

Esercizi sui raccoglimenti

$$(13) \quad 2^8 + 2^{20} = 2^8 (1 + 2^a) \quad a = 12$$

$$2^8 + 2^{20} = 2^8 \cdot 1 + 2^8 \cdot 2^{12}$$

$$(2^5 + 2^9)^2 = 2^{10} + 2^{14} + 2^{13}$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(14) \quad \sqrt[3]{13\sqrt{13}} : \sqrt[4]{13\sqrt[5]{13}} = 13^a$$

$$\sqrt[3]{13^3} : \sqrt[4]{13^6} = 13^{\frac{3}{6}} : 13^{\frac{6}{20}} = 13^{\frac{1}{2} - \frac{3}{10}} = 13^{\frac{1}{5}}$$

$$(5+1)^{20} = 5^{20} (1+5^a)^{20} \quad a = -1$$

$$(5+1)^{20} = \left(5\left(1+\frac{1}{5}\right)\right)^{20} = 5^{20} \left(1+\frac{1}{5}\right)^{20}$$

$$(ab)^{20} = a^{20} \cdot b^{20}$$

In alternativa: fare dall'inizio la radice 20-esima:

$$5+1 = 5(1+5^a) = 5+5^{a+1} \rightsquigarrow a+1=0 \rightsquigarrow a=-1$$

$$(15) \quad \sqrt[6]{13} + \sqrt[7]{13} = \sqrt[7]{13} (\sqrt[9]{13} + 1) \rightsquigarrow a = 42$$

Espando a dx : $\sqrt[6]{13} + \cancel{\sqrt[7]{13}} = \cancel{\sqrt[7]{13}} + \sqrt[7]{13} \sqrt[9]{13}$

$$\rightsquigarrow 13^{\frac{1}{6}} = 13^{\frac{1}{7} + \frac{1}{a}} \rightsquigarrow \frac{1}{6} = \frac{1}{7} + \frac{1}{a} \rightsquigarrow \frac{1}{a} = \frac{1}{6} - \frac{1}{7} = \frac{1}{42}$$

$$\sqrt{7^4 + 7^5} = 49 \sqrt{1+7^a}$$

$$a=1$$

$$\sqrt{7^4 + 7^5} = \sqrt{7^4(1+7)} = \sqrt{7^4} \cdot \sqrt{1+7} = 49 \sqrt{1+7}$$

$$(16) \quad \sqrt[3]{\sqrt{2} + \sqrt[3]{2}} = \sqrt[6]{2} \sqrt[3]{1+2^a} \quad \leadsto a = -\frac{1}{6}$$

Raccolgo $\sqrt{2}$ a sinistra: $\sqrt[3]{\sqrt{2} + \sqrt[3]{2}} = \sqrt[3]{\sqrt{2} \left(1 + \frac{\sqrt[3]{2}}{\sqrt{2}}\right)}$

$$= \sqrt[6]{2} \cdot \sqrt[3]{1 + \frac{\sqrt[3]{2}}{\sqrt{2}}} = 2^{\frac{1}{6}} \cdot 2^{\frac{1}{3} - \frac{1}{2}} = 2^{-\frac{1}{6}}$$

$$\sqrt[3]{\sqrt{2} + \sqrt{3}} = \sqrt[6]{2} \sqrt[3]{1 + \sqrt{a}} \quad a = \frac{3}{2}$$

$$\sqrt[3]{\sqrt{2} + \sqrt{3}} = \sqrt[3]{\sqrt{2} \left(1 + \frac{\sqrt{3}}{\sqrt{2}}\right)} = \sqrt[6]{2} \sqrt[3]{1 + \frac{\sqrt{3}}{\sqrt{2}}} = \sqrt{\frac{3}{2}}$$

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EQUAZIONI CON ESPONENZIALI

Strategia: arrivare a scriverle come

$$\begin{array}{ccc} \text{CANE} & & \text{GATTO} \\ a & = & a \\ \uparrow & & \uparrow \\ \text{stessa base} & & \end{array}$$

A quel punto CANE = GATTO

EQUAZIONI CON LOGARITMI

Strategia: arrivare a scriverle come $\log_a \text{CANE} = \log_a \text{GATTO}$

$\nwarrow$ stessa base $\nearrow$

A quel p.to CANE = GATTO ma devo imporre CANE > 0
e GATTO > 0 (basta uno dei due essendo uguali)

Otteniamo quindi un sistema misto

$$\begin{cases} \text{GATTO} > 0 \\ \text{CANE} = \text{GATTO} \end{cases}$$

Esercizi ① $4 \cdot 2^{x^2} = 8^x$

Portiamo tutto in base 2: $2^2 \cdot 2^{x^2} = (2^3)^x$ cioè

$$2^{2+x^2} = 2^{3x} \leadsto 2+x^2 = 3x \leadsto x^2-3x+2=0$$
$$\leadsto x=1 \text{ e } x=2$$

② $3 \cdot 2^{x^2} = 1 \leadsto 2^{x^2} = \frac{1}{3}$

Per def. di logaritmo $x^2 = \log_2 \frac{1}{3} = \log_2 3^{-1} = -\log_2 3 < 0$
 $\leadsto$ Nessuna soluzione

Fatto generale: se $a > 1$, allora $\log_a 1 = 0$
 $\log_a x > 0$ se $x > 1$
 $\log_a x < 0$ se $x \in (0, 1)$

④ $4^x - 2^{x+2} + 3 = 0$

$$4^x = (2^2)^x = 2^{2x} = (2^x)^2$$

Cerco di isolare 2^x : $(2^x)^2 - 4 \cdot 2^x + 3 = 0$

Pongo $y = 2^x \leadsto y^2 - 4y + 3 = 0 \leadsto (y-3)(y-1) = 0$

$$\leadsto y=1 \text{ e } y=3 \leadsto 2^x=1 \text{ e } 2^x=3$$
$$\leadsto x=0 \text{ e } x=\log_2 3$$

⑤ $2^x = 3^x$

1° modo Faccio a dx e sx il $\log_{||}$:

$$\log_{||} (2^x) = \log_{||} (3^x) \leadsto x \cdot \log_{||} 2 = x \cdot \log_{||} 3$$

$$x \underbrace{(\log_{||} 2 - \log_{||} 3)}_{\neq 0} = 0 \leadsto x = 0$$

2° modo Divido: $\frac{2^x}{3^x} = 1 \rightsquigarrow \left(\frac{2}{3}\right)^x = 1$

$$\rightsquigarrow x = \log_{\frac{2}{3}} 1 = 0$$

⑥ $\log_2 (x-1) + \log_2 (x+1) = 3$

$$\log_2 [(x+1)(x-1)] = 3 \rightsquigarrow \log_2 (x^2-1) = \log_2 8$$

$$\rightsquigarrow x^2-1=8 \rightsquigarrow x^2=9 \rightsquigarrow x = \begin{matrix} \nearrow 3 \\ \searrow -3 \end{matrix} \quad \begin{matrix} \rightsquigarrow \text{OK} \\ \rightsquigarrow \text{NO perché} \\ \text{log (roba neg.)} \end{matrix}$$

⑦ $3 \log_3 x + 2 \log_3 (x^2) = 21$

$$3 \log_3 x + 4 \log_3 x = 21 \rightsquigarrow \log_3 x = 3 \rightsquigarrow x = 27$$

⑧ $[\log_2 (x+2)]^2 + 3 \log_2 (x+2) = 4$

$$y = \log_2 (x+2) \rightsquigarrow y^2 + 3y - 4 = 0 \rightsquigarrow (y+4)(y-1) = 0$$

$$\rightsquigarrow y = -4 \quad \text{e} \quad y = 1 \rightsquigarrow \log_2 (x+2) = -4, \text{ cioè } x+2 = \frac{1}{16}$$
$$\rightsquigarrow \log_2 (x+2) = 1, \text{ cioè } x+2 = 2$$

⑨ $\log_2 \sqrt{x} \cdot \log_2 \sqrt[4]{x} = 2^{-1}$

$$\frac{1}{2} \log_2 x \cdot \frac{1}{4} \log_2 x = \frac{1}{2} \rightsquigarrow [\log_2 x]^2 = 4 \rightsquigarrow \log_2 x = \pm 2$$

$$\nearrow \log_2 x = 2 \rightsquigarrow x = 4$$

$$\rightsquigarrow \log_2 x = -2 \rightsquigarrow x = \frac{1}{4}$$

$$\textcircled{10} \log_2 x = \log_4 (6x)$$

Andiamo in base 2 :

$$\log_2 x = \frac{\log_2 (6x)}{\log_2 (4)} = \frac{\log_2 (6x)}{2} = \frac{1}{2} \log_2 (6x) = \log_2 \sqrt{6x}$$

$$\leadsto x = \sqrt{6x} \leadsto x^2 = 6x \leadsto x = \begin{matrix} \nearrow 0 \\ \searrow 6 \end{matrix}$$

$\leadsto$ NO

$\leadsto$ OK

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