

Esempio $\cos^6 x + \sin^6 x$

$1 = \cos^2 x + \sin^2 x \rightsquigarrow$ elevo al cubo

$$1 = (\cos^2 x + \sin^2 x)^3 = \cos^6 x + 3\cos^4 x \sin^2 x + 3\cos^2 x \sin^4 x + \sin^6 x$$

$$\rightsquigarrow \cos^6 x + \sin^6 x = 1 - 3\cos^2 x \sin^2 x (\cos^2 x + \sin^2 x)$$

$$= 1 - 3\cos^2 x \sin^2 x$$

$$= 1 - \frac{3}{4} \cdot \frac{4\cos^2 x \sin^2 x}{\sin^2(2x)}$$

$$= 1 - \frac{3}{4} \sin^2(2x)$$

$\downarrow$
volendo lo potei ridurre a $\cos(4x)$

$$\cos(2\alpha) = 1 - 2\sin^2 \alpha \rightsquigarrow \cos(4x) = 1 - 2\sin^2(2x)$$

$$\rightsquigarrow \sin^2(2x) = \frac{1 - \cos(4x)}{2}$$

Nota x , come calcolo $\sin$ e $\cos$ di $\frac{x}{2}$

Si parte da $\cos(2x) = 2\cos^2 x - 1$

$$\cos(2x) = 1 - 2\sin^2 x$$

Se metto $x = \frac{\alpha}{2}$, riesco a trovare $\cos^2 \frac{\alpha}{2}$ e $\sin^2 \frac{\alpha}{2}$ in funzione di $\cos \alpha$.

Occhio che nel ricavare $\cos(\frac{\alpha}{2})$ e $\sin(\frac{\alpha}{2})$ bisogna scegliere il segno a seconda del quadrante.

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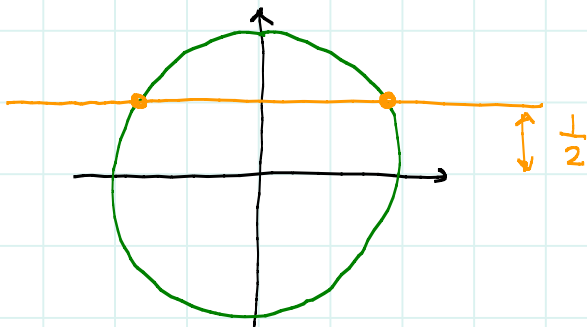
Equazioni trigonometriche BASIC

Es. 1

$$\sin x = \frac{1}{2}$$

Trovare le soluzioni in $[0, 2\pi]$
e le soluzioni in $[0, 3\pi]$

SLOGAN: guardare il cerchio



In $[0, 2\pi]$ le soluz. sono

$$\frac{\pi}{6} \quad e \quad \frac{5\pi}{6}$$

In $[0, 3\pi]$ sono $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$$2\pi + \frac{\pi}{6}$$

$$\uparrow$$

$$2\pi + \frac{5\pi}{6}$$

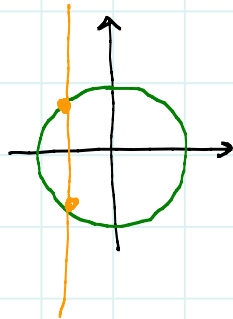
In $[-2\pi, 0]$ le soluzioni sono $-\frac{11}{6}\pi, -\frac{7}{6}\pi$

$$-2\pi + \frac{\pi}{6}, -2\pi + \frac{5\pi}{6}$$

ES. 2

$$\cos x = -\frac{\sqrt{2}}{2}$$

in $[0, 2\pi]$



Sono $\frac{3\pi}{4}$ e $\frac{5\pi}{4}$

Se le volessi in $[0, 3\pi]$: $\frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}$

$$= 2\pi + \frac{3\pi}{4}$$

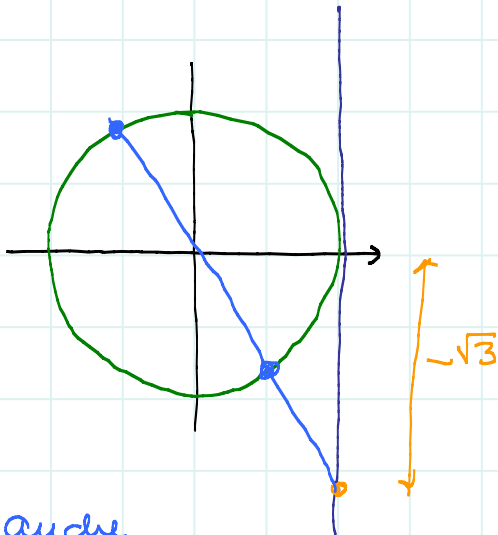
Es. 3

$$\tan x = -\sqrt{3}$$

in $[0, 3\pi]$

Le soluzioni sono

$$\frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}$$



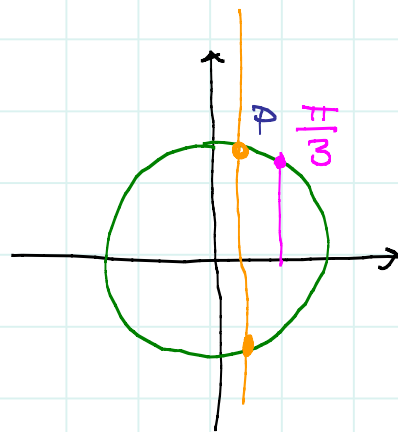
In generale : $\frac{2\pi}{3} + k\pi$ con k intero, anche negativo

Es 4 $\cos x = \frac{1}{7}$ soluzioni in $[0, 2\pi]$?
 soluzioni in $[\frac{\pi}{3}, \frac{2\pi}{3}]$?

Ci sono 2 sol. in $[0, 2\pi]$

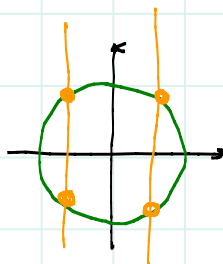
Domanda: il p.to P è prima o dopo $\frac{\pi}{3}$?

Quindi c'è una sol. in $[\frac{\pi}{3}, \frac{2\pi}{3}]$



Es 5 $2 \cos^2 x = 1$ in $[0, 2\pi]$

$\cos^2 x = \frac{1}{2}$ $\cos x = \begin{matrix} \nearrow \frac{1}{\sqrt{2}} \\ \searrow -\frac{1}{\sqrt{2}} \end{matrix}$



$\leadsto$ 4 soluzioni: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ $\leadsto$ in generale $\frac{\pi}{4} + k \frac{\pi}{2}$

Es 6 $\cos x = \cos(2x)$

1° modo Formula per $\cos(2x) \leadsto \cos x = 2 \cos^2 x - 1$

$\leadsto 2 \cos^2 x - \cos x - 1 = 0 \leadsto (2 \cos x + 1)(\cos x - 1) = 0$

$\leadsto \cos x = 1 \leadsto x = 0 (+ 2k\pi)$

$\leadsto \cos x = -\frac{1}{2} \leadsto x = \frac{2\pi}{3} \text{ e } x = \frac{4\pi}{3} \text{ (volendo sempre } + 2k\pi)$

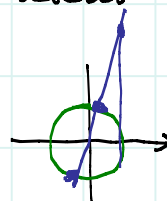
2° modo $\cos(\text{CANE}) = \cos(\text{GATTO})$

$\leadsto \cos(\text{CANE}) - \cos(\text{GATTO}) = 0$

$\leadsto$ sum to product e diventa + semplice

Es 7 $\sin x = 27 \cos x$ in $[0, 2\pi]$ ha 2 soluzioni

Divido $\frac{\sin x}{\cos x} = 27 \leadsto \tan x = 27$

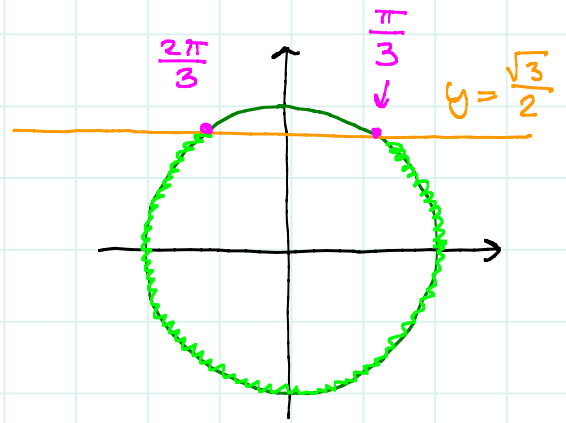


(posso dividere perché non ci sono soluz con $\cos x = 0$)

Disuguaglianze trigonometriche

Es 1 $\sin x \leq \frac{\sqrt{3}}{2}$ in $[0, 2\pi]$

Soluzioni: $[0, \frac{\pi}{3}] \cup [\frac{2\pi}{3}, 2\pi]$



In generale, senza la cond. $x \in [0, 2\pi]$, si può scrivere come

$$\bigcup_{k \in \mathbb{Z}} \left[-\frac{4\pi}{3} + 2k\pi, \frac{\pi}{3} + 2k\pi \right]$$

$\frac{2\pi}{3} \leq x \leq \frac{4\pi}{3}$ NO!!

oppure $\bigcup_{k \in \mathbb{Z}} \left[\frac{2\pi}{3} + 2k\pi, \frac{7\pi}{3} + 2k\pi \right]$

Es 2 $2 \sin^2 x \leq \sin x$

$$2 \sin^2 x - \sin x \leq 0 \quad \sin x (2 \sin x - 1) \leq 0$$

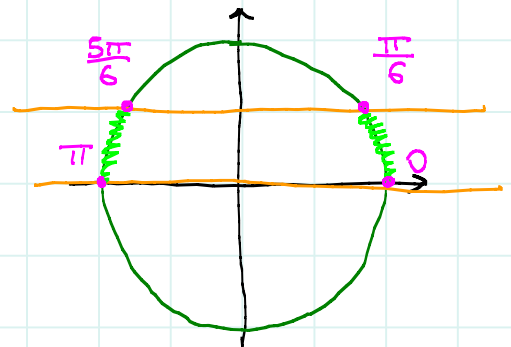
ora studio separatamente i due fattori

2° modo Pongo $y = \sin x \rightsquigarrow 2y^2 \leq y \rightsquigarrow 2y^2 - y \leq 0$
 $\rightsquigarrow y \in [0, \frac{1}{2}] \rightsquigarrow 0 \leq y \leq \frac{1}{2}$

Torno in $\sin x \rightsquigarrow 0 \leq \sin x \leq \frac{1}{2}$ e ora guardo il cerchio

Soluzioni in $[0, 2\pi]$:

$$\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right] \cup \{2\pi\}$$



Es 3 $\sin x + \cos x < 1$

Divido per $\sqrt{2}$: $\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x < \frac{1}{\sqrt{2}}$

$$\sin \frac{\pi}{4} \sin x + \cos \frac{\pi}{4} \cos x < \frac{1}{\sqrt{2}}$$

$$\cos\left(x - \frac{\pi}{4}\right) < \frac{1}{\sqrt{2}}$$

Soluzione:

$$\frac{\pi}{4} < x - \frac{\pi}{4} < \frac{7\pi}{4}$$

$$\Rightarrow \frac{\pi}{2} < x < 2\pi$$

(anche questa si ripete con periodo 2π).

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