1)

X	V	W	$X \stackrel{?}{=} V \oplus W$	Proj_V	Proj_W
$\mathbb{R}^2$	$(2,3)$	$(1,1)$			

$V \in W$ LIN. IND. $\Rightarrow$ $V \cap W = \{0\}$ $V \oplus W = X$

$P_{PROJ,V}$: $A = \begin{pmatrix} 2 & 0 \\ 3 & 0 \end{pmatrix}$ $A' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$\mathcal{E} \to \mathcal{L}$

$P_{PROJ,W}$: $B = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ $B' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$\mathcal{E} \to \mathcal{L}$

CAMBIO BASE $M = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$ $M^{-1} = \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}$

$\mathcal{E} \to \mathcal{L}$

$P_{PROJ,V}$: $A \cdot M^{-1} = \begin{pmatrix} -2 & 2 \\ -3 & 3 \end{pmatrix}$ $P_{PROJ,W}$: $B \cdot M^{-1} = \begin{pmatrix} 3 & -2 \\ 3 & -2 \end{pmatrix}$

$\mathcal{L} \to \mathcal{L}$ $\mathcal{L} \to \mathcal{L}$

2)

X	V	W			
$\mathbb{R}^2$	$x + 3y = 0$	$3x + y = 0$			

$V = \text{SPAN}\{(3,-1)\}$ $W = \text{SPAN}\{(1,-3)\}$ $\Rightarrow$ $V \in W$ LIN. IND. $\Rightarrow$ $V \cap W = \{0\}$

$V \oplus W = \mathbb{R}^2$

$P_{PROJ,V}$: $A = \begin{pmatrix} 3 & 0 \\ -2 & 0 \end{pmatrix}$ $P_{PROJ,W}$: $B = \begin{pmatrix} 0 & 1 \\ 0 & -3 \end{pmatrix}$

$\mathcal{E} \to \mathcal{L}$ $\mathcal{E} \to \mathcal{L}$

CAMBIO BASE $M = \begin{pmatrix} 3 & 1 \\ -1 & -3 \end{pmatrix}$ $\Rightarrow$ $M^{-1} = +\frac{1}{8}\begin{pmatrix} +3 & +1 \\ -1 & -3 \end{pmatrix}$

$\mathcal{E} \to \mathcal{L}$ $\mathcal{L} \to \mathcal{E}$

$P_{PROJ,V}$: $AM^{-1} = \frac{1}{8}\begin{pmatrix} 8 & 8 \\ -8 & -8 \end{pmatrix}$ $P_{PROJ,W}$: $BM^{-1} = \frac{1}{8}\begin{pmatrix} -1 & -3 \\ 3 & 9 \end{pmatrix}$

$\mathcal{L} \to \mathcal{L}$ $\mathcal{L} \to \mathcal{L}$