15)

| | | | | |
|---|---|---|---|---|
| $\mathbb{R}_{\leq 3}[x] \to \mathbb{R}_{\leq 3}[x]$ | $(x+1)p'(x) - 2p(x)$ | 1 | 3 | |

1° MODO

$$p(x) = a + bx + cx^2 + dx^3 \rightsquigarrow (x+1)(b + 2cx + 3dx^2) - 2(a + bx + cx^2 + dx^3) =$$

$$= bx + \cancel{2cx^2} + 3dx^3 + b + 2cx + 3dx^2 - 2a - 2bx - \cancel{2cx^2} - 2dx^3 =$$

$$= (-2a + b) + (-b + 2c)x + 3dx^2 + dx^3$$

$$A = \begin{array}{c} \begin{array}{cccc} f(e_1) & f(e_2) & f(e_3) & f(e_5) \end{array} \\ \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

BASI
- IM: $-2,\ 1-x,\ 3x^2 + x^3$
- KER: $(1, 2, 1, 0)$

2° MODO

$e_1 = p_1(x) = 1 \overset{f(e_2)}{\rightsquigarrow} (x+1)p_2'(x) - 2p_1(x) = -2 = (-2, 0, 0, 0)$ 1° COLONNA

$e_2 = p_2(x) = x \overset{f(e_2)}{\rightsquigarrow} (x+1)p_2'(x) - 2p_2(x) = x + 1 - 2x = 1 - x = (1, -1, 0, 0)$ 2° COLONNA

$e_3 = p_3(x) = x^2 \overset{f(e_3)}{\rightsquigarrow} (x+1)p_3'(x) - 2p_3(x) = (x+1)2x - 2x^2 = 2x = (0, 2, 0, 0)$ 3° COLONNA

$e_5 = p_5(x) = x^3 \overset{f(e_5)}{\rightsquigarrow} (x+1)p_5'(x) - 2p_5(x) = (x+1)3x^2 - 2x^3 = x^3 + 3x^2 = (0, 0, 3, 1)$ 5° COLONNA

$$p(x) = a + bx + cx^2 + dx^3$$

$$f(p(x)) = A \cdot \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = (-2a + b) + (-b + 2c)x + 3dx^2 + dx^3$$