

DISEQUAZIONI CON ESPONENZIALI E LOGARITMI

$$\begin{array}{ccc} \text{CANE} & & \text{GATTO} \\ a & > & a \end{array} \rightsquigarrow \text{CANE} > \text{GATTO}$$

↑
ok se $a > 1$

(se $a \in (0, 1)$ $\rightsquigarrow$ CANE $<$ GATTO)

$$\log_a \text{CANE} > \log_a \text{GATTO} \rightsquigarrow \text{CANE} > \text{GATTO} > 0$$

↑
ok se $a > 1$

(se $a \in (0, 1)$ si inverte il verso)

Esercizi ① $2^{3x+1} \geq 4^{x-1}$

$$2^{3x+1} \geq 2^{2x-2} \rightsquigarrow 3x+1 \geq 2x-2 \rightsquigarrow x \geq -3 \quad \ddot{\smile}$$

② $25^{-x} \geq 5^{2x-3} \rightsquigarrow 5^{-2x} \geq 5^{2x-3} \rightsquigarrow -2x \geq 2x-3$
 $\rightsquigarrow 4x \leq 3 \rightsquigarrow x \leq \frac{3}{4}$

Alternativa: $\frac{1}{25^x} \geq 5^{2x-3} \rightsquigarrow 1 \geq 25^x \cdot 5^{2x-3}$
 ↑
 $25^x > 0$ quindi posso moltiplicare
 $\rightsquigarrow 1 \geq 5^{2x} \cdot 5^{2x-3} \rightsquigarrow 1 \geq 5^{4x-3} \rightsquigarrow 5^0 \geq 5^{4x-3} \rightsquigarrow \dots$

③ $\underbrace{2^{x+4}}_{\text{sempre } > 0} \geq - \underbrace{\frac{1}{2^{x+5}}}_{\text{sempre } < 0}$ Vera per ogni $x \in \mathbb{R}$

Fatto generale: a Mostro > 0 per ogni $a > 0$ e per ogni Mostro $\in \mathbb{R}$

④ $6^x \geq 2^x \cdot 3^{2x-1}$ $2^x \cdot 3^x \geq 2^x \cdot 3^{2x-1}$
 posso perché $2^x > 0$ sempre

$3^x \geq 3^{2x-1} \leadsto x \geq 2x-1 \leadsto x \leq 1$

⑤ $4 \cdot 9^x + 5 \cdot 3^{x+1} \leq 4$ $4 \cdot 3^{2x} + 15 \cdot 3^x - 4 \leq 0$

Pongo $y = 3^x \leadsto 4y^2 + 15y - 4 \leq 0$

$y = \frac{-15 \pm \sqrt{225 + 64}}{8} = \frac{-15 \pm 17}{8}$ $\nearrow \frac{1}{4}$
 $\searrow -4$

$\leadsto$ valori intermedi $-4 \leq y \leq \frac{1}{4}$, cioè $-4 \leq 3^x \leq \frac{1}{4}$
 GRATIS

resta $3^x \leq \frac{1}{4} \leadsto$ facciamo $\log_3$ a dx e sx

$\log_3(3^x) \leq \log_3\left(\frac{1}{4}\right) \leadsto x \leq \log_3 \frac{1}{4} = -2 \log_3 2$

In realtà è come fare $3^x \leq \frac{1}{4} = 3^{\log_3 \frac{1}{4}} \leadsto x \leq \log_3 \frac{1}{4}$

⑥ $4 \cdot 2^x - 8^x \geq 0$

1° modo $2^2 \cdot 2^x - (2^3)^x \geq 0 \leadsto 2^{2+x} - 2^{3x} \geq 0 \leadsto 2^{x+2} \geq 2^{3x}$
 $\leadsto x+2 \geq 3x \leadsto 2x \leq 2 \leadsto x \leq 1$

2° modo Pongo $y = 2^x \leadsto 4y - y^3 \geq 0 \leadsto y(4 - y^2) \geq 0$

-2	0	2	
-	-	+	y
-	+	+	$4 - y^2$
+	-	-	Prodotto

$\leadsto y \leq -2$ oppure $0 \leq y \leq 2$

$2^x \leq -2$ oppure $0 \leq 2^x \leq 2 \leadsto 2^x \leq 2^1 \leadsto x \leq 1$
 $\uparrow$ MAI $\uparrow$ sempre

Con i logaritmi

$$\boxed{1} \quad \log_2(x+3) \leq 4 \rightsquigarrow \log_2(x+3) \leq \log_2 16$$

$$\rightsquigarrow 0 < x+3 \leq 16$$

↑↑

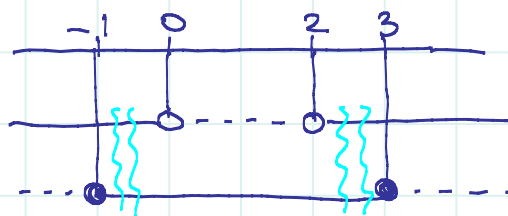
$$\begin{cases} x+3 > 0 \\ x+3 \leq 16 \end{cases} \quad \begin{cases} x > -3 \\ x \leq 13 \end{cases}$$

Soluzioni: $[-3, 13]$

$$\boxed{2} \quad \log_3(x^2-2x) \leq 1 \rightsquigarrow \log_3(x^2-2x) \leq \log_3 3$$

$$\rightsquigarrow 0 < x^2-2x \leq 3 \rightsquigarrow \begin{cases} x^2-2x > 0 \\ x^2-2x \leq 3 \end{cases} \rightsquigarrow \begin{cases} x^2-2x > 0 \\ x^2-2x-3 \leq 0 \end{cases}$$

$$\rightsquigarrow \begin{cases} x < 0 \text{ oppure } x > 2 \\ -1 \leq x \leq 3 \end{cases}$$



Soluzioni: $[-1, 0) \cup (2, 3]$

$$\boxed{3} \quad \log_7(3x-2) \geq \log_7(2x-3)$$

$$\rightsquigarrow 3x-2 \geq 2x-3 > 0$$

$$\begin{cases} 3x-2 \geq 2x-3 \\ 2x-3 > 0 \end{cases}$$

$$\begin{cases} x \geq -1 \\ x > \frac{3}{2} \end{cases}$$

Soluzioni: $(\frac{3}{2}, +\infty)$

$$\boxed{4} \quad \log_2(x+1) + \log_2 x < 1$$

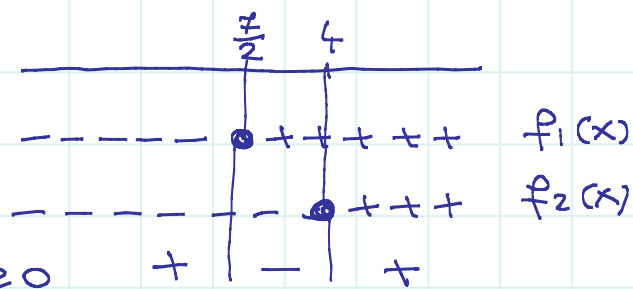
$$\log_2[x^2+x] < \log_2 2$$

$$\begin{cases} x^2+x < 2 \\ x+1 > 0 \\ x > 0 \end{cases} \quad \begin{cases} x^2+x-2 < 0 \\ \boxed{x > -1} \text{ INUTILE} \\ x > 0 \end{cases}$$

$$\begin{cases} x \in (-2, 1) \\ x > 0 \end{cases} \rightsquigarrow x \in (0, 1)$$

$$0 < x < 1$$

$$\boxed{5} \quad \underbrace{(2x-7)}_{f_1(x)} \underbrace{\log_3(x-3)}_{f_2(x)} \geq 0$$



Studio $f_2(x)$. Vediamo dove è ≥ 0

$$\log_3(x-3) \geq 0 \leadsto \log_3(x-3) \geq \log_3 1 \leadsto x-3 \geq 1 > 0$$

$$\leadsto x \geq 4 \quad \uparrow \text{GRATIS}$$

$$\text{Soluzione: } (-\infty, \frac{7}{2}] \cup [4, +\infty)$$

NO! NO! E POI NO!!!

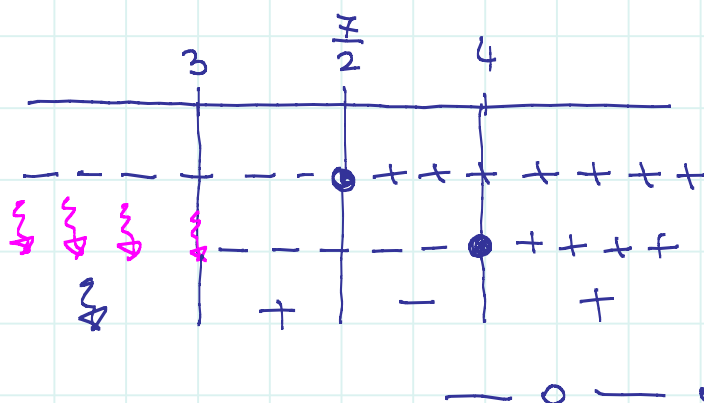
Devo studiare anche $f_2(x) \leq 0 : \log_3(x-3) \leq 0$

$$\leadsto \log_3(x-3) \leq \log_3 1 \leadsto 0 < x-3 \leq 1$$

$$\quad \quad \quad \uparrow \text{NO GRATIS}$$

$$\leadsto \begin{cases} x-3 > 0 \\ x-3 \leq 1 \end{cases} \quad \begin{cases} x > 3 \\ x \leq 4 \end{cases} \leadsto (3, 4]$$

Quindi il vero grafico è



Soluzione vera:

$$(3, \frac{7}{2}] \cup [4, +\infty)$$