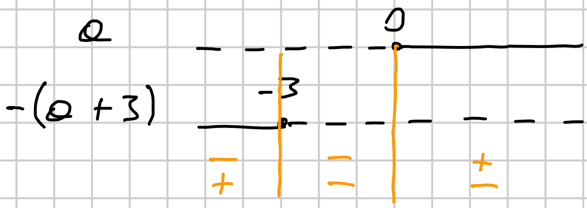
(l) $x^2 + ay^2 + 3z^2 + 4w^2 + 2xz + 4zw + 2ayz$

(a) $x^2 - y^2 + az^2 - w^2 \rightsquigarrow$ INDEFINITA $\forall a \in \mathbb{R}$

(b) $x^2 + ay^2 - w^2 \rightsquigarrow$ INDEFINITA $\forall a \in \mathbb{R}$

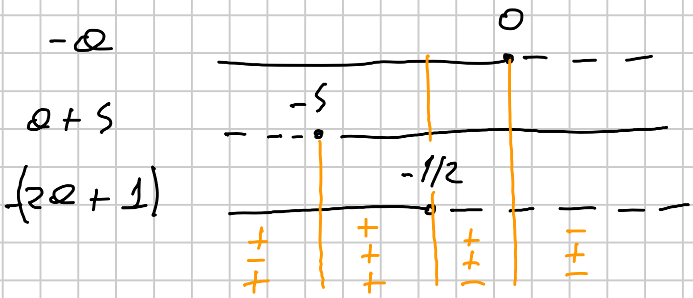
(c) $ax^2 - y^2 - (a+3)z^2 - w^2$

$$\begin{cases} -3 \leq a \leq 0 & \text{DEF. NEG.} \\ a \in (-\infty,-3) \cup (0,+\infty) & \text{INDEF} \end{cases}$$



(d) $-x^2 - ay^2 + (a+4)z^2 - (2a+1)w^2$

$\rightsquigarrow$ INDEFINITA $\forall a \in \mathbb{R}$



(e) $x^2 - y^2 + 2z^2 - w^2 - 2xz + ayw =$

$$\begin{cases} = (x-z)^2 - z^2 + 2z^2 - \left(y - \frac{a}{2}w\right)^2 + \frac{a^2}{4}w^2 - w^2 = \\ = (x-z)^2 + z^2 - \left(y - \frac{a}{2}w\right)^2 + \left(\frac{a^2}{4} - 1\right)w^2 \end{cases}$$

$\rightsquigarrow$ INDEFINITA $\forall a \in \mathbb{R}$

VER. $x^2 + z^2 - 2xz + z^2 - y^2 + ayw - \cancel{\frac{a^2}{4}}w^2 + \cancel{\frac{a^2}{4}}w^2 - w^2$

(f) $az^2 + 2y^2 + 3z^2 - x^2 + 2axy + 2yw =$

$$\begin{cases} = (a+3)z^2 - (x-ay)^2 + a^2y^2 + 2y^2 + 2yw = \\ = (a+3)z^2 - (x-ay)^2 + \left(\sqrt{2+a^2}\, y + w/\sqrt{2+a^2}\right)^2 - \frac{w^2}{2+a^2} \end{cases}$$

$\rightsquigarrow$ INDEFINITA $\forall a \in \mathbb{R}$

(g) $2x^2 + 3y^2 + 4w^2 + axz =$

$= 2\left(x + \frac{a}{4}z\right)^2 - \frac{a^2}{8}z^2 + 3y^2 + 4w^2 \rightsquigarrow \begin{cases} \text{DEF. POSITIVA} & a = 0 \\ \text{INDEFINITA} & a \neq 0 \end{cases}$

(h) $x^2+ay^2+w^2+2yz =$

$$\begin{cases} = x^2+w^2+a(y+z/a)^2-\dfrac{z^2}{a} & a\neq 0 \\ = x^2+w^2+2yz = x^2+w^2+\frac{1}{2}(y+z)^2-\frac{1}{2}(y-z)^2 & a=0 \end{cases}$$

$\rightsquigarrow$ INDEFINITA $\forall a\in\mathbb{R}$

(i) $ax^2+w^2+2ayz+2xz =$

$$= a\left(x+\frac{z}{a}\right)^2-\frac{z^2}{a}+2ayz+w^2 = a\left(x+\frac{z}{a}\right)^2-a\left(\frac{z}{a}-ay\right)^2+a^3y^2+w^2$$

VER. $ax^2+\cancel{\frac{z^2}{a}}+2zx-\cancel{\frac{z^2}{a}}-\cancel{a^3y^2}+2ayz+\cancel{a^3y^2}+w^2$ $\rightsquigarrow$ INDEFINITA $a\neq 0$

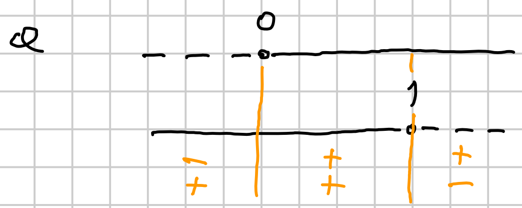
$a=0 \rightsquigarrow w^2+2xz = w^2+\frac{1}{2}(x+z)^2-\frac{1}{2}(x-z)^2 \rightsquigarrow$ INDEFINITA

(l) $x^2+ay^2+3z^2+5w^2+2xz+5zw+2ayz =$

$$= (x+z)^2+a(y+z)^2+5\left(w+\frac{z}{2}\right)^2-z^2-az^2-z^2+3z^2 =$$

$$= (x+z)^2+a(y+z)^2+5(w+z/2)^2+(1-a)z^2$$

VER. $x^2+z^2+2xz+ay^2+\cancel{az^2}+2ayz+5w^2+z^2+5wz+z^2-\cancel{az^2}$



$\rightsquigarrow \begin{cases} \text{DEF. POSITIVA} & 0\leq a\leq 1 \\ \text{INDEFINITA} & a\in(-\infty,0)\cup(1,+\infty) \end{cases}$

4. Consideriamo la seguente forma quadratica in $\mathbb{R}^3$:

$$q(x, y, z) = ax^2 + 2y^2 + 4z^2 + 2xy + byz.$$

Determinare per quali valori dei parametri reali a e b la forma risulta semidefinita positiva e nulla sul sottospazio generato da $(-2, 2, 1)$.

$$v = \delta(-2,2,1) \rightsquigarrow q(v) = 4a\delta^2 + \cancel{8\delta^2} + 4\delta^2 - \cancel{8\delta^2} + 2b\delta^2 = (4+4a+2b)\delta^2 = 0$$

$$\rightsquigarrow 2+2a+b=0 \qquad b = -2-2a$$

$$q(x,y,z) = ax^2 + 2y^2 + 4z^2 + 2xy - (2+2a)yz = a(x+y/a)^2 - y^2/a +$$

$$+[2z-(1+a)y/2]^2 - \frac{(1+a)}{4}y^2 + 2y^2 =$$

$$= \underset{>0}{a}(x+y/a)^2 + [2z-(1+a)y/2]^2 + \underset{=0}{[2-1/a-(1+a)/4]}\, y^2$$

$$\begin{cases} 2 - \frac{1}{a} - \frac{1+a}{4} = \frac{8a-4-a+a^2}{4a} = 0 \rightsquigarrow a^2+7a-4=0 \\ a = \frac{-7\pm\sqrt{49+16}}{2} = \frac{-7\pm\sqrt{65}}{2} \end{cases} \rightsquigarrow \begin{cases} a = \frac{-7+\sqrt{65}}{2} > 0 \\ b = -2+7-\sqrt{65} = 5-\sqrt{65} \end{cases}$$